Lingua Mathematica as an Antidote for Digital Exclusion

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Abstract

Authors of this paper discuss the role of mathematics in contemporary education and working life, as well as present situation. Authors suggest that mathematics should be seen as an elementary building block in contemporary business communication as well as a basic citizen skill and present motivation and potential benefits for this approach. Abstract mathematical thinking and logical reasoning are essential skills in human-computer-interaction as well as interaction between IS designers as well as between end users. Main goal is not just in being able to pass elementary math courses, or in understanding simple accounting. These are basic abilities required to operate freely in a contemporary society and thus are important. Recent PISA results from Europe indicate that these skills are deteriorating already at a very early stage, and this is extremely alarming. Authors are calling for action, and refer to the big picture, where mathematics is a way of thinking and communicating abstract ideas and principles as well as a way of making sense of our surroundings and mental landscape.

Keywords

Inclusion, Digital Divide, Accessibility, Scientific literacy, Abstract thinking, Mathematics, Digitalization

1. Introduction

Active participation in contemporary society requires technological understanding of different systems. Access as well as participation as fully recognized member of contemporary society requires understanding of a technology that is used for providing and delivering all services. It is not that uncommon to see situations where machines, or IT systems provide the very same services autonomously, that were previously provided by clerks or service personnel. While selfservice systems and usability have been in the focus for a while along with cognitive processes required to operate different systems [1], the language of understanding and communicating has not been that clearly discussed.

We are interacting with other people all the time. While we are communicating and exchanging ideas with other people, we are doing that not only face-to-face but also using different mediating technologies. By dictionary definition, communication is [2]: "*a process by which information is exchanged between individuals through a common system of symbols, signs, or behavior*". In this very process of exchanging information, we are using different means of communication. Spoken language, written texts, signs, pictures, different gestures or sounds are all means for human communication. However, in our everyday life we are no longer interacting only with natural human beings, but in addition we are in constant interaction with different systems, communication applications, vehicles, etc. This interaction takes many forms and the interfaces for interaction are manyfold. Although we do not know about the future that is awaiting for us, the

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value system of socio-technical design can be of great help for us [3]. Lyytinen & Newman [3, p. 609] suggest in their study, where PSIC mode of IS change was developed, that IS change is a "*subtle interplay between technologies, actors, organizational relationships, and tasks at multiple levels*" and it can be either incremental or punctuated. The socio-technical approach considers both technological as well as human centered phenomena and it suits particularly well for critically discussing the present course advanced societies have taken [4].

This paper focuses on analyzing the problematic domain of e-inclusion and more precisely the concept of access that is a building block in conceptual digital divide definition. While access related issues are nicely covered in the literature, the focus has been on the level of executing actions or possessing tangible resources instead of understanding and communicating [5], [6], [7]. It seems that language has been either self-evidently left out or it is considered as merely an availability issue (i.e. localization or similar). This paper suggests that the focus should be directed back to the basics, i.e. the focus should be directed back to the language that is needed and used, and on what it represents. Underlying logic here is that to understand intelligent artifacts, the ability of abstract mathematical thinking is necessary. As such, mathematics should be a Lingua Franca, that is needed for successful digital interaction and is should be seen as one.

As such, this paper is not limited to accessibility, nor simple usability related questions alone, but digs in deeper. Focus is on arguing for *the need of common language, the actual universal way of communicating in a human-computer interaction as well as between humans, when using, designing or analyzing systems or phenomenon at hand.* For this we already have many different modelling languages and tools (UML, OPM, SysML, etc.), as well as programming languages, etc. However, to be able to use these effectively, the ability to create abstractions as well as mathematical thinking is necessary, and this is not delimited only on systems designers but also to the actual users.

The interaction is not just about communicating and/or using advanced IT systems, but instead of understanding the way systems operate and take input as well as present their output. How is it possible to create a solid theory-in-use, when the means for understanding, interpreting as well as communicating intentions is lacking common and general language? So, this is a position paper, where the importance of mathematical thinking is highlighted in contemporary knowledge society. It is worth noticing that this is essential for multiple reasons, and not the least because AI-enhanced technology is becoming more common at an increased pace, while the performance of younger generations in STEM is going in an alarming direction.

