Intelligent System and Technology for Optimized Object Placement in Medical and Biological Applications

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Abstract

Intelligent systems for optimized object placement in medical and biological applications leverage artificial intelligence advanced data fusion techniques to enhance precision, efficiency, and patient outcomes. These systems tackle a range of issues, including the positioning of surgical tools, deployment of sensors, and analysis of diagnostic images. Advanced mathematical modeling has become essential in healthcare and biological research, driving innovative solutions for treatment planning and spatial arrangements. This paper introduces an intelligent system aimed at optimizing the placement of geometric objects in medical and biological contexts. We employ a universal mathematical model that functions as an intelligent agent, utilizing parameters to adapt to different scenarios and optimize outcomes. We develop mathematical models and advanced algorithms to ensure precise placement, achieving the desired therapeutic or research outcomes while minimizing adverse effects. The mathematical model is formulated as a knapsack problem and expressed as Mixed Binary Non-Linear Programming (MBNLP). Problems related to optimized object placement can be addressed by selecting different model parameters. Several implementations demonstrate this approach, including Gamma Knife radiosurgery, laser coagulation, brachytherapy, and chromosome territory modeling.

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Keywords

Intelligent system, intelligent technology, optimized geometric design, nonlinear programming, phi-function, cylinder, ellipse, polyhedron, sphere, cuboid,

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Introduction

AI-powered intelligent systems are being implemented in the medical and biological fields to achieve improved object placement through artificial intelligence, the Internet of Things, and sophisticated data integration methods. The systems are highly adaptable, addressing various practical challenges such as the strategic placement of surgical tools, efficient sensor deployment, and thorough analysis of diagnostic images.

The utilization of intelligent systems within the medical field is experiencing a marked increase, particularly in medical assessment and treatment design. These systems are invaluable for medical professionals, helping them make more accurate decisions, reduce errors, and improve the effectiveness of therapeutic interventions [1]. Specifically, intelligent systems are used in various tasks such as detailed medical image analysis, personalized treatment planning, epidemic prediction and modeling, and aiding in drug discovery processes [2].

By adjusting the model's parameters, such systems function as intelligent agents, adapting to various scenarios and optimizing their performance. This adaptability and optimization capability are key principles of artificial intelligence, demonstrating how these systems leverage AI techniques to enhance precision and efficiency [3]. The effectiveness of these systems is significantly enhanced

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by mathematical modeling, providing the foundation for simulating complex scenarios and refining decision-making processes [4].

In medicine, mathematical modeling supports diagnostic processes and enhances therapeutic approaches. Differential equation-based models simulate biological systems, offering insights into disease progression and guiding treatment choices [5]. On the other hand, statistical models analyze patient data to forecast disease outcomes and identify the most effective treatments [6]. Advances in this field have significantly transformed healthcare and expanded biological knowledge. More sophisticated systems are now used to develop personalized treatment plans, analyze medical images, and manage workflows, thereby enhancing clinical outcomes and research productivity [7,8].

Automating treatment design marks a significant leap in enhancing the precision, speed, and effectiveness of medical protocols. Healthcare professionals can develop optimized treatment strategies, shorten planning times, and improve patient outcomes. Mathematical modeling in treatment planning is widely used across various medical and biological fields. Automated treatment systems use complex algorithms and mathematical models to define therapeutic targets, ensuring precise delivery of treatments while minimizing harm to healthy tissues.

Gamma Knife radiosurgery is a non-invasive radiotherapy used to treat brain and upper spine conditions. It employs computer-controlled planning to deliver targeted gamma rays to specific areas, minimizing damage to surrounding tissues. This therapy is particularly effective for small brain tumors, vascular malformations, and trigeminal neuralgia. Due to its precision, patients usually require only one treatment session, reducing the need for multiple rounds of radiation therapy. The role of automation and artificial intelligence in radiation therapy planning is further explored in reference [9].

Laser coagulation, also known as laser photocoagulation, is a surgical technique used to treat various eye conditions. It works by cauterizing blood vessels within the eye, commonly used for issues like diabetic retinopathy and retinal tears. The procedure involves using a laser to create tiny burns in the targeted tissues, promoting scar tissue formation that seals the edges of tears and prevents detachment. Laser coagulation effectively slows the progression of retinal disorders, reducing the risk of future vision loss. The article [10] discusses the use of artificial intelligence in diagnostic screening, predicting disease progression, and assessing treatment effectiveness through quantitative methods.

