Development of a model selection tool based on numerical set analysis for rehabilitation using robotic technologies

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Abstract

Decision-making systems are considered in which the task of selecting a model arises as a tool for significant information compression. The problem of substantiating and creating quantitative artificial intelligence (AI) tools for automatic model selection preceding approximation is analyzed. The sequence of actions of the process of evaluating mutually related data sets is formed, which is brought to co-dimensional conditions that are determined by the accuracy class of the devices, the measurement method, and the method of reducing models to a single form. The generalized model reduces alternative models of the selection set to a single form. Analytically determined characteristic constants allow us to study, compare them between groups, and classify the dynamics of assessing reliability and adequacy as integral indicators before the approximation begins. This tool for analyzing the suitability of the model for description, which is based only on the results of data from related numerical sets, is an AI tool. An analytically quantitative assessment of the interval of existence of the permissible value of characteristic constants is presented. The relationship between the magnitude of their boundaries for the model within the permissible values of the relative error suitable and unsuitable for description based on the properties of quadratic norms, which determines the maximum possible error, has been established. The formed toolkit for quantitative proof of the best membership of the model type was analyzed for the description of experimental data. The examples show that according to a set of definitions, the deviations of the characteristic constants are synchronous with the relative error and adequacy of the model. The high resolution of the adequacy indicator of 10^{-3} and the range, which is more than 40-60 times the max/min ratio, will be useful for program algorithms as an indicator.

Keywords

robotic rehabilitation, approximation, selection tool, best model type, characteristic constants, permissible interval, reliability, adequac

1. Introduction

The problem of creating effective decision support systems and automated control systems is associated with the need to develop large databases, bases of models, and algorithms. The latter significantly exacerbates the need for automatic model construction and expert assessment using a single integral indicator [1]. The urgent need to process large amounts of data has also initiated the search for new means of using AI in particular long-term medical monitoring [2]. The application and review of the course of the naive Bayesian method and cluster analysis methods DBSCAN, PCA, k-means, identified the main advantages of using an ensemble of methods for generalizing large amounts of data in space of states [3]. Such works may not have ended the discussion about the belonging to the categories of AI tools of a scientifically based analysis algorithm that is automatically implemented and synthesizes a conclusion without operator participation.

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However, they stimulated the search for indicators that, like the concepts of adequacy and distance between clusters, define boundaries in a jump-like manner, and therefore are analytical tools for recording such transitions [4]. Their further development is to increase the efficiency of real-time data compression and transmission based on the automatic creation of suitable models [4]. Their further development to increase the efficiency of model selection only according to the data of related numerical sets becomes relevant in the problems of expressing a big volume of data. The particular importance is the problem of automatic approximation for automatic compression of a large amount of data of monitoring systems or long-term remote recovery, which does not occur in the conditions of the direct presence of a doctor [5].

2. Analysis of literature data and problem statement

In works published in the last decade, machine learning methods have increasingly been used to find key clusters [2]. Their results are one of the examples that demonstrate how to effectively collect and study large volumes of medical data on individual patient characteristics [2]. However, from a general theoretical point of view, their application allows us to determine the distances between cluster instances. The latter forms a quantitative basis for effective qualitative clustering [2]. In addition, they can also serve as an example of finding tools for isolating the boundary of dividing sets into clusters [2].

The results of the study of the applicability of AI methods to the identification of patient medical conditions are presented in [3]. Of particular importance is the study of the transition period between conditions using Data Mining methods [3]. The experience of classifying patient medical conditions based on the results of laboratory and other diagnostic studies demonstrates the advantages of naive Bayesian and cluster analysis methods, in particular DBSCAN, PCA, k-means [3]. The work is an example of the isolation of common features and operations as a set, which is a common part for different methods. With a special generalization of its proposals, it will be clearly classified as a tool for AI analysis in the future. The marginal difference as a jump-like variability of a feature is presented as one of the possible and simpler features of choosing the boundaries of a cluster [4]. This work confirms the transitions from one cluster to another by jumping known complex features. It demonstrates a new AI approach to analysis and selection for pre-formed simple features [4]. The list of features, conditions, operations and procedures for automatically forming unambiguous conclusions is obviously a tool for the fields of application of AI. Their positive experience becomes the basis for the further creation of special AI tools. In addition, there is an increasing need for a well-founded preparation of a decision on the marginal difference, which is proposed to be implemented on the basis of sequential measurement at four points [5]. The idea of sequential calculation of the first and second derivatives at three points forms the basis for registering the dynamics of changes in a medical indicator and derivatives of the second and third orders in the presence of four points. However, to verify the conclusion about the dynamics of changes, a comparative analysis is carried out at the fifth point. The correspondence of the predicted value to the measured one is established in [5], which serves as the basis for data compression. It is especially relevant in the tasks of classification of states and data compression in restorative medicine [5]. Such an intellectual analysis procedure expands the functional capabilities of the system through the use of three-level comparative analysis [6]. The need for such analysis and its applicability to data compression due to the structure, including a recurrent analytical model, was demonstrated in [6]. However, its applicability to variable and discontinuous processes is complex and requires correction.