Most recent OECD Programme for International Student Assessment (PISA) is from 2022 [8]. This report brings to daylight three alarming trends [9]: 1) “One in four students underachieve in mathematics, reading or science”; 2) “Half of students from disadvantaged socioeconomic backgrounds underachieve in mathematics”, and 3) “Less than one in ten students are top performers in any of the PISA subject areas”. Can we really afford this?

Similar signals can be seen in other studies as well. In a study from 2022 about the life-course outcomes of young people and reading and mathematics skills in NZ, the results suggest that students with low measured skills (this refers to low performance in mathematics and/or reading) have less favorable outcomes in many areas, such as further education, employment, average earnings, higher rate of hospitalization and non-admitted secondary care events, as well as higher rates of criminal offending and convictions [10].

If an end user cannot understand the logic of a system, how is it possible to critically evaluate the performance? It becomes even harder when evaluating the results, if everything appears as a “black box” to users because of limited understanding of underlying system logic. An excellent example is the recent emergence of Large Language Models (LLM), where the focus seems to be mostly focused on miraculous results on producing answers and generating text, pictures, videos or even music at an astonishing speed. However, it seems that the understanding of used technology, the main logic behind magic like appearance, is either missing or simply considered obsolete. If we take two concepts, rigor and relevance, and use those for analyzing the situation we soon notice that this reasoning is not sustainable.

Authors argue that mathematics should be seen as an elementary building block in contemporary communication skill set as well as a basic citizen skill and present motivation and potential benefits for this approach. The reason is not just being able to pass elementary math courses, or in understanding simple accounting. Authors are referring to the big picture, where mathematics is a way of thinking and communicating abstract ideas and principles. At present, in digitalized environments mathematical thinking is a part of the design and creation of artefacts, such as systems engineering or information systems development (ISD) practices. But what about the users of these systems and tools? Is it a baseline requirement that users must be trained engineers before being able to fully utilize technology?

2. Accessibility and Digital Inclusion

2.1. The Fear of Missing Out

Contemporary societies are utilizing advanced technologies on every level of society. Participation and access to contemporary society requires the use of digitized services. How can services that are needed for participation be used, if these very services are impossible to comprehend? These digital services are key elements in accessing the resources of society and play the role of a gatekeeper in a sense of dividing citizens into those who have access and to those who do not have access. Another, maybe even more important, question is to figure out who the ones are who can have access and who cannot have access, and here the language used for communication plays a crucial role.

It is noteworthy that human beings are biological entities, whereas IT systems are artificial constructs—man-made artifacts. However, humans do not behave like natural objects, in contrast to IT systems, which tend to operate in a predictable and deterministic manner which as such provides a solid foundation for the possibility of understanding their behavior. A practical example can be found in design trends, which often assume that users will behave in a certain way—an assumption that is understandable, as without it, the target (i.e., the human) could not be "captured" or modeled in any meaningful way. Some of these assumptions are based on the notion of the user as a regularly behaving object.

Conversely, practices aimed at improving accessibility often consider human factors within user groups or construct personas—user groups with shared characteristics—based on data related to human abilities, in sensory perception, cognition, and functional operations. However, it is important to recognize that when we create user groups, we simultaneously categorize people into different segments. Consequently, there is a risk that our attention will primarily focus on the larger groups, as they appear to be dominant. On the other hand, we may consciously choose to focus on smaller groups in an effort to act ethically and to consider them equally alongside the larger groups. However, such action implies a voluntary commitment to "act ethically well," which in turn reveals that we continue to perceive these smaller groups as minorities.

All would be well if IT artifacts were designed in such a way that they presented no accessibility or usability issues. This can be interpreted as a distress signal in response to the challenges of interacting with poorly designed IT. However, accessibility issues remain and a certain level of proficiency in mathematical modes of thinking is required to comprehend IT — their operational logic — and thereby avoid being marginalized or excluded from their use.