Brachytherapy is a type of internal radiation therapy that treats cancer by placing radioactive materials directly in or near the affected tissue. This method delivers high doses of radiation to the tumor while protecting healthy tissues from excessive exposure. Brachytherapy is used for various cancers, such as prostate, cervical, and breast cancer. Treatments can be temporary or permanent, depending on the type of cancer and the treatment plan. A study in article [11] describes a genetic algorithm that optimizes the placement of radiation seeds, ensuring complete coverage of the prostate and reducing radiation 'hotspots' in the urethra. The accuracy of placing cylindrical radioactive capsules in brachytherapy depends on their orientation and distance from the target tissue. These factors are essential for delivering the radiation dose precisely to the tumor while minimizing exposure to healthy tissues. Proper alignment of the capsules directs the radiation to the tumor, avoiding unnecessary exposure of healthy tissues and improving treatment effectiveness. Additionally, the distance between the capsule and the tumor significantly affects the radiation dose distribution.

Chromosome territory modeling studies the 3D arrangement of chromosomes in the cell's nucleus during interphase. Chromosomes occupy specific areas called chromosome territories and usually arrange themselves in a radial pattern within the nucleus. This organization varies by cell and tissue type and is a conserved trait across evolution. A research paper [12] explores the spatial organization of CTs in mammalian cell nuclei, highlighting the non-random, probability-driven nature of CT arrangement. Researchers model chromosome territories to study their spatial arrangement in the nuclear space. Packing algorithms can adjust the arrangement of overlapping ellipses representing chromosome territories, helping to simulate random or non-random chromosome distribution patterns. This approach enhances understanding of genomic regulation and function.

Packing problems, particularly those requiring optimal arrangement of items within containers without any overlap, frequently rely on nonlinear optimization techniques [13]. These approaches are beneficial for dealing with the complex limitations inherent in these problems. They are designed to determine numerical solutions for arranging various shapes, including circles, spheres, ellipses, and ovals. Due to the inherent intricacy of such packing scenarios, finding completely accurate solutions is typically unfeasible. Consequently, researchers and practitioners focus on deriving approximate or numerical solutions.

Employing heuristic approaches, which encompass strategies like genetic algorithms, simulated annealing, and tree search methods, is a common practice to refine the quality of numerical results [14]. These heuristic approaches capitalize on specific problem knowledge and operational guidelines to identify approximate solutions for packing scenarios. They are exceptionally useful in handling the intricate nature and computational hurdles of applying non-overlap and containment requirements.

Whether linear or nonlinear, mixed-integer programming models address both the continuous and discrete facets inherent in packing problems [15]. They integrate diverse methodologies such as constraint programming and tailored heuristics to ascertain optimal or near-optimal solutions, specifically for standard allocation, cutting, and packing applications.

This paper introduces an intelligent system designed to simulate the arrangement of geometric entities. This system makes use of a universal model grounded in the phi-functions method [16]. Expressly, normalized phi-functions allows calculating distance between these objects. The method considers object orientation, thereby affording fine-grained control over positioning. By adjusting the model's parameters, the system operates as an intelligent agent, adapting to various scenarios and optimizing object placement. Furthermore, by modulating the model's parameters, users can simulate object placement at defined distances or achieve carefully managed overlaps. The intelligent system's ability to refine and optimize based on input parameters aligns with AI methodologies, providing a robust tool for complex medical and biological applications. This approach transforms placement challenges into the framework of MBNLP [13, 15].

Examples of the system's applicability include optimizing the positioning of radioactive seeds in brachytherapy treatments, planning the arrangement of laser spots in laser coagulation procedures, and modeling the spatial organization of chromosome territories.

Special Universal Mathematical Model and its Characteristics

The foundation of the proposed intellectual system is a distinctive universal mathematical model of optimization geometric design constructed with specialized intellectual means of modeling this category of problems. These intellectual means encompass specific functions designated as "phi-functions" [16]. These functions facilitate the construction of a generalized universal mathematical model in the form of a nonlinear optimization problem.