In this regard, an analysis of the works was conducted, most of which are devoted to the search for general approaches to the choice of the type of model and uniform approximation with the conditions that are basis-forming [7]. Their authors focus on substantiating and studying the regularities of approximation of one of the types of models [7]. Thus, to ensure uniform approximation, the possibilities of a piecewise-continuous approach to approximation by splines are investigated [8]. In recent works, using different concepts of measure and methods of quantitative measurement: metrics, a review and assessment of many years of experience have been carried out. Thus, in work [9], types of metrics and distances in the hierarchy of categorical semantics and functions, which are key in mathematics, including approximation theory, are considered. The work presents applications to practical approximations of functions and to the theory of graphs in general and trees in particular [9]. The effectiveness of approximation of functions that are difficult to calculate in conditions of limited resources of single-board computers arises with the development of monitoring and recovery systems. For them, it is necessary to choose simple and analytical expressions that will not be inferior in accuracy to spline approximation [10]. This work also considers and proves the uniqueness of the best approximation by generalized polynomials. No less important are the proposals using approximation to simplify operator and nonlinear differential equations [11]. Its author demonstrates the applicability of approximation to the solution of differential and integral equations by using approximation for the iterative-approximate solution of equations with analytical conditions [11]. Another example from a series of works that investigate the possibilities of approximating the combination of the sum of a polynomial and an exponent is work [12]. The author proves that a sufficient condition for the existence of a uniform approximation by the sum of a polynomial and an exponent is the continuity of the function and the boundedness of its derivative at the beginning and end of the definition interval [12]. The order of the polynomial and the unknown constants in it and two additional constants (the multiplier at the exponent and the constant in the exponent) provide the best uniform approximation of the function. Such a model with an accurate reproduction of its value at the extreme points of the segment is suitable for constructing continuous minimax spline approximations [12]. However, as the analysis of works [10–12] shows, the issue of choosing and proving the best suitability of the model for approximation when considering several types of models is not even raised in them.

The application of approximation for the construction of models in dynamic programming problems or for problems in which the models are given by nonlinear differential equations was made in work [13]. The fundamentality of the idea of piecewise linear approximation, regardless of the time of its emergence, which was highlighted in it, gives its positive results for the formation of models in the form of a convergent sequence. However, the application of these results to the development of numerical methods has slowed down the development and implementation of analytical models, including empirical ones.

Examples of further development are piecewise quadratic and piecewise cubic recurrence approximations, which have expanded the applicability of approximation to the structure of nonlinear models in the form of analytical solutions of nonlinear algebraic equations and systems and nonlinear differential equations and systems and recurrent networks [14]. The obtained recurrence models have analytical forms and allow fast calculation, which makes them especially attractive for solving applied problems in complex nonlinear systems [14]. However, despite these results, the question of the evidential choice of the model type was not raised and therefore the choice among the types of approximation was not made at the first stage. To analyze the quality of models using multi-criteria assessments, for example, such as adequacy, it is necessary to carry out an approximation and determine its constants. The latter, according to the results of using several types of approximation for the analysis of alternatives, significantly increases the complexity of solving model selection problems [14].

A review of the works, the results of which are presented, allows us to state that uncertainty, as a feature of the structure of model types, is increasingly dominant [15]. One example of the implementation of systems whose models are described by fuzzy sets is a system for supporting and making decisions in automated process control in marine technologies. The experience of such a system will serve as an alternative prototype for further developments in which AI tools and means of simplified representation of complex models are used [15]. However, the lack of analyticity of such models hinders their further spread.

The implementation of the concept of creating a modular cyber-physical system will be accompanied by a rapid growth in the volume of data transmission and storage [16]. As the authors argue, early diagnostics will become a priority in the operation of industrial equipment based on Industry 4.0 standards and should be included in the algorithm of its operation as an integral structural element. In particular, it is expected that Internet of Thinqs methodologies will also play their role in the formation of new requirements for compression and will contribute their positive role in the process of information protection [16]. As a stable trend, the need for the use of neural-fuzzy browsers [17] is observed. Their development will require the formation of new forms of fuzzy model structures and the creation of computer libraries for operations with asymmetric membership functions [17].

Theoretically important are the results of work [18], which proved theorems on the estimation of the approximation of polynomial approximations for segment-integrable functions.