2.2. Dividing Access

According to Eurostat [11], "*Digital divide refers to the distinction between those who have access to the internet or other digital technologies and are able to make use of online services, and those* *who are excluded from these services.*" In their definition [11]: "*The digital divide can be classified according to criteria that describe the difference in participation according to gender, age, education, income, social groups or geographic location.*"

Van Dijk [12, pp. 20–21] conceptualizes access to digital technologies into four different types of stages, where: First stage "Motivational access" refers to personal motivation towards the use of digital technology; Second stage "Material or physical access" refers to possession or permission to use digital appliances; Third Stage "Skills access" refer to persons digital literacy, and; Fourth stage "Usage access" refers to the amount and types, as well as use of applications. Van Dijk also argues that accessibility problems located in first two stages in the model will little by little sidle towards remaining two.

Van Dijk [12, p. 22] suggests that digital skills should not be seen only as general IT skills but also as skills to find, process and apply information as well as ability to use this information for one's own benefit in society. Helsper [13, p. 29] conceptualized digital inclusion to cover four categories; (1) use, (2) attitudes, (3) access and (4) skills, as well as related lower level indicators.

In both models the role of abstract thinking and language does not seem to be emphasized. When Van Dijk refers to skills access and Helsper refers to skills, these do not seem to fully cover mathematical abstract thinking ability, nor used shared language. These things are important, so essential that those might be seen as so elementary that there is no need to express those at all. We argue that when using this type of approach, caution should be exercised.

2.3. Digital End User Skills

When we are discussing digital interaction, we soon notice that programming skills alone is not a solution. Neither is artificially superimposed familiarization on the operating logic of each individual application or system. These can be seen as parts of the solution, but first we need to understand the real problem at hand. When we are using different systems, what are we really doing and how are we doing it? It can be argued that when we have a very clear goal, the machines are expected to help us in reaching that goal, right? If this does not happen, what are the root causes? Do we understand what needs to be done and how – or is this simply unnecessary knowledge?

In many European policy papers regarding education, the importance of digitalization has been emphasized. The Council of the European Union notes, that [14]: "*Every European citizen should have access to digital education which enables them to develop the knowledge, skills and competences needed for active participation in today's increasingly digital societies.*" In the very same document, it is also stated, that: "*... in a fast-developing world, it is vital to continuously improve the effectiveness and efficiency of education and training and support new teaching and learning approaches, including through existing and emerging digital solutions. Moreover, it is vital that learners understand the functioning of underlying technologies, and develop skills and competences for a creative, safe, ethical and responsible use of digital technologies*"[14].

This council recommendation is a plethora of initiatives and good ambitions, however even though it is explicitly indicated that EU citizens should have competence and understanding to be able to participate in contemporary digital society, the paper is mostly concerned about technologies and utilization of these. Our main concern, that is deteriorating mathematical skills is covered in the recommendations by focusing on learning environments, not the actual problem itself: "*...building science, technology, engineering, arts and mathematics (STEAM) learning environments that can foster digital pedagogy through an interdisciplinary approach...*"[14]. We argue that science and engineering, and even arts, do need a firm foundation in the form of language, that is mathematics.

In recent policy papers regarding the education of Artificial Intelligence in schools the approach seems to be extremely utilitarian, where focus is on the use of applications without focusing on the underlying technology. How can students understand or even assess the results, if it is unclear what logic the systems use and what sort of training data has been used? Same applies to participation in the digitalized society. For example in a AI competency framework for students by UNESCO four different competency aspects were presented: 1. Human-centered mindset, 2. Ethics of AI, 3. AI techniques and applications and 4. AI system Design [15]. While the framework seems to be nicely building up from human-centered mindset all the way to AI system design, this is futile if mathematical thinking is not included in this framework. This should not be a separate thing or element, but instead it should be merged in all steps. So, when learner's skills get better the amount and requirements in mathematical thinking should also increase respectively. Now this aspect seems to be missing from the framework and there is huge a leap in AI competencies directly to a challenging level, where the ability to comprehend abstract mathematics and to think logically is essential.

