

Computational technology for uncertain extended energy-saturated objects magnetic silence control

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Abstract

Work presents the development of computational technology for prediction and control by uncertain extended energy-saturated object magnetic silence based on geometric inverse magneto static problem solution with consideration of magnetic characteristics uncertainty to improve magnetic silence of extended energy-saturated object. Geometric inverse problem solution reduced to vector game solution with Wolfram Mathematica software package calculated payoff vector. Game solution calculated based on heuristic optimization algorithms from Pareto-optimal solutions taking into account binary preference relations local games solutions. Based on developed computational technology results of prediction and control by uncertain extended energy-saturated object magnetic silence presented.

Keywords

energy-saturated extended technical objects, magnetic field silencing, prediction, measurements, uncertainty, geometric inverse magneto static problem, heuristic optimization algorithms

1. Introduction

1.1. Motivation

An important scientific and technical problem of modern magnetism of technical objects is implementation of strict requirements for external magnetic field level [1, 2, 3]. This problem is especially acute for magnetism of spacecraft, naval vessel and submarines [4, 5, 6]. The success of solving the problem of magnetism of these technical objects is largely determined by the adequacy of mathematical models of the external magnetic field (MF) to the real values of the magnetic characteristics of these objects [7, 8, 9]. To measure the real characteristics of the MF of spacecraft, military ships and submarines, special magnetodynamic measuring stands have been developed, one of which is located at the Anatolii Pidhornyi Institute of Power Machines and Systems of the National Academy of Sciences of Ukraine.

Based on the experimentally measured values of the MF components on the bench, a mathematical model of the MF of technical object designed [10, 11, 12]. Then, based on the mathematical model of the technical object designed on the basis of measurements of the near MF, the values of the MF parameters in the far zone are calculated. This is the task of magnetic field prediction [1, 13]. Then, based on the calculated values of the MF parameters in the far zone, the problem of calculating the parameters and coordinates of the location in the space of the technical object of the compensating sources of the MF is solved to meet the requirements for the parameters of the MF of the technical object [14, 15].

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1.2. Magnetic silence – state of the art

The most widely used sources of MF are point sources, the MF of which is described in a spherical coordinate system (SCS). The mathematical model of technical objects is often adopted in the form of a Multiple Dipole Model (MDM) [2, 3]. The parameters of the dipoles and the coordinates of their location in the space of the technical object are determined in the course of solving the geometric inverse problem of magnetostatics from the condition of minimizing the error between the measured and predicted by the model values of the parameters of the external MF at the specified points of measurement of space [5, 6].

Despite the fact that the shape of military ships and submarines has a "cigar-shaped" appearance of elongated technical objects, mathematical models of the MF of such objects are also often adopted in MDM form. In the works [1, 2] the expediency of using mathematical models in the form of elongated ellipsoidal sources (EES) of MF, describing the parameters of the MF in prolate spheroidal coordinate systems (PSCS), is shown for such elongated technical objects.

On magnetodynamic stands, measurements of magnetic characteristics of technical objects are usually measured in Cartesian coordinate systems (CCS) related to the center of technical objects. In MDM of the magnetic field of technical objects, the positions of dipole sources of the MF are also specified in CCS related to the center of technical objects. In addition, on magnetodynamic stands, magnetic characteristics of component units of electrical equipment of technical objects are often measured, which are also, as a rule, measured in CCS related to the center of these component units of electrical equipment of technical objects [10, 11].

However, mathematical models of concentrated MF sources are calculated in SCS associated with the centers of these sources. Mathematical models of elongated magnetic field sources are calculated in the form of prolate spheroidal magnetic field sources in PSCS associated with the centers of these sources [5, 6].

In classical works on electrodynamics [1, 2], solutions of the Laplace equation for the scalar potential of a magnetic field in a SCS and in a PSCS are known. Accordingly, these solutions are written in terms of SCS and PSCS [16, 17]. But for practice, it is often necessary to work in terms of a CCS [18, 19]. In addition, it is not the scalar potential that is practically important, but the projections of the magnetic induction.

In modern works, for example, related to the magnetic cleanliness of spacecraft and magnetic silence of naval vessel and submarines, based on solutions of the Laplace equation for the scalar potential of the MF outside the source, analytical formulas for the projections of magnetic induction in terms of the SCS and PSCS are obtained. Moreover, in the case of the SCS, these formulas are obtained only for several first spherical harmonics (up to 4) and for these formulas, the associated Legendre polynomials are written out, thereby obtaining rather cumbersome formulas [1, 2].