Let $O_i \in \mathbf{R}^d$ ($d = 2,3$) be objects with given metric characteristics $\mathbf{m}_i$, $i \in I_N = \{1, 2, ..., N\}$. We define the location of objects in Euclidean space as $\mathbf{u} = (\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_N)$ where $\mathbf{u}_i = (\mathbf{v}_i, \mathbf{\Theta}_i)$, $\mathbf{v}_i = (x_i, y_i)$ (or $\mathbf{v}_i = (x_i, y_i, z_i)$) are coordinates of the poles of O_i and $\mathbf{\Theta}_i$ are angles, specifying orientations of O_i, $i \in I_N$. We denote the object O_i with placement parameters $\mathbf{u}_i$ as $C_i(\mathbf{u}_i)$, $i \in I_N$.

The placement region P is specified by given metric characteristics $\mathbf{m}$. Objects O_i, $i \in I_N$ should be packed in P in one of two ways:

- At the minimum admissible distances $d_{ij} \geq 0$, $1 \leq i < j \in I_N$, between themselves and the minimum admissible distances $d_i \geq 0$, $i \in I_N$ to the frontier of P
- With allowing overlap of objects, regulated by parameters $d_{ij} < 0$, $1 \leq i < j \in I_N$, and allowing objects to extend beyond the frontier of P, regulated by parameters $d_i < 0$, $i \in I_N$.

We aim to define a subset from the set O_i, $i \in I_N$, that maximizes the total volume of the objects when placed in P. The mathematical model of the problem is as follows:

$$V^* = \max \sum_{i \in I_N} t_i V(O_i) \quad \text{s.t.} \ \mathbf{u} \in G \tag{1}$$

where

$$t_i = \begin{cases} 1 \text{ if } \Phi_i(\mathbf{u}_i) \ge d_i, \\ 0 \text{ otherwise,} \end{cases} \tag{2}$$

$$G = \left\{ \mathbf{u} \in \mathbf{R}^{(2d-1)N} : t_i t_j \Phi_{ij}(\mathbf{u}_i, \mathbf{u}_j) \ge d_{ij},\ 1 \le i < j \in I_N \right\}. \tag{3}$$

Here, t_i, $i \in I_N$, are binary variables that determine whether an object belongs to P. The inequality $\Phi_i(\mathbf{u}_i) \ge d_i$ specifies whether the object O_i satisfies the placement condition relative to the frontier of P. At the same time, the inequality $\Phi_{ij}(\mathbf{u}_i, \mathbf{u}_j) \ge d_{ij}$ checks whether the conditions for the mutual placement of objects hold.

To solve the problem (1) – (3), it is necessary to construct normalized phi-functions. Generally, this is a complex task, but researchers have already developed such phi-functions for some basic objects [17,18].

MBNLP problems is inherently complex due to the combination of continuous and discrete variables and nonlinear constraints. Solving such problems often involves techniques such as branch-and-bound, which systematically explores the solution space by dividing it into smaller subproblems. However, given the number of variables and constraints, such exhaustive enumeration is impractical. Therefore, we employ heuristic approaches, selecting subsets of objects from the given set that meet the objective function criteria. Subsequently, a block optimization algorithm is applied, which has significantly lower computational complexity compared to branch-and-bound algorithms.

The problem (1) – (3) divides into two stages. In the first stage, we enumerate subsets from the set of all objects. In the second stage, the placement of each subset in P . Then, the placement with the best objective function value is an approximate solution of the problem (1)–(3). According to the typology of Cutting and Packing Problems [19], the problem relates to Knapsack Problem or Identical Item Packing Problem depending on the metric characteristics of the objects. Therefore, to obtain a solution, a sequential addition scheme [20,21] is usually performed, also known as block optimization [22,23]. A method to solve the Knapsack Problem considered in [24] allows for collective rearrangement within the sequential addition scheme. Another challenge is the presence of angles, which specify the orientation of the objects.

Next, we implement the model for some applications in medicine and biology.

Applications in medicine and biology

Planning of Gamma Knife radiosurgery therapy

Gamma knife treatment involves directing beams to a common center to create a radiation dose. The primary geometric difficulty in this treatment involves precisely positioning a series of spheres within a three-dimensional tumor of varying shapes. Significant sphere overlap can lead to excessive dosages, whereas controlled, minor overlap is generally acceptable.