2.4. Language of technical thoughts

The language used behind design and implementation of different systems is often based on mathematics. The same applies to business-critical workplace communication. Because of this, it is essential to understand the basic mathematical thinking patterns and principles, to successfully make it in working life. So, how can users without adequate mathematical skills create a working theory-in-use of systems or applications they try to use, or understand the principles of business operations, if they do not understand the substance of communication at hand?

Authors argue that mathematics should be seen as a language for constructing a common ground, as well as interface for human and device interaction. In addition, mathematics should be seen as a crucial element in daily management communication, e.g. an essential building block of successful work life communication. From a communicational viewpoint mathematical skills can be seen as essential citizen skills (i.e. communicational meta skill) for contemporary knowledge society. Alarming observation is the fact that students' mathematical skills have deteriorated in Finland, and in other countries as well, while at the same time the need for math skills is even more necessary than before.

2.5. Abstraction and Mathematical Thinking

According to Platonism "*mathematical statements, and theorems of mathematical theories in particular, are about abstract objects forming a domain that those theorems describe*" [16, p. 1]. However, philosophers have not managed to agree where to draw the line between abstract and concrete objects [16, p. 2].

It is generally believed, that the emerge of abstract mathematical thinking is a direct consequence of the evolving language [17, p. 5]. In mathematics education, the concept of Abstract Thinking refers to the ability to think "*beyond physical representations or instances of a mathematical concept, in a process of extracting the underlying essence of an idea*" [18, p. 1]. Here the thinking evolves into abstract level, when it is separated from the originating context – a contrary to Concrete Thinking where physical representations play an important role [18]. At present in digitalized environments mathematical thinking is a part of the design and creation of artefacts. A good example is systems engineering or ISD practices. But what about the users of these systems and artefacts?

Certain challenges associated with learning mathematics have been identified, many of which relate to the learner's individual characteristics. For instance, a learner may experience dyscalculia (difficulty in perceiving numerical quantities and relationships between numbers), visuospatial challenges (difficulty in spatial reasoning and in perceiving shapes and spatial relationships), weaknesses in working memory (difficulty in simultaneously processing multiple pieces of information), linguistic difficulties (regardless of the language in question), or challenges in cognitive information processing (such as cognitive overload during information processing).

To overcome these challenges, various support methods are employed in teaching, aiming to concretize abstract concepts, for example, through visual or physical modeling. These approaches provide learners with better opportunities to grasp abstract ideas, thereby fostering the development of abstract thinking itself.

2.6. Possible biases in digital literacy?

Inadequate understanding and skills within mathematical thinking will unfortunately lead to several biases that influence the way the surrounding environment as well as society is being perceived. The relationship between overconfidence and ability in the task domain was originally coined by Kruger and Dunning [19]. If a person is overconfident about something this person is not that literate in, this is referred to as Dunning-Kruger Effect (DKE). Dunning and Krueger simply summarize their point by stating that: "… *those with limited knowledge in a domain suffer a dual burden: Not only do they reach mistaken conclusions and make regrettable errors, but their incompetence robs them of the ability to realize it*" [19, p. 1132].

Dunning and Kruger studied incompetence and poor performance in their famous study in 1999 and noticed that incompetence causes poor performance, but in addition it is also a reason for not being able to notice person's own poor performance. When analyzing the reason for this, Dunning and Kruger noticed that less skilled participants were lacking in their metacognitive skills, however when participants' metacognitive skills were improved, this improved the accuracy of self-evaluations as well. Dunning and Kruger noticed that paradoxically when people are made competent on substance area in question, they also recognize their own incompetence. [19].

DKE has been studied also in the context of IT use among other things. In the study by Giggs et al. an interesting implication was reported, where rapid development and changes in IT were considered as a possible reason for creating a delay in end-users perception of their own knowledge in the studied domain [20]. Then again, DKE can reach relatively extreme levels. A stunning example is a study conducted by Arroyo-Barrigüete et al. [21], who studied Spanish flat-earthers (i.e. those who believe that earth is flat) and based on their findings they suggest that low scientific literacy and high overconfidence together will result highest levels of flat earth belief. However, because of the study design, they point out that sample of population is not representative and it cannot present causalities, but instead only correlation.