However, to date there is no generalization of the formula for the case of the n -harmonic. Additional difficulties are the need to transform coordinates from the CCS to the SCS, and then the obtained projections of magnetic induction from the SCS to the CCS (a similar situation is with the PSCS). For the PSCS, the situation is even more complicated: in the formulas for the projections of magnetic induction, it is necessary to take derivatives of the scalar potential with respect to the PSCS.

The peculiarity of the considered energy-saturated elongated objects is the inaccurate knowledge of magnetic characteristics and their change in different operating modes. Such objects are called uncertain objects [16, 17, 18, 19].

1.3. Objectives

This work is devoted to the development of computational technology for prediction and control by uncertain extended energy-saturated object magnetic silence based on geometric inverse magneto static problem solution with consideration of magnetic characteristics uncertainty to improve magnetic silence of extended energy-saturated object.

2. Definition of forward magnetostatics problem for spheroidal sources

Consider analytical formulas for projections of magnetic induction in CCS for spheroidal harmonics of MF in PSCS. Consider multispheroidal model of original magnetic field of energy-saturated extended technical object in PSCS. Let us assume that initial magnetic field of extended energy-saturated object generated using I spheroidal magnetic field sources located at technical object space points with coordinates (x_i, y_i, z_i) in CCS associated with the center of technical object as shown in Figure 1.

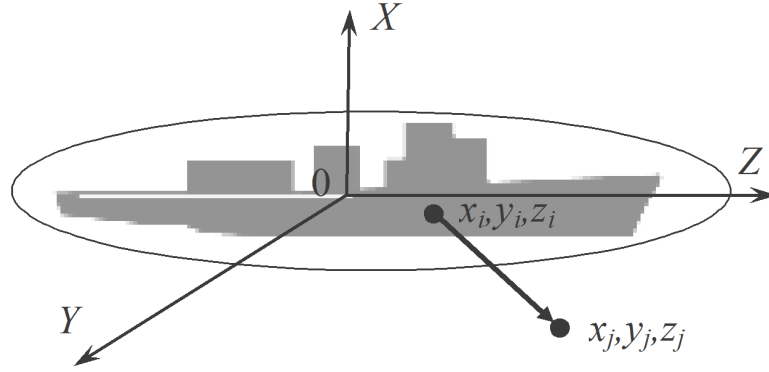


Figure 1: Energy-saturated extended technical object.

The relationship between the right triple of unit vectors x, y, z of the DSC and the triple ξ, η, ϕ of the VSSC has the form [13, 14]: where c is half the focal length of the spheroid whose foci lie on the z -axis at the points $\pm c$. From a geometric point of view, the triple ξ, η, ϕ is a family of prolate spheroids ($= \text{const}$), two-sheeted hyperboloids ($= \text{const}$) and half-planes ($= \text{const}$) passing through the z -axis.

The solution of the Laplace equation in the PSCS with respect to the scalar potential of the magnetic field for the external region outside the sources has the form [1, 2]:

$$U = \frac{1}{4\pi} \sum_{n=1}^{\infty} \sum_{m=0}^n Q_n^m(\xi) (c_n^m \cos m\phi + s_n^m \sin m\phi) P_n^m(\eta), \quad (1)$$

where P_n^m, Q_n^m are the associated Legendre functions of the first and second kind, respectively, with degree n and order m ; c_n^m, s_n^m are constant coefficients characterizing the magnetic field in the PSCS.

The scalar potential $U(\xi, \eta, \phi)$, presented in the PSCS (1), can also be considered in the CCS $U(x, y, z)$, expressing ξ, η, ϕ through x, y, z . To do this, it is necessary to solve equations respectively, with respect to ξ, η, ϕ .

We can find the projections of magnetic induction using the known relationship $B = -\text{grad}U$. Moreover, it should be borne in mind that when taking partial derivatives with respect to x, y, z , the function $U(x, y, z)$ should be perceived as a complex $U[(x, y, z), (x, y, z), (x, y)]$ and act in accordance with the differentiation of a complex function.