According to the problem (1) – (3), $O_i = S_i \in \mathbf{R}^3$ are spheres with given radius r_i, $i \in I_N = \{1, 2, ..., N\}$. $\mathbf{u}_i = \mathbf{v}_i = (x_i, y_i, z_i)$. There is no need to account for rotation angles $\mathbf{\Theta}_i$. We set the parameters $d_{ij} < 0$, $1 \le i < j \in I_N$, and $d_i < 0$, $i \in I_N$ and consider the placement region P as a convex polyhedron defined by a system of inequalities $A_l x + B_l y + C_l z + D_l \ge 0,\ l \in L$. Here, $A_l x + B_l y + C_l z + D_l = 0,\ l \in L$, are the normal equations of planes.

The problem (1) – (3) takes the following form:

$$V^* = \frac{4}{3}\pi \max \sum_{i \in I_N} t_i r_i^3 \quad \text{s.t. } u \in G \tag{4}$$

where

$$t_i = \begin{cases} 1 \text{ if } \Phi_i(\mathbf{u}_i) \ge d_i, \\ 0 \text{ otherwise,} \end{cases} \tag{5}$$

$$G=\left\{\mathbf{u}\in\mathbf{R}^{3N}:t_it_j\Phi_{ij}(\mathbf{u}_i,\mathbf{u}_j)\ge d_{ij},\ 1\le i<j\in I_N\right\},\tag{6}$$

$$\Phi_i\left(v_i,r_i\right)=\min\left\{A_lx_i+B_ly_i+C_lz_i+D_l-r_i,\ l\in L\right\},$$

$$\Phi_{ij}(\mathbf{u}_i,\mathbf{u}_j)=\sqrt{(x_i-x_j)^2+(y_i-y_j)^2+(z_i-z_j)^2}-(r_i+r_j).$$

The sequential addition scheme is realized to solve the problem (4) – (6).

We consider a polyhedron with 12 vertices. Table 1 provides the coordinates of the vertices.

Table 1
Coordinates of vertices of P

No.	1	2	3	4	5	6	7	8	9	10	11	12
x	12.14	4.64	-4.64	12.14	-15	-12.14	-4.64	4.64	12.14	15	0	0
y	7.5	-7.5	-7.5	-7.5	7.5	-7.5	7.5	7.5	7.5	-7.5	16.8	-16.8
z	8.82	14.63	14.63	8.82	0	8.82	-14.63	-14.63	-8.82	0	0	0

The number of faces of P is $|L|=20$. Faces are defined by three vertices with numbers 1-11-3, 3-11-5, 5-11-7, 7-11-9, 9-11-1, 1-3-2, 2-3-4, 3-5-4, 4-5-6, 5-7-6, 6-7-8, 7-9-8, 8-9-10, 9-1-10, 10-1-2, 2-12-10, 4-12-2, 6-12-4, 8-12-6, 10-12-8. In this example, we set parameters $d_1=d_2=-3.5$ to ensure controlled overlapping of spheres and their overhanging beyond the treatment area.

Table 2 presents the radii and coordinates of the spheres. Figure 1 illustrates the placement of 15 spheres.

Table 2
Radii and coordinates of spheres in P

No.	r_i	x_i	y_i	z_i
1	6.0000	6.0862	9.4494	13.1205
2	6.0000	14.4555	13.8876	6.5106
3	4.0000	11.1704	3.2400	9.5100
4	4.0000	15.4769	7.7476	12.7806
5	4.0000	8.7643	18.0687	11.7612
6	4.0000	5.8567	15.6307	5.8312
7	4.0000	7.9100	8.9659	3.2636
8	4.0000	13.3247	13.9842	15.2371
9	3.0000	13.5530	6.7428	2.1862
10	3.0000	1.8558	15.6172	10.5151
11	3.0000	7.8793	15.8433	17.4366
12	3.0000	15.9303	6.1139	6.858569
13	3.0000	12.1263	7.7394	17.9496
14	3.0000	2.37207	9.8437	6.0495
15	3.0000	5.72614	4.5232	6.9294