The findings of Arroyo-Barrigüete et al. [21] are alarming, because they show that if people do not have adequate understanding, and when this is combined with high overconfidence, they are vulnerable for different influences. When poor mathematical skills are combined with cognitive bias, such as DKE, we are facing a situation where anything is seen possible. It does not matter whether something is logically impossible or simply erroneous, it is still considered as a viable solution. So, when we start to study students' declining skills in the field of STEM, we soon notice that we are entering an insecure soil. These skills are the foundation for tackling more advanced cognitive challenges, and thus not to be neglected.

2.7. Changing Role of (Mathematical) Communication

This paper refers to mathematics as Lingua Franca, that is "… *a language or way of communicating which is used between people who do not speak one another's native language*" [22]. However, we are constantly interacting with people with differing native languages, and sometimes we do not have any common language both parties are skilled enough to use in communication. For situations like this the universal principles of mathematics can act as a pidgin [23]: “*a simplified speech used for communication between people with different languages*”. We suggest that this very same approach would be applicable also in Human-Computer Interaction.

Conceptual models can be seen as supporting “heuristic devices” by describing components, relationships and processes for the study [24, p. 47]. This paper utilizes this idea and combines mathematical thinking as an integral part of communication process. For describing communication process, this paper applies Berlo’s communication model (SMCR), that consists of four components: Source, Message, Channel and Receiver [25].

According to Petersons and Khalimzoda [25] *Source* (or sender) in the SCMR model refers to the sender that is the origin of thought. They explain that in this model, ideas of the sender are transferred to *Receiver*, so that the ideas are encoded into understandable words, and here it is essential that both parties share the same knowledge, social system, culture, attitude and skills in communication. According to them, these factors are important for successful communication, in both encoding and decoding.

Petersons and Khalimzoda [25] emphasize that in Berlo’s model common understanding is seen as an extremely important part of communication. they also note, that although Berlo’s model has been criticized for lacking feedback option as well as barriers, filters or feedback, it has made an important contribution by presenting the idea that “*meanings are not in the message, they are in the message users, and therefore communicators must be explored from perspective of their background*” [25, pp. 426–427].

3. Discussion

While it has been argued that mathematics is not everyone’s piece of cake, it most surely has become one. The approach on mathematics in education should alter from plain compulsory calculation exercises on few obligatory courses in mathematics included in degree curricula into more comprehensive mathematical thinking. If the goal is to bridge the risk of creating a digital divide gap and to foster digital inclusion, the skills category plays an essential role.

While the number of sources of information are steadily increasing and access seems to be easier, the responsibility for staying informed is gradually shifting increasingly to the users. Important information as well as critical insights are easily buried under useless digital noise produced by influencers that are mostly concerned about promoting their own personal brand without any fact check. With all these new technological playmates we really do need a common language, so that we can communicate our intentions and requirements in an unambiguous way so that we get what we want. In addition, we need to understand what we get, whether it is what we want or what we need. How can AI determine what we need, while the basic logic is to provide what we seem to want?

European PISA results are indicating a trend that seems alarming. Declining skills in STEM disciplines seem to be a general trend. The reason for this is worth studying, because this is something that should be reversed. Elementary skills in these areas are essential for citizens in a contemporary digital society. If poor skills in these areas are combined with possible cognitive biases, such as DKE or confirmation bias, a sensible source criticism is no longer a viable option. Instead, anything can be taken for granted, because skills to analytically review received information are simply missing.

We also argue that mathematical thinking is important in human-to-human communication as well as in human-computer interaction. When communication process is analyzed using Berlo’s SCMR model, it can be clearly seen that in encoding and decoding phases mathematical thinking is an integral part of the process. Message must be communicated in such a way that it can be decoded in the receiving end without errors. Mathematical literacy is essential for those using and designing advanced systems. The required mathematical literacy is not likely high for ordinary end users, but the ability to operate and interpret results as well as recognize possible malfunctions or reasons for malfunctioning require ability for abstract mathematical thinking.