After simplification and grouping relative to functions $Q_n^m(\xi)$ and $P_n^m(\eta)$ to reduce calculation time, we obtain the final formula for B_x :

$$B_x(x, y, z) = -\frac{\mu_0}{4\pi} \sum_{n=1}^{\infty} \sum_{m=0}^n \left[\begin{aligned} & (m\phi'_x (s_n^m \cos m\phi - c_n^m \sin m\phi) - (n+1) \left(\frac{\eta'_x \eta}{\eta^2-1} + \frac{\xi'_x \xi}{\xi^2-1} \right) \times \\ & \times (c_n^m \cos m\phi + s_n^m \sin m\phi) (P_n^m(\eta) Q_n^m(\xi) + (n-m+1)) \times \\ & \times \left(\frac{\eta'_x}{\eta^2-1} P_{n+1}^m(\eta) Q_n^m(\xi) + \frac{\xi'_x}{\xi^2-1} P_n^m(\eta) Q_{n+1}^m(\xi) \right) \times \\ & \times (c_n^m \cos m\phi + s_n^m \sin m\phi) \end{aligned} \right] \quad (2)$$

Similarly, we obtain formulas for B_y, B_z (note that in the case of B_z the first term in the curly brackets

is zero, since $\phi'_z = 0$):

$$B_y(x, y, z) = -\frac{\mu_0}{4\pi} \sum_{n=1}^{\infty} \sum_{m=0}^n \left[\begin{aligned} & m\phi'_y (s_n^m \cos m\phi - c_n^m \sin m\phi) - \\ & - (n+1) \left(\frac{\eta'_y \eta}{\eta^2-1} + \frac{\xi'_y \xi}{\xi^2-1} \right) (c_n^m \cos m\phi + s_n^m \sin m\phi) \times \\ & \times P_n^m(\eta) Q_n^m(\xi) + (n-m+1) \times \\ & \times \left[\frac{\eta'_y}{\eta^2-1} P_{n+1}^m(\eta) Q_n^m(\xi) + \frac{\xi'_y}{\xi^2-1} P_n^m(\eta) Q_{n+1}^m(\xi) \right] \times \\ & \times (c_n^m \cos m\phi + s_n^m \sin m\phi) \end{aligned} \right] \quad (3)$$

Note, that all the formulas (2)–(3) given above are for the case when the technical object is extended along the z axis. However, a more familiar coordinate system is also often considered, when the technical object is extended along the x axis. If the technical object is extended along the x-axis, then the CCS must be rotated relative to the PSCS so that the x -axis takes the place of the z-axis, y takes the place of x, and z takes the place of y. In this case, the following replacement must be made in formulas (5)–(8): $x \rightarrow y; y \rightarrow z; z \rightarrow x$. And in the right-hand parts of formulas (2)–(3): $B_x \rightarrow B_y; B_y \rightarrow B_z; B_z \rightarrow B_x$.

3. Definition of forward magnetostatics problem for spherical sources

Let us consider analytical formulas for projections of magnetic induction in CCS using spherical harmonics. The relationship between the right triple of unit vectors x, y, z in CCS and triple r, θ, ϕ in SCS has form [1, 2].

The solution of the Laplace equation in the SCS with respect to the scalar potential of the MF for the region outside the sphere $r > R_0$, where the sources of this field are contained, has the form [1, 2]:

$$U = \frac{1}{4\pi} \sum_{n=1}^{\infty} \frac{1}{r^{n+1}} \sum_{m=0}^n (g_n^m \cos m\phi + h_n^m \sin m\phi) P_n^m(\cos\theta) \quad (4)$$

where g_n^m, h_n^m are constant coefficients characterizing the magnetic field in the SCS.

For simplicity, we write the relationship between x, y, z and $r, \cos\theta, \phi$, [1, 2].

Proceeding in a similar manner as for spheroidal coordinates, using connection $B = -\text{grad}U$, we write the x - projection B. After simplifying and grouping relative functions $P_n^m(\cos\theta)$ to reduce calculation time, we obtain the final formula for B_x :

$$B_x(x, y, z) = -\frac{\mu_0}{4\pi} \sum_{n=1}^{\infty} \frac{1}{r^{n+2}} \times \sum_{m=0}^n \left[\begin{aligned} & m\phi'_x r (h_n^m \cos m\phi - g_n^m \sin m\phi) - (n+1) \times \\ & \times \left(r'_x + \frac{(\cos\theta)'_x r \cos\theta}{\cos^2\theta-1} \right) (g_n^m \cos m\phi + h_n^m \sin m\phi) \times \\ & \times P_n^m(\cos\theta) + (n-m+1) \frac{(\cos\theta)'_x r}{\cos^2\theta-1} (g_n^m \cos m\phi + h_n^m \sin m\phi) \times P_{n+1}^m(\cos\theta) \end{aligned} \right] \quad (5)$$