The result of the work is analytical expressions for the estimates of deviations for approximations of a function and its derivatives [18].

Analysis of recent works on approximation shows that its application has been extended to solutions of nonlinear differential equations by mixed finite-element approximation of solutions in Hilbert space, in particular for the completely nonlinear Hamilton-Jacobi-Bellman equation [19]. The paper proposes to use indirect means of informing about the error values exceeding the limits during the solution, which is a sign for grid correction [19]. However, the indicator used has practical significance for grid selection, but does not assess the adequacy of the model creation result. A priori and a posteriori limits on the approximation error are proven. Contributions from the a posteriori error estimator can be used as refinement indicators in the adaptive grid correction algorithm. The convergence of this procedure is proven and empirically investigated in numerical experiments [19].

Assessment of adequacy through statistical analysis of the stability of achieving a specific goal as a mandatory step at the stage of its development is presented in [20]. The process of forecasting using a model and recognizing errors is presented as a set of tests in the course of developing more reliable and accurate models. The paper argues that assessment of the adequacy of models is possible only by combining several statistical tests and a proper study of the achieved goals [20]. One of the new modern views on the development of the method and software and tool is presented by the work that artificially generates biomedical image databases [21]. Working with such a database also requires evaluation and compression, for which, as shown in [22], it is provided by methods of comparative analysis and fuzzy sets together with adequacy assessment. The latest CNN Stacking Model for medical image classification and medical visualization of anatomical organs on medical images is presented in [23]. It is expected that the implementation of the latest results [24] together with the use of parallel algorithms and interpolation tools using Bezier curves and B-splines for the restoration of medical data, including fragmentarily lost ones. All these works taken together [21-24] emphasize the relevance of the need and importance of the use of approximation for the representation, compression of biomedical information, including for healthcare communication technologies. A special role will obviously be played by methods and tools of hybrid classification based on the idea of Two ML classifiers and prediction [25].

Thus, the latest results in the above works investigate and substantiate the properties and possibilities of forming individual types of models. However, as can be seen from the analysis, the existing approximation methodology does not contain tools for evidential selection of the best model based on the data sets formed during the experiment.

In this regard, the main unsolved problem is the justification and creation of quantitative AI tools that are used to select a model before the approximation stage begins.

3. Purpose and objectives of the study

The purpose of the work is to substantiate and build quantitative AI tools for the analysis of connected numerical sets, which will provide an evidential choice of the type of model for the available experimental data before the approximation stages. This will make it possible to reduce the complexity and time required to create a model by carrying out approximation and quality assessment only for one model that best describes the numerical data of connected sets.

To achieve this goal, the following tasks were formulated:

– to form a sequence of actions for estimating the interval of permissible values of a vector function for determining integral indicators;

– to reduce alternative models to linear forms with characteristic constants as functions of the index - the number of the element in the array (hereinafter, the characteristic constant will be called the constant whose deviation most affects the model error);

– to form intervals of existence of permissible values, which are determined by the methodology of direct and indirect measurement;

– to form tools for quantitatively proving the best fit of the data description model before starting the approximation.

4. Materials and methods of research

The general task of the research is to increase the efficiency of automatic decision-making in management tasks while ensuring the given reliability and adequacy. The objects of the research are decision-making support systems. The subject of the research is the decision-making process in model selection problems using the method of determining their adequacy. The hypotheses of the research are: the existence of a limiting interval of permissible values of integral indicators; the existence of invariant properties, which is established by the combined application of qualitative provisions of the theory of similarity and quantitative analysis tools. First of all, models according to the theory of similarity are reduced to co-measured conditions, using the coordinate rectification method, and statistical methods determine the parameters of the samples, which determine the permissible intervals using discrete quadratic norms. For an arbitrary identifier and several neighboring points of the definition set, characteristic constants are analytically found, i.e. those that affect the increment of the function when the arguments change. The application of functional analysis methods, operator expansion, quadratic norms and Bunyakovsky-Cauchy inequalities sets boundaries for characteristic constants and indicator estimates. The latter forms a closed system of actions for automatically proving the best fit of the data description model before the approximation begins, which makes it possible to increase the efficiency of decision-making. For modeling, the Python 3.12.0 programming language was used to build statistical data processing programs and determine characteristic constants, the Microsoft Excel 2007 environment, Math-Cad 14 (USA). The task that was solved during the modeling was to evaluate and compare examples of models based on the results of approximation. Also, for examples, preceding the approximation as such, using the developed sequence of actions, to study the synchronicity of changes in indicators, the quality of the model was investigated: relative error and adequacy. For technical support of the study, a DESKTOP based on the Intel(R) Core(TM) i7-4600U CPU processor was used in the research process. The study is based on analytical expressions for calculating the general integral indicator of the reliability and adequacy of the model according to certain criteria [1] for assessing the quality of the model. In addition, it is based on conclusions about the need to develop reliable and unambiguous criteria for separating classes [2, 3], assessments of the success of works that developed approximation approaches taking into account the analysis of literary sources [7-13]. The results of the search for automatic decision-making algorithms, with their examples, stimulate the search for new AI tools [6, 15-24].