As a remedy, mathematical thinking does not necessarily need to be increased, but instead it should be embedded. In practice this means that all subjects should have mathematical elements included, and depending on the subjects this can be implemented in different ways. The common principle with all disciplines should be that there are high-level objectives in mathematical thinking that are clearly defined. If we fail to implement these, or even worse, if we fail to recognize these, the ability to gain necessary competencies for global education are hard to reach.

Declaration on Generative AI

The author(s) have not employed any Generative AI tools.

References

[1] J.-P. Mäkipää, R. Rousi, and T. Vartiainen, "Exploring Effects of Uncertainty Avoidance in Self-Service Technology User Interface Design in Japan and Finland," in *Information Modelling and Knowledge Bases XXXVI*, IOS Press, 2025, pp. 76–95. doi: 10.3233/FAIA241575.

[2] Merriam-Webster Dictionary, "Definition of COMMUNICATION." Accessed: Feb. 19, 2025. [Online]. Available: https://www.merriam-webster.com/dictionary/communication

[3] E. Mumford, "The story of socio-technical design: reflections on its successes, failures and potential," *Information Systems Journal*, vol. 16, no. 4, pp. 317–342, Oct. 2006, doi: 10.1111/j.1365-2575.2006.00221.x.

[4] K. Lyytinen and M. Newman, "Explaining information systems change: a punctuated sociotechnical change model," *European Journal of Information Systems*, vol. 17, no. 6, pp. 589– 613, Dec. 2008, doi: 10.1057/ejis.2008.50.

[5] D. M. B. Paiva, A. P. Freire, and R. P. De Mattos Fortes, "Accessibility and Software Engineering Processes: A Systematic Literature Review," *Journal of Systems and Software*, vol. 171, p. 110819, Jan. 2021, doi: 10.1016/j.jss.2020.110819.

[6] K. Mack, E. McDonnell, D. Jain, L. Lu Wang, J. E. Froehlich, and L. Findlater, "What Do We Mean by 'Accessibility Research'?: A Literature Survey of Accessibility Papers in CHI and ASSETS from 1994 to 2019," in *Proceedings of the 2021 CHI Conference on Human Factors in Computing Systems*, Yokohama Japan: ACM, May 2021, pp. 1–18. doi: 10.1145/3411764.3445412.

[7] J.-P. Mäkipää, J. Norrgård, and T. Vartiainen, "Factors Affecting the Accessibility of IT Artifacts: A Systematic Review," *Communications of the Association for Information Systems*, vol. 51, no. 1, Nov. 2022, doi: 10.17705/1CAIS.05129.

[8] OECD, *PISA 2022 Results (Volume I): The State of Learning and Equity in Education*, vol. 2023. Paris: OECD Publishing. Accessed: Jun. 03, 2025. [Online]. Available: https://doi.org/10.1787/53f23881-en

[9] M. Montanari, "PISA 2022 and the EU: three thought-provoking trends," European Commission, European School Education Platform. Accessed: Jun. 03, 2025. [Online]. Available: https://school-education.ec.europa.eu/en/discover/expert-views/pisa-2022-and-eu-three-thought-provoking-trends

[10] L. Meehan, G. Pacheco, and T. Schober, *Reading and mathematics skills and the life-course outcomes of young people in NZ: Evidence from PISA and linked administrative data.* Auckland, NZ: NZ Work Research Institute. Accessed: Sep. 29, 2025. [Online]. Available: https://workresearch.aut.ac.nz/__data/assets/pdf_file/0007/689938/PISA-trajectoriesNov-2022.pdf