Similarly, we obtain formulas for B_y, B_z (note that in the case of B_z the first term in the curly brackets is zero, since $\phi'_z = 0$):

$$B_y(x, y, z) = -\frac{\mu_0}{4\pi} \sum_{n=1}^{\infty} \frac{1}{r^{n+2}} \times \sum_{m=0}^n \left[\begin{aligned} & m\phi'_y r (h_n^m \cos m\phi - g_n^m \sin m\phi) - (n+1) \times \\ & \times \left(r'_y + \frac{(\cos\theta)'_y r \cos\theta}{\cos^2\theta-1} \right) (g_n^m \cos m\phi + h_n^m \sin m\phi) \times \\ & \times P_n^m(\cos\theta) + (n-m+1) \frac{(\cos\theta)'_y r}{\cos^2\theta-1} (g_n^m \cos m\phi + h_n^m \sin m\phi) \times P_{n+1}^m(\cos\theta) \end{aligned} \right] \quad (6)$$

$$B_z(x, y, z) = -\frac{\mu_0}{4\pi} \sum_{n=1}^{\infty} \frac{1}{r^{n+2}} \times \sum_{m=0}^n \left[\begin{aligned} & - (n+1) \left(r'_z + \frac{(\cos\theta)'_z r \cos\theta}{\cos^2\theta-1} \right) (g_n^m \cos m\phi + h_n^m \sin m\phi) \times \\ & \times P_n^m(\cos\theta) + (n-m+1) \frac{(\cos\theta)'_z r}{\cos^2\theta-1} (g_n^m \cos m\phi + h_n^m \sin m\phi) \times \\ & \times P_{n+1}^m(\cos\theta) \end{aligned} \right] \quad (7)$$

It is quite simple to calculate the magnetic field created by several, for example N_1 , spheroidal sources with coordinates x_i, y_i, z_i relative to the center of the technical object $x_0, y_0, z_0 = 0, 0, 0$ and several, for example N_2 , spherical sources that compensate for the magnetic field in a given area, with coordinates x_j, y_j, z_j relative to the center of the technical object. For this, we use the superposition principle and obtain, for example, for the projection

$$B_x^{result}(x_p, y_p, z_p) = \sum_{i=1}^{N_1} B_{xi}(x_p - x_i, y_p - y_i, z_p - z_i) + \sum_{j=1}^{N_2} B_{xj}(x_p - x_j, y_p - y_j, z_p - z_j), \quad (8)$$

where B_{xi} is calculated with its parameters i, c_{ni}^m, s_{ni}^m and B_{xj} calculated with its parameters g_{nj}^m, h_{nj}^m . The same is true for other projections.

Thus, using formulas (2)–(4) and (6)–(8) based on superposition principle, it is possible to calculate the magnetic field at an arbitrary point in the region outside the spherical and spheroidal sources. The advantage of these formulas over the known ones [1, 2] are: 1) the projections of the magnetic induction in the CCS are explicitly written due to taking direct derivatives with respect to the CCS coordinates; 2) their generalization to the case of the n-harmonic; 3) there is no need to transform from one coordinate system to another, which is especially important in the case of calculating the MF from several spherical and spheroidal sources; 4) the relative compactness of the formulas.

Verification of the correctness of formulas (2)–(4) and (6)–(8) was confirmed by comparison with the results obtained by taking numerical partial derivatives with respect to the coordinates x, y, z . Another verification was performed using COMSOL modeling of magnetic field of an ellipsoid of revolution. The COMSOL model allows specifying direction of ellipsoid magnetization, which made it possible to verify the correctness of formulas (1)–(2) for the case of first harmonics.

4. Definition of prediction geometric inverse magnetostatics problems

Prediction problem implies design of mathematical model of magnetic field of technical object based on experimentally measured values of magnetic field components, as a rule, in near zone of technical object [15, 16]. The vast majority of mathematical models of magnetic field of various technical objects — spacecraft, naval vessel and submarines — are multidipole models.

The main advantage of multidipole models is ease of calculating components of magnetic field generated by each magnetic dipole as a source of magnetic field in rectangular coordinate system connected to center of technical facility. The main disadvantage of multi-dipole models is large number of dipoles required to adequately simulate magnetic field of technical object to actually measured values of magnetic field on magnetodynamic stand. This is especially typical for modeling magnetic field of elongated energy-saturated technical objects.