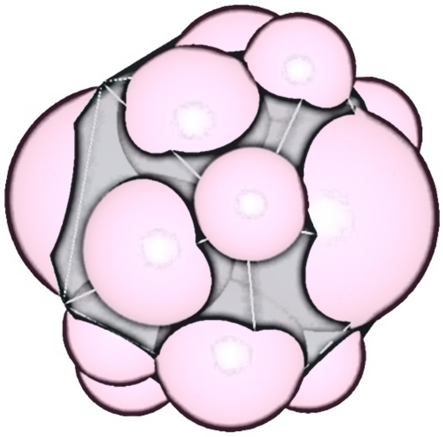

Figure 1: Illustration of target placement

Planning of laser coagulation treatment

Accurately placing photocoagulates (microburns) on the retina is a key geometric task. The photocoagulates must be evenly distributed within the edematous area, avoiding contact with blood vessels and healthy regions. Significant overlap of photocoagulates can lead to excessive dosages, whereas controlled, minor overlap is acceptable.

We model microburns on the retina as equal circles with a given radius and specify a minimum allowable distance between the circles. The placement region consists of convex polygons. This way, the problem can be reduced to solving several subproblems.

According to the problem (1) – (3), $O_i = C_i \in \mathbf{R}^2$ are equal circles with given radius r, $i \in I_N = \{1,2,...,N\}$. The vectors $\mathbf{u}_i = \mathbf{v}_i = (x_i, y_i)$ do not involve $\boldsymbol{\Theta}_i$. The parameters $d_{ij} = d_1 \geq 0$, $1 \leq i < j \in I_N$, represent the minimum admissible distance between circles whereas the parameter $d_i = 0$. We consider the placement region P as a convex polygon defined by the system of inequalities $A_l x + B_l y + C_l \geq 0,\ l \in L$ where $A_l x + B_l y + C_l = 0,\ l \in L$ are the normal equations.

The problem (1) – (3) takes the following form:

$$V^* = \pi r^2 \max \sum_{i \in I_N} t_i \quad \text{s. t. } u \in G \tag{7}$$

where

$$t_i = \begin{cases} 1 \text{ if } \Phi_i(\mathbf{u}_i) \geq 0, \\ 0 \text{ otherwise,} \end{cases} \tag{8}$$

$$G = \left\{ \mathbf{u} \in \mathbf{R}^{2N} : t_i t_j \Phi_{ij}(\mathbf{u}_i, \mathbf{u}_j) \geq d,\ 1 \leq i < j \in I_N \right\}, \tag{9}$$

$$\Phi_i(v_i, r_i) = \min\left\{ A_l x_i + B_l y_i + C_l - r_i,\ l \in L \right\},$$

$$\Phi_{ij}(\mathbf{u}_i, \mathbf{u}_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} - 2r.$$

To solve the problem (7) – (9), we implement the sequential addition scheme. The selected placement subregion is quadrilateral with vertices (10,0), (90,5), (69,40), (10,45). Figure 2 shows the treatment region. We set the parameter $d_1 = 1$ to avoid closely spaced microburns. Figure 3 illustrates the placement of 21 circles within the marked convex polyhedron in the placement region shown in Figure 2. Table 3 provides the radii and coordinates of the placed circles.