[11] Eurostat, “Glossary:Digital divide.” Accessed: Mar. 13, 2025. [Online]. Available: https://ec.europa.eu/eurostat/statistics-explained/index.php?title=Glossary:Digital_divide
[12] J. A. G. M. van Dijk, *The Deepening Divide : Inequality in the Information Society*. California, London, New Delhi: SAGE Publications, Inc, 2005. Accessed: Sep. 24, 2025. [Online]. Available: ProQuest Ebook Central
[13] E. J. Helsper, *Digital inclusion: an analysis of social disadvantage and the information society*, vol. 2008. London: UK: Department for Communities and Local Government. Accessed: Sep. 24, 2025. [Online]. Available: http://eprints.lse.ac.uk/26938/
[14] European Union, “Council Recommendation of 23 November 2023 on the key enabling factors for successful digital education and training,” *Official Journal of the European Union*, no. C/2024/1115, Accessed: Sep. 24, 2025. [Online]. Available: http://data.europa.eu/eli/C/2024/1115/oj
[15] UNESCO, *AI competency framework for students*. Paris: UNESCO, 2024. Accessed: Sep. 24, 2025. [Online]. Available: https://doi.org/10.54675/JKJB9835
[16] M. Panza and A. Sereni, *Plato’s Problem: An Introduction to Mathematical Platonism*. London, UNITED KINGDOM: Palgrave Macmillan UK, 2013. Accessed: Sep. 24, 2025. [Online]. Available: ProQuest Ebook Central
[17] C. B. Boyer and U. C. Merzbach, *A History of Mathematics*. Newark, UNITED STATES: John Wiley & Sons, Incorporated, 2011. Accessed: Mar. 18, 2025. [Online]. Available: ProQuest Ebook Central
[18] K. K. McComas, “Abstract Thinking,” in *The Language of Mathematics Education: An Expanded Glossary of Key Terms and Concepts in Mathematics Teaching and Learning*, S. W. Dingman, L. B. Kent, K. K. McComas, and Cynthia C Orona, Eds., Boston: BRILL, 2019, p. 1. Accessed: Sep. 24, 2025. [Online]. Available: Proquest Ebook Central
[19] J. Kruger and D. Dunning, “Unskilled and unaware of it: How difficulties in recognizing one’s own incompetence lead to inflated self-assessments.,” *Journal of Personality and Social Psychology*, vol. 77, no. 6, pp. 1121–1134, Dec. 1999, doi: 10.1037/0022-3514.77.6.1121.
[20] S. Gibbs, K. Moore, G. Steel, and A. McKinnon, “The Dunning-Kruger Effect in a workplace computing setting,” *Computers in Human Behavior*, vol. 72, pp. 589–595, Jul. 2017, doi: 10.1016/j.chb.2016.12.084.
[21] J. L. Arroyo-Barrigüete, C. Bellón Núñez-Mera, J. Labrador Fernández, and V. L. De Nicolas, “Dunning–Kruger effect and flat-earthers: An exploratory analysis,” *Public Underst Sci*, vol. 32, no. 7, pp. 835–844, Oct. 2023, doi: 10.1177/09636625231166255.
[22] Collins English Dictionary, “Lingua Franca,” LINGUA FRANCA definition and meaning | Collins English Dictionary. Accessed: Jun. 10, 2025. [Online]. Available: https://www.collinsdictionary.com/dictionary/english/lingua-franca
[23] Merriam-Webster Dictionary, “Definition of PIDGIN.” Accessed: May 07, 2025. [Online]. Available: https://www.merriam-webster.com/dictionary/pidgin
[24] R. Craig T., “Constructing theories in communication research,” in *Theories and Models of Communication*, vol. 2013, P. Cobley and P. J. Schulz, Eds., Basel/Berlin/Boston, GERMANY: De Gruyter, Inc., 2013, pp. 39–57. Accessed: Jun. 09, 2025. [Online]. Available: ProQuest Ebook Central
[25] A. Petersons and I. Khalimzoda, “Communication Models and Common Basis for Multicultural Communication in Latvia,” *SOCIETY. INTEGRATION. EDUCATION. Proceedings of the International Scientific Conference*, vol. 4, pp. 423–433, May 2016, doi: 10.17770/sie2016vol4.1555.