A significant simplification of modeling magnetic field of elongated energy-saturated technical objects achieved by using elongated spheroidal magnetic field sources in prolate spheroidal coordinate system. Moreover, to obtain required adequacy of mathematical model to actually measured characteristics of magnetic field number of elongated spheroidal magnetic field sources may be required tens or even hundreds of times less compared to number of dipole magnetic field sources [20, 21, 22].

The obtained formulas (2)–(4) and (6)–(8) allow us to solve forward problem of magnetostatics. Using these formulas calculated components of magnetic field induction in CCS at any point in space generated by spheroidal and spherical sources of MF. Naturally, in this case, coordinates of spatial location of these MF sources and their harmonics are specified.

The convenience of using these formulas lies in fact that components of projections of resulting MF in CCS are equal to sums of corresponding projections of magnetic field induction of same in CCS, generated by all MF spheroidal and spherical sources.

Consider formulation of geometric inverse problem of design mathematical model of MF based on results of experimental measurements of magnetic field. Introduce vector G of uncertainties in magnetic characteristics of technical object, due to inaccurate knowledge of initial values of magnetic

characteristics of blocks of technical object, as well as changes in these magnetic characteristics in different operating modes [23, 24, 25]. Typically, magnetic field measurements are carried out in CCS associated with center of technical object. Let us introduce vector $YM(G)$ of measured magnetic field components [26, 27].

Consider design of mathematical model of elongated energy-saturated object in form of set of spheroidal magnetic field sources. Let us introduce vector X_P of desired parameters components of which are coordinates of spatial location and spatial harmonics of magnetic field of these spheroidal magnetic field sources. Then, vector $Y_C(X_P, G)$ calculated values of magnetic field calculated based on (2)–(4) and (6)–(8).

Then solution of predictions geometric inverse problem of magnetostatics reduced to solution of vector game [28, 29, 30].

5. Definition of control geometric inverse magnetostatics problems

The problem of controlling the magnetic silence of technical object is design of spatial arrangement and spatial harmonic sources of compensating magnetic field. With the help of these compensating magnetic field sources resulting MF of elongated energy-saturated technical object generated in such a way those stringent requirements for magnetic silence of energy-saturated technical object satisfied.

Note that the requirements for magnetic silence of technical object are usually imposed in the far zone. In particular, for military ships and submarines, requirements are imposed on magnitude and rate of change of magnetic field components at control depth when an object moves at given speed.

The designed predictive mathematical model of magnetic field of elongated energy-saturated object calculated based on experimental measurements of magnetic field in near zone. Based on this prediction model of magnetic field in near zone values of characteristics of magnetic field of technical object calculated in far zone, which limited to meet requirements of magnetic silence of technical object.

Introduced uncertainty vector GC of magnetic characteristics of energy-saturated technical object [8, 9]. Then, based on the designed predictive model vector $B(G)$ of initial values of magnetic characteristics of technical object, which determines its magnetic silence calculated.

To compensated original magnetic field of technical object introduced dipole sources of compensating magnetic field. Introduced vector XC of required parameters for solving control geometric inverse problem of magnetostatics components of which are coordinates of spatial location and spatial harmonics of compensating dipoles.

Then vector $B_C(X_C, G)$ of calculated characteristics of magnetic silence of technical object calculated based on solution of forward problem of magnetostatics (6)–(8) for spherical magnetic field sources. Then solution of control geometric inverse problem of magnetostatics reduced to solution of vector game [29, 30, 31]:

$$B_R(X_C, G_C) = B(G_C) + B_C(X_C) \quad (9)$$

To calculate payoff vector game (11) it is necessary to repeatedly solved forward problem of magnetostatics (6)–(8) for spherical magnetic field sources.

6. Inverse magnetostatics problems solution method

Solutions of both vector games (10) and (11) calculated by particle multi-swarm nonlinear optimization algorithms [28, 29]. Number of swarms calculated by number of components in vectors games (10)–(11), so that with help of each swarm solution of scalar game calculated.

Each swarm j contained two types particles i . Position $x_{ij}(t)$ and movement velocity $v_{ij}(t)$ for first type particles calculated from conditions of minimizing payoff game along vectors X_P and X_C of

desired parameters and described by following expressions

$$v_{ij}(t+1) = w_{1j}v_{ij}(t) + c_{1j}r_{1j}(t) \times H(p_{1ij}(t) - \varepsilon_{1ij}(t)) [y_{ij}(t) - \dots - x_{ij}(t)] + c_{2j}r_{2j}(t) H(p_{2ij}(t) - \varepsilon_{2ij}(t)) [y_j^*(t) - x_{ij}(t)] \quad (10)$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \quad (11)$$

Moreover, the best local $y_{ij}(t)$ and global $y_j^*(t)$ position of particle determined from condition of minimizing game vector along vectors X_P and XC of desired parameters for games (10) and (11) respectively.

