Capturing Bitcoin Market Dynamics: Assessing Advanced Permutation Entropy Metrics as Early-Warning Indicators

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Abstract

Permutation entropy (PEn) is a widely adopted nonlinear statistical measure for quantifying complexity in time series data. Despite its conceptual clarity and computational efficiency, classical PEn has notable limitations, particularly its disregard for amplitude variations in time series data and the simplistic handling of sequences containing equal-valued observations. Although modified PEn methods exist, their potential as early-warning indicators for cryptocurrency market crashes remains largely unexplored. This paper addresses these limitations by conducting a comparative analysis of classical PEn and three enhanced methods: weighted permutation entropy (WPEn), amplitude-aware permutation entropy (AAPEn), and uniform quantization-based permutation entropy (UPEn). Specifically, these entropy metrics are employed to analyze the Bitcoin market crash from December 2017 to February 2018, utilizing a sliding window approach. Empirical results demonstrate that amplitude-enhanced entropy methods effectively capture nuanced market dynamics and fluctuations, offering more precise and more reliable signals of impending market instability. This study confirms the value of advanced entropy measures in cryptocurrency markets and underscores their potential as robust indicators for detecting and forecasting financial crashes.

Keywords

permutation entropy, weighted permutation entropy, amplitude-aware permutation entropy, uniform quantization-based permutation entropy, complexity measures, cryptocurrency market crash, Bitcoin, early warning indicators, market instability

1. Introduction

Quantifying the complexity inherent in temporal data offers profound insights into the underlying dynamics of complex systems, such as cryptocurrency markets [1, 2]. Despite its significance, complexity lacks a universally accepted definition [3, 4]. Among various methods proposed, entropy-based metrics have emerged as particularly effective in assessing complexity, given their conceptual clarity and computational efficiency [5]. Entropy encapsulates complexity by measuring the degree of randomness or unpredictability in time series data. These entropy methods can be applied across diverse types of data, including deterministic, chaotic, stochastic, stationary, and nonstationary processes [6].

Cryptocurrency markets, especially Bitcoin, exhibit pronounced volatility, high noise levels, and nonlinearity, making classical linear analytical techniques insufficient for comprehensive market analysis [7, 8]. Entropy-based approaches provide a viable alternative to traditional methods such as fractal dimension [9], Lyapunov exponent [10], or Lempel-Ziv complexity [11], particularly due to their robustness when dealing with short, noisy, and nonstationary data. Previous research has successfully demonstrated the efficacy of information-theoretic entropy measures in analyzing complex financial time series [12, 13, 14].

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The efficient market hypothesis (EMH), initially formulated by Fama [15], postulates that market prices rapidly incorporate all available information, leading to random walk-like behavior in asset price fluctuations. Under EMH conditions, informational efficiency implies maximum entropy states, where no predictable profit opportunities remain due to information symmetry among market participants. However, empirical observations suggest real-world cryptocurrency markets exhibit varying degrees of efficiency, with entropy levels fluctuating over time due to market sentiment, regulatory news, technological developments, or speculative trading activities [16]. Entropy metrics thus provide intuitive and practical tools for capturing shifts in market efficiency regimes, highlighting their utility in detecting impending market disruptions.

Cryptocurrency market crashes, particularly in Bitcoin, are characterized by complex, nonlinear interactions and rapid transitions from relatively stable states towards chaotic regimes [17, 18, 19]. Understanding these crashes demands a nuanced examination of their emergent properties, including increased correlation among market participants, evolving self-organized patterns, and heightened systemic risk. Permutation entropy (PEn), a powerful nonlinear complexity metric, and its various modified forms are promising tools to investigate these dynamics.

This study focuses specifically on the Bitcoin market crash occurring between December 2017 and February 2018 [2], a significant event marked by the bursting of a speculative bubble. Using classical permutation entropy and several enhancements thereof, we employ a sliding time-window methodology to observe temporal changes in complexity. This approach reveals patterns and trends indicative of impending market crises. The identification of early-warning signals based on permutation entropy measures provides substantial benefits not only to traders and investors but also to policymakers and regulatory authorities. Recognizing precursor signals of market crashes enables stakeholders to implement proactive measures, mitigate systemic risks, and formulate informed short- to long-term strategies.

2. Permutation entropy methodology

2.1. Classical Permutation Entropy

Permutation entropy (PEn) is a complexity measure that quantifies the predictability of a time series by analyzing the frequency distribution of its ordinal (permutation) patterns [20]. Inspired by Claude Shannon's information entropy [21], PEn has proven effective for various real-world data analysis applications, particularly in finance and economics [22, 23].

Shannon entropy (ShEn) quantifies the uncertainty associated with a discrete random variable X having a probability distribution $p(x)$ as follows:

$$H(X) = -\sum_{x \in \chi} p(x) \log p(x), \tag{1}$$

where χ denotes the set of possible outcomes for X. ShEn measures the number of bits needed to encode information, thus reflecting the unpredictability of outcomes. Variations of ShEn, such as Rényi entropy and joint entropy, have also been successfully employed in different fields to characterize random processes [24, 25].