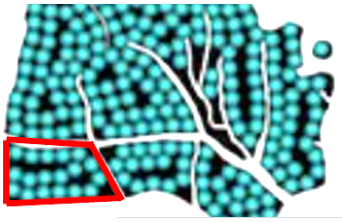

Figure 2: Illustration of the treatment region

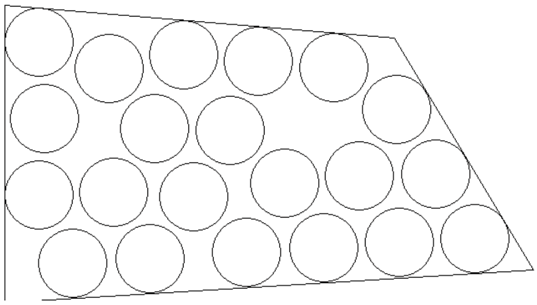

Figure 3: Location of 21 circles

Table 3
Radii and coordinates of circles

No.	5.200	48.505	36.507
1	5.200	26.626	16.626
2	5.200	25.906	35.356
3	5.200	63.776	19.136
4	5.200	16.106	27.791
5	5.200	52.391	18.006
6	5.200	32.094	6.603
7	5.200	44.190	25.944
8	5.200	69.379	29.226
9	5.200	59.874	35.537
10	5.200	37.129	37.436
11	5.200	75.223	19.426
12	5.200	69.710	9.105
13	5.200	46.687	7.618
14	5.200	15.215	16.268
15	5.200	20.309	5.952
16	5.200	15.209	39.319
17	5.200	32.780	26.242
18	5.200	58.312	8.247
19	5.200	81.126	9.663
20	5.200	38.695	15.928
21	5.200	48.505	36.507

Planning of brachytherapy

To achieve precise placement of cylindrical radioactive capsules during brachytherapy, it is necessary to evaluate their location and orientation relative to the target tissue. Correct positioning ensures that the radiation is concentrated on the tumor, avoiding unnecessary exposure of healthy tissues and improving treatment effectiveness.

According to the problem (1) – (3), $O_i = C_i \in \mathbf{R}^3$ are equal cylinders with given radius r, and height r, $i \in I_N = \{1,2,...,N\}$, $\mathbf{u}_i = (\mathbf{v}_i, \mathbf{\Theta}_i)$, $\mathbf{v}_i = (x_i, y_i, z_i)$, $\mathbf{\Theta}_i = (\varphi_i, \omega_i)$. We set the parameters

$d_{ij} = d_1 > 0$, $1 \le i < j \in I_N$, as the minimum admissible distance between the cylinders and $d_i = d_2 > 0$, $i \in I_N$, as the minimum admissible distance to the frontier of P. The placement region P is a convex polyhedron defined by the inequality system $A_l x + B_l y + C_l z + D_l \ge 0,\ l \in L$, where $A_l x + B_l y + C_l z + D_l = 0,\ l \in L$, are the normal equations.

The problem (1) – (3) takes the following form:

$$V^* = \pi r^2 h \max \sum_{i \in I_N} t_i \quad \text{s.t. } u \in G \tag{10}$$

where

$$t_i = \begin{cases} 1 \text{ if } \Phi_i(\mathbf{u}_i) \ge d_2, \\ 0 \text{ otherwise,} \end{cases} \tag{11}$$

$$G = \left\{ \mathbf{u} \in \mathbf{R}^{3N} : t_i t_j \Phi_{ij}(\mathbf{u}_i, \mathbf{u}_j) \ge d_1,\ 1 \le i < j \in I_N \right\}. \tag{12}$$

For the problem (10) – (12), we realize the sequential addition scheme as well. We approximate the cylinders with convex polyhedrons and use the normalized phi-functions [17,25]. Figure 4 illustrates the placement of 20 cylinders in a cuboid with dimensions 13.88x12.03x13.65.

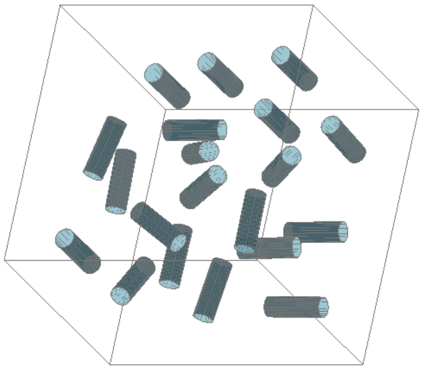

Figure 4: Location of 20 cylinders

Modeling of chromosome territories

Chromosome territories can be represented as overlapping ellipses within the nucleus, which is approximated by a convex polygon. To model these territories, it is necessary to accurately simulate the spatial distribution and interactions of these ellipses. This involves evaluating their positions and overlaps to reflect the actual behavior of chromosomes during interphase.