Position $g_{ij}(t)$ and movement velocity $u_{ij}(t)$, $z_{ij}(t)$ for second type particles calculated from conditions of minimizing payoff game along vectors G and GC of magnetic characteristics uncertainty and described by following expressions

$$u_{ij}(t+1) = w_{2j}u_{ij}(t) + c_{3j}r_{3j}(t) H(p_{3ij}(t) - \varepsilon_{3ij}(t)) [z_{ij}(t) - \delta_{ij}(t)] + \dots + c_{4j}r_{4j}(t) H(p_{4ij}(t) - \varepsilon_{4ij}(t)) [z_j^*(t) - \delta_{ij}(t)] \quad (12)$$

$$g_{ij}(t+1) = \delta_{ij}(t) + u_{ij}(t+1) \quad (13)$$

Moreover, the best local $z_{ij}(t)$ and global $z_j^*(t)$ position of particle calculated from condition of minimizing game vector along vectors G and GC of magnetic characteristics uncertainty for games (10) and (11) respectively.

To narrow Pareto set of optimal solutions in (10)–(11) binary preference relations of local games used [30].

7. Simulation results

Let us consider the results of magnetic field modeling of elongated energy-saturated technical object 200 m long and 40 m wide, for which the magnetic silence requirements are set at a control depth of 19 m and 60 m. The initial magnetic field was modeled using 16 dipole sources of the technical object's magnetic field, the measurement of which was performed at 909 points.

For this example, we will consider checking the correctness and efficiency of applying formulas (2)–(4) and (6)–(8). We will check the correctness and efficiency of the formulas on the values of spherical and spheroidal harmonics obtained as a result of optimization.

These values were obtained on the basis of solving the prediction of the geometric inverse problem of magneto static's by minimizing the sum of the squares of the differences in the projections of the real magnetic field and the magnetic field models: for spheroidal (2)–(4) and spherical (6)–(8) MF sources up to and including the third harmonics.

The results of calculating the signatures of initial MF (solid lines) with models based on spheroidal (dotted lines) and spherical (dash-dotted lines) harmonics for projections B_x – red, B_y – green, B_z – blue are shown in Figure 2 – Figure 4, respectively, for three cases: $Y = -20$ m, $Y = 0$ m and $Y = 20$ m. Since the technical object is extended, the magnetic field model based on spheroidal harmonics gives better results in approximating the original magnetic field.

The correctness of formulas (6)–(8) for spherical harmonics verified by comparing them with the results obtained by taking numerical partial derivatives with respect to the coordinates x , y , z . The results of these calculations are practically identical.

In the case of spheroidal harmonics, the difference in the calculation speed between the analytical and numerical methods is not as pronounced as for spherical ones, but it still takes place – by 4 or more times. Moreover, in all cases, with an increase in the order of the harmonic, this difference only increases. Let us now proceed to checking the correctness of formulas (2)–(4).

The results of comparing the calculation using these formulas with the results calculated by taking numerical partial derivatives with respect to the coordinates x , y , z are also consistent and differ by a deviation of about $10 \div 11$ nT.