PEn applies this concept to time series data by investigating ordinal patterns, thus capturing temporal dynamics and predictability. Consider a univariate time series $\{x_t\}_{t=1}^{N}$ with N data points. To identify ordinal patterns, the series is segmented into embedding vectors defined by two parameters: embedding dimension d_E (length of the subsequences) and time delay τ. For each time t, embedding vectors are constructed as follows:

$$\vec{X}_t^{d_E,\tau} = \left(x_t, x_{t+\tau}, \ldots, x_{t+(d_E-1)\tau}\right), t = 1, \ldots, N - \left(d_E - 1\right)\tau. \tag{2}$$

Each embedding vector $\vec{X}_t^{d_E,\tau}$ is mapped onto one of $d_E!$ possible ordinal patterns $\{\pi_i\}_{i=1}^{d_E!}$ based on the relative ordering of its elements. Specifically, the ordinal pattern represents the permutation required to sort vector components into ascending order. For example, given a time series segment

$(5,8,4)$ with $d_E=3$ and $\tau=1$, the ordinal pattern is classified as $\pi_i=(2,0,1)$ since the order of indices corresponding to ascending values is $(3,1,2)$.

Table 1 summarizes all possible ordinal patterns for an embedding dimension of $d_E=3$:

Table 1
Possible ordinal patterns for embedding dimension $d_E=3$

Ordinal Pattern	(x_a, x_b, x_c)	Condition
π_1	$(3,2,1)$	$x_t > x_{t+\tau} > x_{t+2\tau}$
π_2	$(3,1,2)$	$x_t > x_{t+2\tau} > x_{t+\tau}$
π_3	$(2,3,1)$	$x_{t+\tau} > x_t > x_{t+2\tau}$
π_4	$(2,1,3)$	$x_{t+2\tau} > x_t > x_{t+\tau}$
π_5	$(1,3,2)$	$x_{t+\tau} > x_{t+2\tau} > x_t$
π_6	$(1,2,3)$	$x_t < x_{t+\tau} < x_{t+2\tau}$

The probability of each ordinal pattern π_i occurring in the time series is computed by

$$p(\pi_i)^{d_E,\tau} = \frac{¿\{\vec{X}_t^{d_E,\tau} \vee \vec{X}_t^{d_E,\tau}\, corresponds ¿\, pattern\, \pi_i\}}{N-(d_E-1)\tau}, i=1,\ldots,d_E!. \quad (3)$$

Finally, the permutation entropy for the time series is defined as

$$PEn(X)^{d_E,\tau} = \frac{-1}{\ln d_E!} \sum_{i=1}^{d_E!} p(\pi_i)^{d_E,\tau} \ln p(\pi_i)^{d_E,\tau}, \quad (4)$$

where the normalization term $(\ln d_E!)^{-1}$ ensures that the entropy values range between 0 (completely predictable series) and 1 (completely random series), thus facilitating meaningful comparisons across different time series and applications.

2.2. Weighted Permutation Entropy

While classical permutation entropy effectively captures complexity by analyzing ordinal patterns, it disregards amplitude-related information inherent in the original time series data. This limitation can lead to several drawbacks: (i) significant amplitude differences between data points within ordinal patterns are ignored, potentially losing critical information; (ii) patterns with substantial amplitude variations and those resulting from minor fluctuations (noise) contribute equally to the permutation entropy measure, diminishing the method's sensitivity; and (iii) ignoring amplitude may reduce the discriminative power of permutation entropy when applied to real-world data, such as financial or physiological signals.

To address these limitations, Fadlallah et al. [26] introduced weighted permutation entropy (WPEn), which integrates amplitude information by assigning different weights to each ordinal pattern based on the local variance or energy of the corresponding subsequences. The main idea behind WPEn is to emphasize ordinal patterns derived from subsequences with more considerable amplitude variations, thus incorporating valuable amplitude-related information.

Formally, for each embedding vector $\vec{X}_t^{d_E,\tau}$, weight w_t is defined using the variance of the elements within the subsequence as

$$w_t = \frac{1}{d_E} \sum_{k=1}^{d_E} \left(x_{t+(k-1)\tau} - \langle \vec{X}_t^{d_E,\tau} \rangle \right)^2, \quad (5)$$

where $\langle \vec{X}_t^{d_E,\tau} \rangle$ represents the arithmetic mean of the subsequence:

$$\left\langle \vec{X}_t^{d_E,\tau} \right\rangle = \frac{1}{d_E} \sum_{k=1}^{d_E} x_{t+(k-1)\tau}. \tag{6}$$

Once the weights are computed, the weighted probability of each ordinal pattern π_i is given by

$$p_w(\pi_i)^{d_E,\tau} = \frac{\sum_{t:\vec{X}_t^{d_E,\tau} \in \pi_i} w_t}{\sum_t w_t}, i = 1, \ldots, d_E!, \tag{7}$$

where the denominator ensures normalization, preserving the probabilistic interpretation $\sum_{i=1}^{d_E!} p_w(\pi_i)^{d_E,\tau} = 1$.