According to the problem (1) – (3), $O_i = E_i \in \mathbf{R}^2$ are ellipses with half-axes a_i and b_i, $i \in I_N = \{1,2,...,N\}$, $\mathbf{u}_i = (\mathbf{v}_i, \mathbf{\Theta}_i)$, where $\mathbf{v}_i = (x_i, y_i)$ and $\mathbf{\Theta}_i = \varphi_i$. We set the parameters $d_{ij} = d < 0$, $1 \le i < j \in I_N$, and $d_i = 0$, $i \in I_N$, and consider the placement region P as a convex polygon defined by the system of inequalities $A_i x + B_i y + C_i \ge 0,\ l \in L$. Here, $A_i x + B_i y + C_i = 0$ is the normal equation.

The problem (1)–(3) takes the following form:

$$V^* = \pi \max \sum_{i \in I_N} t_i a_i b_i \quad \text{s.t. } u \in G \tag{13}$$

where

$$t_i = \begin{cases} 1 \text{ if } \Phi_i(\mathbf{u}_i) \ge d_2, \\ 0 \text{ otherwise,} \end{cases} \tag{14}$$

$$G = \left\{ \mathbf{u} \in \mathbf{R}^{3N} : t_i t_j \Phi_{ij}(\mathbf{u}_i, \mathbf{u}_j) \ge d_1,\ 1 \le i < j \in I_N \right\}. \tag{15}$$

Table 4
Coordinates and orientation angles for cylinders placed in P

No.	x_i	y_i	z_i	φ_i	ω_i
1					
2	12.64				
	5	8.588	5.777	0.014	1.547
3	9.554	5.454	9.421	0.723	-1.206
4	13.63				
	7	5.593	13.998	1.585	4.719
5	3.802	5.874	4.351	-1.585	1.578
6	6.797	11.150	10.248	0.643	-1.777
7	4.788	12.608	13.977	-3.142	1.557
8	9.651	5.593	13.656	1.557	1.307
9	3.782	8.463	9.300	0.057	3.142
10	12.65				
	1	12.333	4.371	0.000	-1.557
11	13.65				
	8	11.584	13.000	1.519	-34.559
12	4.786	4.587	9.257	-3.142	3.364
13	9.782	11.601	13.992	-1.585	1.307
14	12.65				
	1	12.608	8.506	3.142	1.557
15	12.75				
	1	7.885	9.820	0.792	-1.645
16	8.627	4.587	5.086	-3.142	-0.995
17	3.782	11.660	5.467	-2.519	0.000
18	7.641	8.601	5.357	-3.132	-0.010
19	13.65				
	8	4.608	5.357	-3.156	0.000
20	5.691	7.601	13.998	-1.558	61.254

We set the parameters to $d_1 = -3$ to allow controlled overlapping of ellipses. Papers [17,18] consider the construction of normalized phi-functions. For the problem (13) − (15), we also implement the sequential addition scheme with collective rearrangement.

Figure 5 illustrates the placement of 10 ellipses within the polyhedron P, with their coordinates presented in Table 5. We consider P as a composition of two convex polyhedrons. Table 6 shows the coordinates and orientation angles of ellipses illustrated in Figure 5.

Table 5
Coordinates of vertices of P

No.	1	2	3	4	5	6	7	8	9	10
x	22	37	47	42	95	91	73	59	12	6
y	9	14	12	4	3	9	27	32	33	32