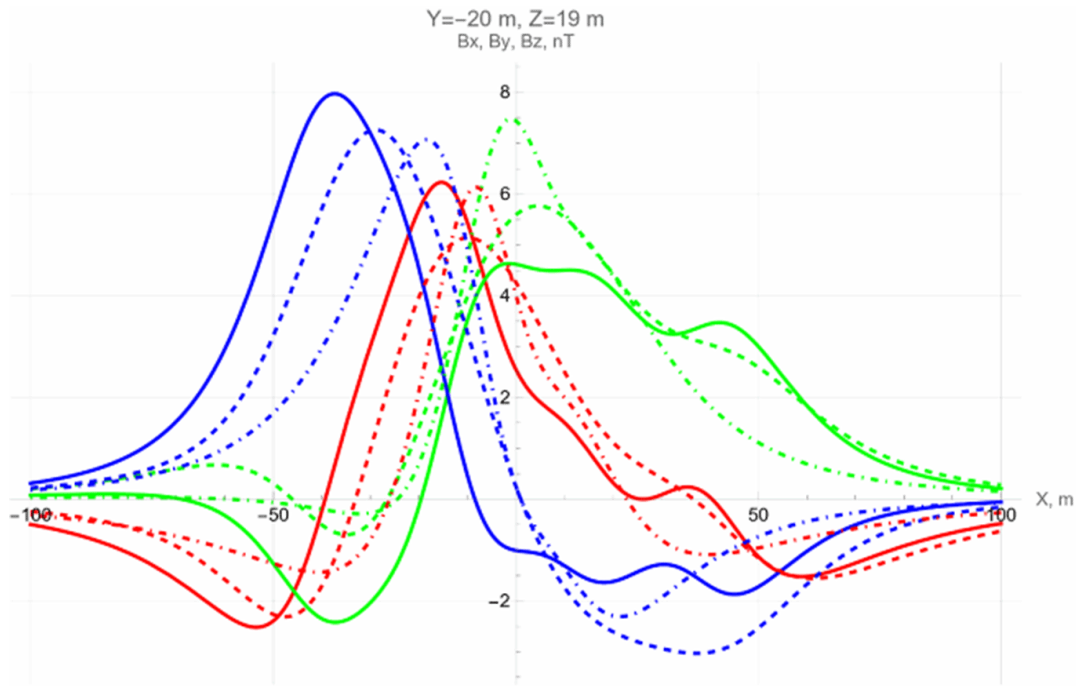


Figure 2: Magnetic signatures of original, model spherical and model spheroidal magnetic fields for $Y = -20$ m.

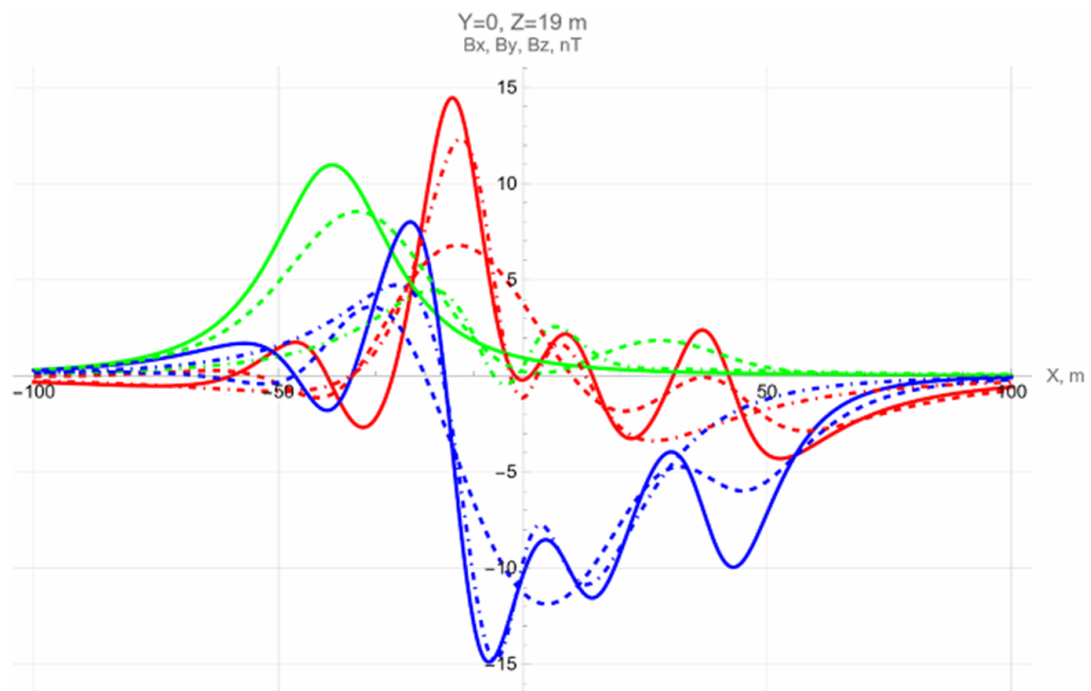


Figure 3: Magnetic signatures of original, model spherical and model spheroidal magnetic fields for $Y = 0$ m.

Another way of checking is to consider the magnetic field of a dipole and a spheroid (the case of the 1st harmonic) with an equivalent magnetic moment at large distances from the source.