The WPEn is then defined analogously to the ShEn formulation as

$$WPEn(X)^{d_E,\tau} = -\sum_{i=1}^{d_E!} p_w(\pi_i)^{d_E,\tau} \ln p_w(\pi_i)^{d_E,\tau}. \tag{8}$$

WPEn can be seen as an amplitude-sensitive adaptation of weighted Shannon entropy [27], providing a way to measure complexity when outcomes have different importance levels or intensities. Thus, WPEn significantly enhances permutation entropy's utility by effectively combining both ordinal and amplitude information, making it particularly suitable for analyzing complex signals such as those encountered in financial markets and other noisy real-world environments.

2.3. Amplitude-Aware Permutation Entropy

Although WPEn successfully incorporates amplitude variance into the entropy calculation, it still exhibits some limitations. Specifically, WPEn cannot differentiate cases where a constant offset is added to a time series since the variance remains unchanged under such transformations. Additionally, WPEn is less sensitive to scenarios involving minor amplitude shifts or additive constants, potentially limiting its effectiveness in capturing subtle but meaningful amplitude-based information within a signal.

To address these limitations, Azami and Escudero [28] introduced amplitude-aware permutation entropy (AAPEn), a refined entropy measure explicitly designed to capture amplitude information more comprehensively. This method improves upon WPEn by assigning variable contributions to ordinal patterns based on both the absolute amplitude levels and the relative changes between consecutive samples.

To illustrate the shortcomings of standard permutation entropy methods regarding amplitude information:

1. Classical permutation entropy relies solely on ordinal relationships, ignoring amplitude magnitude. For instance, sequences such as $(5,20,8)$ and $(5,12,8)$ share an identical ordinal pattern (021), despite significant amplitude differences. Similarly, sequences $(5,12,8)$ and $(25,37,30)$ also share the same ordinal pattern due to the absence of amplitude considerations.
2. In the presence of equal consecutive values, traditional ordinal analysis may yield ambiguous results. Bandt and Pompe [20] suggested resolving ties based on the order of occurrence or by adding small noise. However, this approach is problematic because, for example, the vectors $(3,9,9)$ and $(3,6,9)$ can both yield ambiguous ordinal patterns. This issue is particularly relevant in discretely sampled or digitized signals.

To mitigate these issues, AAPEn modifies the traditional histogram-based ordinal pattern encoding by introducing amplitude-based weighting. Specifically, each embedding vector contributes a variable amount to the ordinal pattern frequency histogram instead of uniformly incrementing by one:

$$p(\pi_i)^{d_E,\tau} = p(\pi_i)^{d_E,\tau} + L\left(\vec{X}_t^{d_E,\tau}\right), if\ \vec{X}_t^{d_E,\tau}\ corresponds\ ¿\ pattern\ \pi_i, \tag{9}$$

where the amplitude-based adjustment coefficient $L\left(\vec{X}_t^{d_E,\tau}\right)$ is defined as

$$L\left(\vec{X}_t^{d_E,\tau}\right) = \frac{A}{d_E}\sum_{k=1}^{d_E}\left|x_{t+(k-1)\tau}\right| + \frac{1-A}{d_E-1}\sum_{k=2}^{d_E}\left|x_{t+(k-1)\tau} - x_{t+(k-2)\tau}\right|, \tag{10}$$

with $A \in [0,1]$ balancing the relative importance of amplitude magnitudes and consecutive amplitude changes.

The final amplitude-aware probabilities for each ordinal pattern are normalized as follows:

$$p(\pi_i)^{d_E,\tau} = \frac{p(\pi_i)^{d_E,\tau}}{\sum_{t=1}^{N-(d_E-1)\tau} L\left(\vec{X}_t^{d_E,\tau}\right)}. \tag{11}$$

The parameter A allows flexibility in emphasizing either mean amplitude levels or amplitude difference. For anomaly detection tasks, setting $A \ll 0.5$ emphasizes sudden amplitude changes, enhancing sensitivity. Conversely, for tasks like financial crash detection, where both mean amplitude and amplitude fluctuations carry importance, a balanced value ($A = 0.5$) is recommended.

Additionally, the choice of delay parameter τ significantly impacts AAPEn results. While a delay of $\tau = 1$ is typically adequate, certain signal characteristics, such as single-sample spikes versus extended spikes, may benefit from greater delays $(\tau > 1)$. Careful selection of τ helps avoid aliasing-like effects, preserving the integrity of amplitude and frequency characteristics within the signal. For analyses at multiple temporal scales, frameworks such as those proposed by Costa et al. [29] or Azami et al. [30] can further enhance the robustness of AAPEn.

By effectively capturing amplitude dynamics alongside ordinal structure, AAPEn provides a powerful and flexible tool, well-suited for nuanced applications such as cryptocurrency market analysis, anomaly detection, and other complex time series tasks.

2.4. Uniform Quantization-Based Permutation Entropy

Chen et al. [31] introduced uniform quantization-based permutation entropy (UPEn), a refined entropy measure designed to capture amplitude variations and mitigate ambiguities associated with equal-valued data points. Unlike classical PEn, which relies solely on ordinal patterns, UPEn incorporates amplitude information through a quantization-based encoding approach. The method involves two primary steps:

1. Pattern Formation: Embedding vectors are symbolized via uniform quantization.
2. Entropy Estimation: The entropy is calculated based on the distribution of quantized patterns.