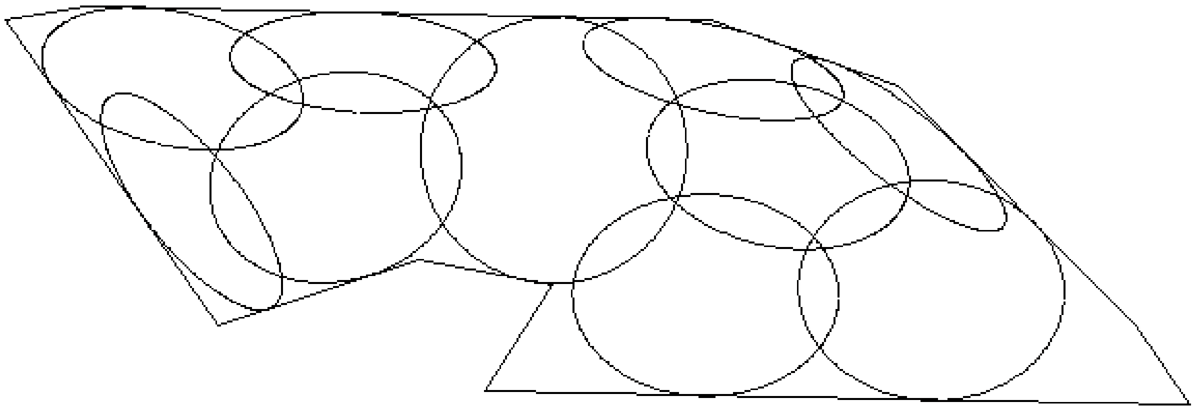

Figure 5. Illustration of placement of 10 ellipses

Table 6
Coordinates and orientation angles for ellipses placed in P

No.	a_i	b_i	x_i	y_i	φ_i
1	9.578	7.786	-0.267	30.764	20.135
2	5.023	10.000	1.788	18.494	27.563
3	3.765	10.000	1.620	32.820	28.783
4	10.000	3.250	3.358	59.166	28.264
5	10.000	10.000	-1.518	47.175	22.163
6	10.000	6.165	3.357	64.030	21.106
7	10.000	2.801	0.661	73.084	22.612
8	10.000	7.582	0.011	58.580	11.271
9	3.571	10.000	2.472	19.935	18.303
10	10.000	8.262	0.065	75.553	11.635

Discussion

Computational experiments have demonstrated the high adaptability and flexibility of the intelligent system, attributed to its parameterization as an intelligent agent. This adaptability allows the system to optimize object placement across various scenarios effectively.

The computational complexity of the algorithm depends not only on the number of objects being placed but also on the type of phi-functions used. For instance, when placing circles or spheres, the phi-functions are relatively simple, enabling the placement and local reorganization of hundreds of objects. In contrast, when describing interactions between ellipses (or ellipsoids) and cylinders, which are non-oriented and whose placement depends on rotation angles (involving trigonometric functions), the phi-functions have a significantly more complex structure and logical operators. This complexity can substantially impact computational efficiency.

Additionally, the geometric shape of the placement region significantly influences the complexity of phi-functions and, consequently, the computational complexity. For example, irregular or complex-shaped regions require more intricate phi-functions to accurately describe the spatial relationships and constraints.

In the case of approximating cylinders with polyhedra, the computational complexity is also affected by the accuracy of the approximation, which depends on the number of faces of the polyhedra. Higher accuracy requires more faces, leading to increased computational demands.

Therefore, the algorithm can handle the placement of dozens of objects when using the sequential addition scheme (block optimization) and local reorganization (optimization) of placements. This approach ensures that the system remains efficient and effective, even when dealing with the intricate nature of three-dimensional space and the associated computational challenges.

The sequential addition scheme (block optimization) is effective for managing the placement of objects, especially when combined with local reorganization. The ability to perform collective rearrangement within this scheme further enhances the system's effectiveness.

Conclusion

This paper puts forth a proposal for the development of intelligent technologies in the domain of geometric design. These technologies utilize advanced methodologies and instruments for automating and optimizing the processes of placing geometric objects in space. Optimization is achieved by applying these technologies in the context of solving applied problems in medicine and biology.

The proposed universal mathematical model, which utilizes normalized phi-functions, encompasses continuous and combinatorial facets of packaging problems. The model's formulation encompasses the movement and orientations of geometric objects, enabling the modeling of object placement at a distance or their controlled overlap. This model possesses characteristics inherent to AI systems, such as adaptability, automation, and intelligent modeling methods and can be applied for decision-making.

The model's capacity to operate with diverse geometric shapes and placement constraints underscores its potential for addressing a broad spectrum of problems. Employing linear and nonlinear mixed-binary programming methods, in conjunction with constraint programming and heuristics, facilitates the identification of near-optimal solutions for cutting and packing problems. The model enhances the efficiency of medical treatment and facilitates a more profound comprehension of biological processes.

Examples of application include optimizing the placement of radioactive seeds in brachytherapy, determining optimal laser impact points in laser coagulation, and predicting the behavior of chromosomes in chromosome territory modeling. These applications demonstrate the practical use of the model in medical and biological contexts, highlighting its potential to improve clinical outcomes and research efficiency.

Declaration on Generative AI

During the preparation of this work, the authors used Grammarly in order to: Grammar and spelling check. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the publication’s content.

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