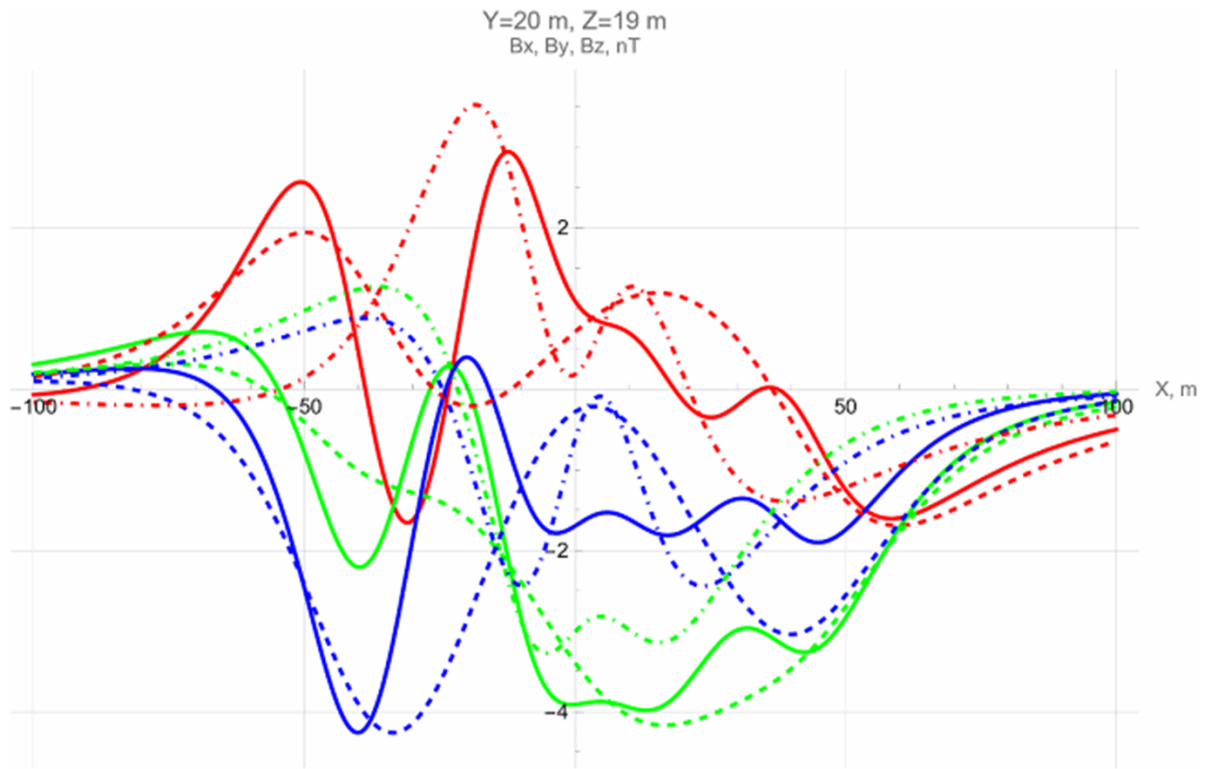


Figure 4: Magnetic signatures of original, model spherical and model spheroidal magnetic fields for $Y = 20$ m.

8. Conclusions

Computational technology for prediction and control by uncertain extended energy- saturated object developed based on development and application of method of analytical calculation of induction of magnetostatics fields of spherical and spheroidal sources in the Cartesian coordinate system based on near-field measurements. Unlike known methods developed method allows calculated magnetic field directly in Cartesian coordinate system without finding magnetic induction projection in prolate spheroidal coordinate system and spherical coordinate system and without their translation from prolate spheroidal coordinate system and spherical coordinate system in Cartesian coordinate system and vice versa.

Forward problems of magnetostatics solved based on developed method of analytical calculation of magnetostatics field induction of spherical and spheroidal sources in Cartesian coordinate system based on near-field measurements. Geometric inverse problems of magnetostatics for solving prediction and control problems of magnetic silence of technical object calculated based on vector games solution. Both vector games payoff calculated as forward problems solutions Wolfram Mathematica software package used.

The possibility of a more than 10 times calculation time reduction of magnetic field induction of magnetic field elongated spheroidal sources and the possibility of a more than 4 times calculation time reduction of magnetic field induction of magnetic field spherical sources when magnetic field calculating of uncertain extended energy-saturated object based on development and application of analytical calculation method of magnetostatics field induction of spherical and spheroidal sources in the Cartesian coordinate system shown based on near-field measurements.

Declaration on Generative AI

The author(s) have not employed any Generative AI tools.

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