Initially, the time series is segmented into embedding vectors $\vec{X}_t^{d_E,\tau}$. The first elements of these embedding vectors $\vec{X}_{t,1}^{d_E,\tau}$ undergo uniform quantization (UQ), transforming the continuous data into discrete symbols. For a time series X, the UQ process assigns each value to one of D quantization levels, as defined by

$$UQ(x) = \lfloor \frac{x - x_{min}}{\Delta} \rfloor, where\ \Delta = \frac{x_{max} - x_{min}}{D}, \tag{12}$$

with x_{min} and x_{max} representing the minimum and maximum values in the series, respectively, and D denoting the discretization level.

After symbolizing the first column of the embedding vectors $S_{t,1}$, the subsequent elements are symbolized relative to the first quantized element. For each embedding vector, the quantized symbols for subsequent elements are computed as follows:

$$S_{t,k}=S_{t,1}+\lfloor \frac{\vec{X}_{t,k}^{d_E,\tau}-\vec{X}_{t,1}^{d_E,\tau}}{\Delta} \rfloor, 1\le t\le N-(d_E-1)\tau, 2\le l\le d_E. \tag{13}$$

This procedure results in a symbolic pattern matrix S, where each row represents a quantized ordinal pattern π_i^U. The probability distribution $p(\pi_i^U)$ of these quantized patterns is calculated by counting occurrences and normalizing by the total number of patterns:

$$p(\pi_i^U)=\frac{¿\{S_t^{d_E,\tau} \vee S_t^{d_E,\tau} \, corresponds ¿ \, pattern \, \pi_i^U\}}{N-(d_E-1)\tau}, i=1,\ldots,D^{d_E}. \tag{14}$$

The UPEn is then computed similarly to ShEn, with normalization to ensure values range between 0 and 1:

$$UPEn(X)^{d_E,\tau,D}=\frac{-1}{\ln D^{d_E}}\sum_{i=1}^{D^{d_E}} p(\pi_i^U)\ln p(\pi_i^U), \tag{15}$$

where the normalization factor $\ln D^{d_E}$ represents the theoretical maximum entropy achievable under a uniform pattern distribution.

Parameter selection is crucial in UPEn analysis. Typically, an embedding dimension $d_E=3$ is employed, balancing computational simplicity and capturing realistic dynamics of most real-world signals. Additionally, a delay parameter $\tau=1$ is chosen to preserve the structural integrity of sequential observations [32]. The discretization level D significantly influences the performance of UPEn. A higher retains more amplitude detail, enhancing sensitivity but also increasing susceptibility to noise and requiring larger sample sizes for stability. Conversely, lower values of D provide noise robustness at the expense of amplitude resolution. Chen et al. [31] recommend a discretization level of $D=4$ for practical applications such as financial crash detection, providing an optimal compromise between detail preservation and robustness.

3. Methods and Empirical Results

To comparatively evaluate classical PEn and its variants, as well as to identify potential early-warning indicators of cryptocurrency market crashes, we specifically focus on the significant Bitcoin market crash period spanning from August 21, 2017, to April 3, 2018. This period includes the well-documented speculative bubble burst at the end of 2017 and early 2018, which provides an exemplary scenario for studying complexity dynamics within cryptocurrency markets.

The analysis utilizes daily Bitcoin price data, transformed into standardized returns to ensure stationarity and comparability across entropy measures. The returns are computed as:

$$G(t)=\frac{x(t+\Delta t)-x(t)}{x(t)}, \tag{16}$$

and subsequently standardized as:

$$g(t)=\frac{G(t)-\langle G\rangle}{\sigma}, \tag{17}$$

where $\langle G\rangle$ denotes the mean and σ the standard deviation of returns G.

All computational analyses in this study were executed using the Python programming language within the Jupyter Notebook interactive environment. Implementation of the entropy calculation methods, including permutation entropy variations, leveraged the Entropy Hub software package [33], ensuring consistency and reproducibility of the results.

A sliding window technique was adopted for calculating entropy values, facilitating a dynamic and temporal assessment of complexity changes. Specifically, the chosen window length was $w=50$ days, determined through preliminary experimentation as optimal for capturing significant complexity fluctuations during the studied Bitcoin crash period. The window was incrementally shifted along the time series with a step of $\Delta t=1$, allowing a comprehensive temporal analysis.

Comparing the dynamics of the actual Bitcoin returns and corresponding entropy measures provides insights into complexity trends that precede and characterize market crashes. Consistent complexity behavior patterns, such as noticeable rises or drops during the pre-crash phase, could serve as reliable precursor indicators for impending market disruptions [34, 35, 36, 37, 38]. These findings contribute to the broader understanding of cryptocurrency market behavior, enhancing predictive capabilities and risk management strategies.

In Figure 1, we present the comparative dynamics of Bitcoin prices (BTC-USD) alongside the classical PEn metric during the critical period spanning from August 21, 2017, to April 3, 2018. The dashed green line marks December 6, 2017, indicating the onset of the major Bitcoin market crash.

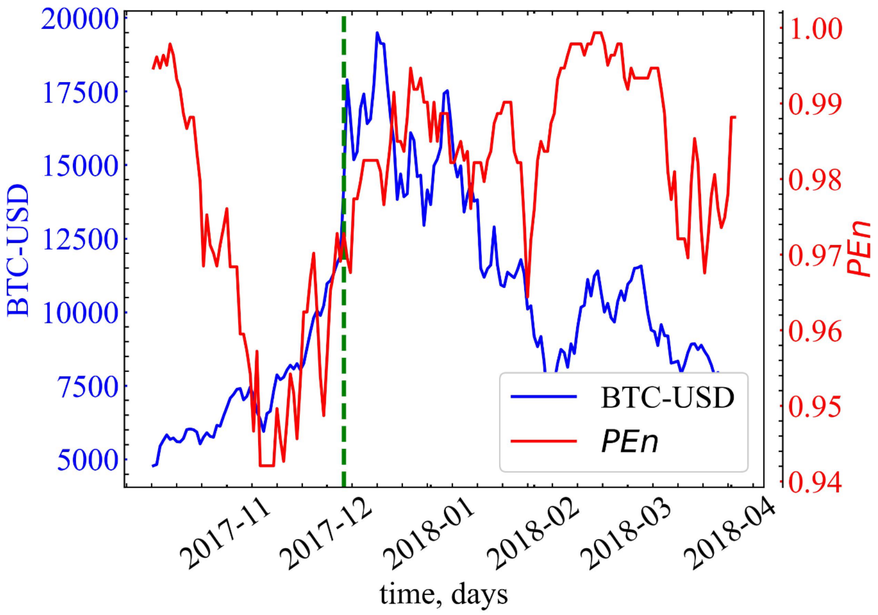

Figure 1: Comparative dynamics of the Bitcoin market crash (2017-2018) and the standard PEn

Initially, from late August to early December 2017, Bitcoin prices exhibit an exponential upward trend, reaching unprecedented highs and reflecting market optimism and speculative interest. During this pre- crash phase, the classical permutation entropy metric remains relatively high, indicative of significant market complexity and unpredictability, characteristic of dynamically healthy cryptocurrency markets. Approaching early December 2017, Bitcoin price growth accelerates sharply, reaching its historical peak. Correspondingly, the PEn measure begins a notable and rapid decrease from its previously elevated values, signaling a crucial shift from a highly complex state to increasingly predictable dynamics. This reduction in entropy clearly precedes the actual crash, highlighting the emergence of ordered patterns within price movements. Such a drop in complexity implies that market participants' behavior is becoming more synchronized and less diverse, reflecting reduced market efficiency and heightened systemic risk.

Following the green dashed line marking December 6, 2017, Bitcoin prices rapidly decline, marking the onset of the cryptocurrency market crash characterized by high volatility and investor uncertainty. During this crash period, permutation entropy continues to decline and reaches its lowest values, underscoring significantly increased predictability and reduced market complexity. This entropy minimum effectively coincides with the deepest market downturns, capturing the peak synchronization of trader behavior indicative of panic-driven selling and herd-like market dynamics.

After the steepest phase of the crash, beginning approximately mid-February 2018, Bitcoin prices start to stabilize and gradually recover, though remaining volatile due to ongoing uncertainty. In parallel, the PEn values gradually recover, reflecting the slow return of market complexity and efficiency. The increasing entropy during this recovery phase suggests that diverse market behaviors and a broader range of trading strategies are slowly being restored, signaling a cautious re-emergence of market resilience.

In summary, Figure 1 emphasizes the potential utility of classical permutation entropy as an early indicator for cryptocurrency market crashes. Its distinctive temporal pattern – high entropy during stable market growth, rapid entropy decrease preceding the crash, minimal entropy at the crash peak, and a gradual entropy recovery afterward – provides valuable insights for traders, investors, and policymakers concerned with predicting and managing risks associated with cryptocurrency market instability.

Figure 2 illustrates the comparative dynamics of Bitcoin prices (BTC-USD) and the WPEn metric during the Bitcoin market crash period from August 21, 2017, to April 3, 2018. The dashed green vertical line denotes December 6, 2017, the identified starting point of the significant crash in Bitcoin prices.

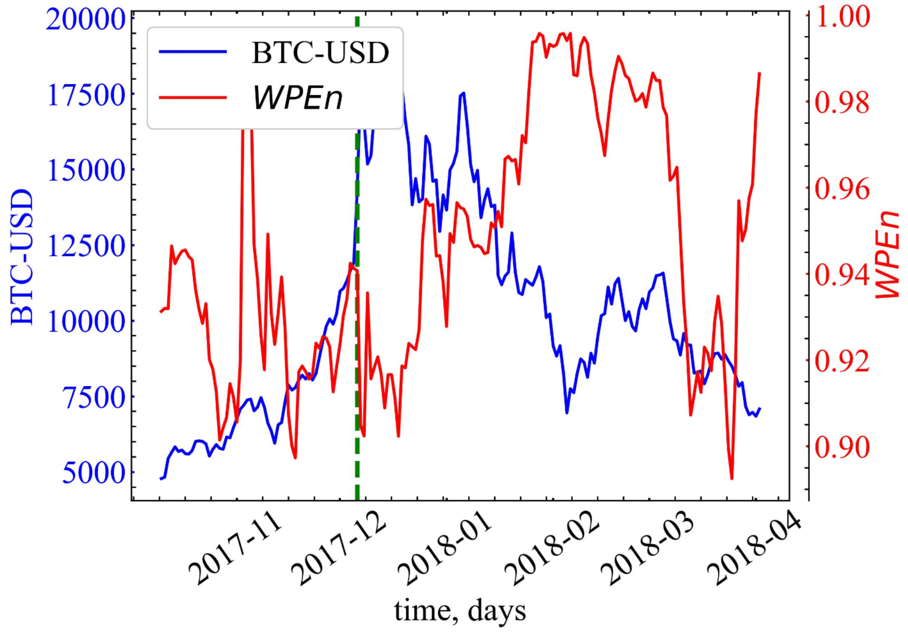

Figure 2: Comparative dynamics of the Bitcoin market crash (2017-2018) and the WPEn

In contrast to classical PEn, WPEn explicitly incorporates amplitude variations, assigning greater importance to patterns derived from subsequences with significant variance or energy. This property enables WPEn to detect and reflect subtle yet crucial fluctuations in market volatility and price amplitude, providing additional depth to complexity analysis in cryptocurrency markets.

During the pre-crash period from late August to early December 2017, Bitcoin prices rose substantially, reaching historical highs amid strong market enthusiasm and speculative activities. WPEn values remained relatively elevated throughout this phase, indicating a highly complex and diverse market environment characterized by dynamic interactions among market participants without dominant or overly coordinated patterns.

As the market approaches early December 2017, WPEn exhibits notable and sharp fluctuations, corresponding closely with significant price movements in Bitcoin. Unlike the gradual decline seen in classical PEn, WPEn demonstrates abrupt drops associated directly with intense volatility events and pronounced amplitude variations. These sudden entropy reductions reflect rapid transitions toward less complex and more predictable market dynamics, capturing critical moments of increased instability immediately preceding and during the early phases of the crash.

At the peak of the crisis (around late December 2017 to January 2018), WPEn values reach their lowest points, aligning precisely with the most severe declines in Bitcoin prices. This pronounced entropy drop illustrates the increased market synchronization and collective investor behavior, typical of panic-driven sell-offs, and highlights WPEn's sensitivity to substantial amplitude and volatility shifts. Following the main phase of the crash, Bitcoin prices enter a volatile recovery period, accompanied by rapid increases and fluctuations in WPEn. The post-crisis recovery shows multiple sharp entropy variations, indicating persistent periods of instability and uncertainty in market dynamics. These fluctuations underscore the continued vulnerability and complexity of the cryptocurrency market as it attempts to regain equilibrium.

In summary, Figure 2 demonstrates WPEn's capability to detect immediate market instabilities and significant amplitude variations effectively. While WPEn does not provide as clear an anticipatory signal as classical PEn, its acute responsiveness to abrupt market fluctuations makes it a powerful analytical tool for identifying and characterizing critical moments of cryptocurrency market instability.

Figure 3 presents a comparative analysis of Bitcoin prices (BTC-USD) alongside the AAPEn metric for the period from August 21, 2017, to April 3, 2018. The green dashed line marks December 6, 2017, denoting the onset of the Bitcoin market crash.

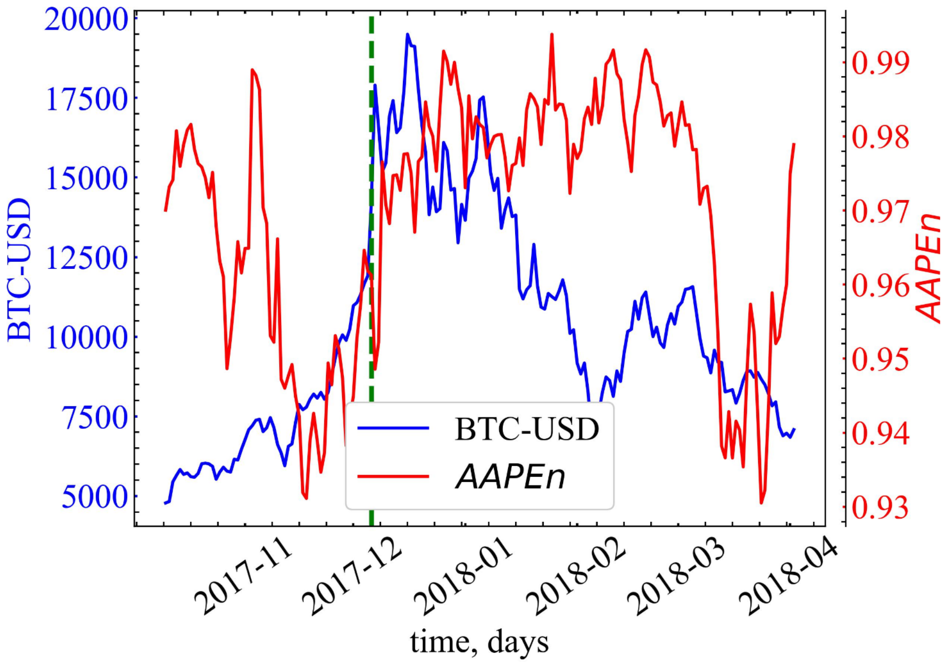

Figure 3: Comparative dynamics of the Bitcoin market crash (2017–2018) and the AAPEn

Unlike classical PEn and weighted permutation entropy (WPEn), the amplitude-aware permutation entropy explicitly considers amplitude differences between consecutive data points, enhancing its sensitivity to detect significant structural shifts and sudden anomalies in market behavior.

In the pre-crash period, spanning from late August to early December 2017, Bitcoin prices experience rapid growth and pronounced volatility. During this phase, AAPEn values remain elevated, indicative of a complex and diverse market state characterized by relatively unsynchronized market participant behavior. High AAPEn values here reflect a healthy market condition without clear early warnings of the impending crash.

As the market approaches early December 2017, AAPEn exhibits more pronounced fluctuations and begins a discernible downward trend. This early entropy decline, particularly noticeable before the actual onset of the crash (marked by the green dashed line), underscores AAPEn's sensitivity and effectiveness in capturing subtle, amplitude-driven market disturbances. Thus, AAPEn provides valuable precursor signals of rising market instability earlier than traditional entropy metrics.

At the crash peak between December 2017 and January 2018, Bitcoin prices sharply decline, and concurrently, AAPEn significantly decreases, reaching its minimum values. This drop clearly illustrates the transition toward more predictable, amplitude-coordinated patterns arising from synchronized panic-driven selling behaviors, characteristic of severe market crises.

In the subsequent recovery phase, from late January to April 2018, AAPEn demonstrates partial recovery toward higher complexity levels, albeit with substantial fluctuations reflecting continued market uncertainty and episodes of heightened volatility. These entropy fluctuations during the recovery phase underscore the lingering instability within the cryptocurrency market as it attempts to regain equilibrium.

Overall, Figure 3 highlights the superior capability of amplitude-aware permutation entropy in detecting and interpreting nuanced market dynamics. Its sensitivity to subtle amplitude fluctuations allows it to serve effectively as both an early-warning indicator and a detailed analytical tool, offering deeper insights into the structural and behavioral complexities of cryptocurrency markets during periods of significant turbulence.

Figure 4 presents the comparative dynamics of Bitcoin prices (BTC-USD) and the UPEn metric from August 21, 2017, to April 3, 2018. The green dashed line indicates December 6, 2017, marking the onset of the significant Bitcoin market crash.

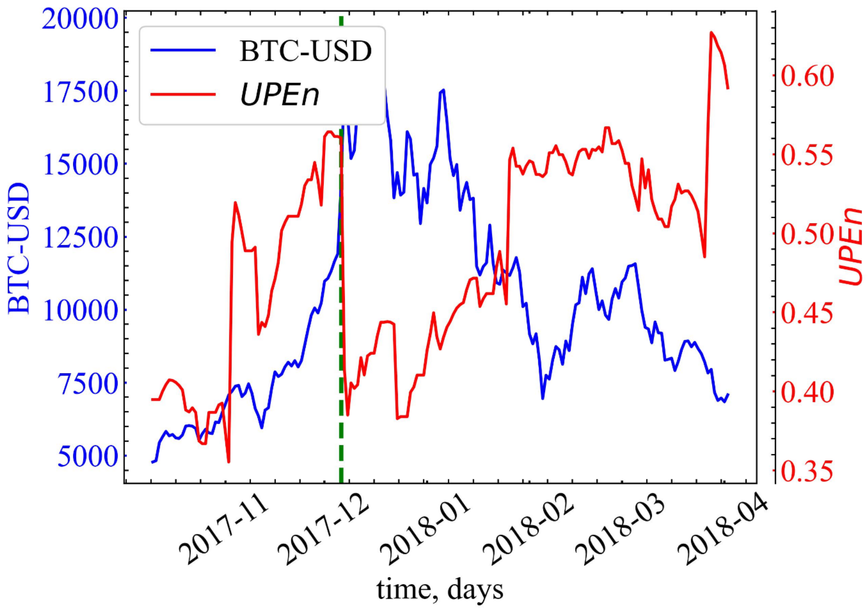

Figure 4: Comparative dynamics of the Bitcoin market crash (2017–2018) and the UPEn

Unlike traditional permutation entropy approaches, UPEn utilizes uniform quantization to explicitly incorporate amplitude information and address the issue of equal-value observations. This allows UPEn to provide a more robust and stable complexity representation by effectively capturing longer-term structural changes in market dynamics while minimizing sensitivity to minor fluctuations.

In the initial period from late August to early December 2017, Bitcoin prices steadily rise amid market optimism and speculative activities, accompanied by moderate volatility. UPEn values during this phase gradually increase, reflecting growing market complexity and active dynamics, though remaining relatively stable and moderate overall. This stability indicates balanced complexity conditions without immediate signs of market distress.

As the Bitcoin market approaches early December 2017, the UPEn metric begins to exhibit a discernible decline, signaling the early emergence of structural instability preceding the crash. Unlike the more volatile behavior seen in classical or amplitude-aware permutation entropy metrics, UPEn's decline is smoother and more gradual, effectively filtering short-term volatility while emphasizing longer-term market changes.

During the peak crash period between December 2017 and January 2018, Bitcoin prices experience rapid declines. Correspondingly, UPEn reaches its lowest point, clearly reflecting diminished market complexity and increased predictability resulting from coordinated, panic-driven selling behavior. This minimum entropy period effectively captures the structural transition from a complex, healthy market to a more ordered but fragile state characteristic of crisis conditions.

In the post-crash recovery phase, beginning around February 2018, UPEn gradually increases, indicating a slow yet consistent restoration of market complexity and stability. Compared to other permutation entropy methods, UPEn shows fewer abrupt fluctuations during this recovery phase, suggesting it effectively emphasizes sustained structural recovery rather than short-term volatility. This characteristic makes UPEn particularly valuable for detecting and interpreting the longer-term complexity evolution in cryptocurrency markets during periods of recovery and restabilization.

Overall, Figure 4 underscores the effectiveness of UPEn as a reliable, robust indicator for capturing structural complexity changes associated with cryptocurrency market crashes. Its capability to highlight gradual complexity shifts and filter short-term noise makes UPEn highly suitable for policymakers, investors, and analysts aiming for stable and long-term market stability indicators.

4. Conclusion

In this paper, we performed a comprehensive comparative analysis of classical permutation entropy (PEn) and its enhanced variants – weighted permutation entropy (WPEn), amplitude-aware permutation entropy (AAPEn), and uniform quantization-based permutation entropy (UPEn) – specifically applied to the Bitcoin market crash from August 21, 2017, to April 3, 2018. Our primary goal was to evaluate the effectiveness of these entropy measures as early-warning indicators of cryptocurrency market instability, overcoming traditional PEn's limitation of disregarding amplitude information.

Our empirical findings underscore the unique strengths of each entropy method in capturing distinct aspects of cryptocurrency market dynamics. The classical PEn measure proved notably effective in detecting a gradual complexity decline prior to the crash, accurately reflecting the transition from a complex and efficient market to a predictable and vulnerable state. Its ability to identify reduced entropy preceding the actual market downturn highlights its robustness as a reliable precursor metric for cryptocurrency market crashes.

The WPEn metric, through its variance-based weighting of ordinal patterns, demonstrated significant sensitivity to abrupt market fluctuations, capturing immediate instability events with notable precision. Although WPEn was less effective in identifying gradual complexity reductions compared to classical PEn, its rapid responsiveness makes it particularly valuable for real-time detection of severe volatility episodes typical in cryptocurrency markets.

AAPEn emerged as exceptionally effective due to its refined incorporation of amplitude differences among consecutive data points. It captured subtle but meaningful market shifts with greater sensitivity and offered more transparant early-warning signals compared to both classical PEn and WPEn. The flexibility in tuning its parameters also enhances its adaptability to diverse cryptocurrency market conditions, improving predictive accuracy and interpretability regarding structural shifts and emerging instabilities. UPEn, leveraging uniform quantization to incorporate amplitude data, provided stable and robust indicators by emphasizing sustained structural changes while effectively filtering out short-term volatility. Although less sensitive to immediate fluctuations compared to WPEn or AAPEn, UPEn was particularly effective in revealing longer-term complexity trends, making it highly suitable for strategic monitoring of cryptocurrency markets over extended periods.

Overall, our analysis confirms the utility of permutation entropy methods, especially amplitude-enhanced variants, as powerful tools for predicting and analyzing cryptocurrency market crashes. While classical PEn continues to serve as a straightforward and reliable early indicator, advanced entropy measures such as WPEn, AAPEn, and UPEn significantly enrich the analytical toolkit by capturing deeper market complexities and subtle signals of impending instability.

Future research directions include applying these entropy methodologies to analyze other cryptocurrency crashes and market anomalies, exploring their applicability across diverse digital assets and market conditions. Integrating these entropy metrics with advanced machine learning algorithms, including deep learning techniques, could further improve forecasting precision and enable the development of sophisticated real-time alert systems for cryptocurrency market monitoring. Additionally, exploring multivariate extensions of these entropy measures may provide deeper insights into interdependencies and collective dynamics among different cryptocurrencies, further enhancing their value as decision-support tools for investors, market analysts, and regulatory authorities. Furthermore, combining entropy-based complexity analysis with clustering techniques may provide novel insights into market regime identification and trading strategy optimization, ultimately leading to better-informed trading decisions and improved risk management practices in cryptocurrency markets [39].

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Declaration on Generative AI

During the preparation of this work, the authors used Grammarly in order to: Grammar and spelling check. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the publication's content.

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