5. Results of the formation of a sequence of actions that will provide an evidential choice of the type and assessment of the model for approximation

5. 1. Formation of a sequence of actions for estimating the interval of admissible values of a vector function for determining integral indicators

The process of forming an empirical model was considered. Thus, at the stage when the experiment was conducted and its statistical processing was carried out, the task of selecting and proving the best suitability of the model among the alternatives for the description was set. Let us assume that for each value of the identifier *I*, are given m values of the components of the vector $\overline{x}$, as a known set in the form of an m + 1 - dimensional array. Further, specifying *N* groups of results of measurements we will denote the vector $\overline{x}\left[I\right]$ and present it in the form of an array:

$$x\left[I,j\right], I=\overline{1,N}, j=\overline{1,m}\Big\}. \quad (1)$$

We also introduce and denote l the component vector function $\overline{Y}\left[I\right]$, as a set of its values also in the form of a l+1-dimensional array of index - identifier *I*. Let us assume that the second index - identifier *j* of these two arrays and their boundaries will differ depending on the number of component vectors $\overline{X}$ and $\overline{Y}$. Since, as a rule, measurements of physical quantities are made directly and indirectly according to the laws of $\Phi(x[I,j])$ for the components of vectors $\overline{X}$, the measurement value for the vector component of the vector-function $\overline{Y}$ is given as an array:

$$Y[I,j] = \Phi(x[I,j]). \tag{2}$$

The results of statistical processing were then assumed to be known and uniquely specified. The results of the experiment as mathematical expectations, the mean square errors of the components of the vectors for each value of the identifier I of the quantities $\overline{X}$ and $\overline{Y}$ were represented by arrays. The suitability of the normal law for describing the probability distribution density was also established. According to the data of the technical passports of the devices and the method of indirect measurement (2), the maximum possible errors were found for each pair of values of the function and argument. The maximum possible error will be estimated by the differentiation method and according to the data of the accuracy classes of the devices and statistical estimates of direct measurements by the quadratic norm:

$$\Delta Y[I,j] = \sqrt{\sum_{j=1}^{m}\left[\frac{\partial \Phi(x[I,j])^{*}}{\partial x_j}\Delta x_j\right]^2}. \tag{3}$$

*Here and further, we will denote the value of the partial derivative $\frac{\partial \Phi(x[I,j])}{\partial x_j}$ at the point $x[I,j]$, and Δx_j will be considered the maximum possible instrumental error as that determined by the accuracy class of the device. Based on such an estimate, for each element of the set $I = \overline{1,N}, j = \overline{1,m}$, it is possible to estimate the interval of values, which is determined by the estimate of the maximum possible error in the form:

$$Y[I,j] = Y[I,j]_{me} \pm \Delta Y[I,j],$$

or:

$$\left[Y[I,j]_{me} - \Delta Y[I,j], Y[I,j]_{me} + \Delta Y[I,j]\right]. \tag{4}$$

Thus, the sequence of actions brings interconnected data sets to the conditions of co-measurableness, by generalizing the definition space as a vector set (1) and by sets of values generalizes the vector of functions as sets (4). The designated interval (4) defines the space and its boundaries in which the possible values of the vector function $\overline{Y}[I]$ change and is the starting point for assessing the reliability and adequacy as an integral indicator.

5. 2. Reduction of alternative models to linear forms with characteristic constants by functions of the identifier

Suppose that the set of models considered as alternatives $F_q(\overline{x}, a_{qk})$ is selected and denoted by a set of numbers K:

$$F_q(\bar{x}, a_{qk}), x[I, j],\ q = \overline{1,Q},\ k = \overline{1,K}. \tag{5}$$

To bring each of the models of the set (5) to the conditions of codimensional analysis, we will use the coordinate straightening method. The justification and further presentation of the main idea of the article will be carried out for a linear form, which we will represent by a vector function, which is represented by a matrix-vector product:

$$\bar{Y}[I,l] = \lceil A_{l,j} \rceil \bar{x}[I,j] + [B_l]. \tag{6}$$

The component of this vector function contains unknown constants of type $A_{l,j}$ and B_l and will take the form for next I+1 point:

$$\bar{Y}[I+1,l] = \sum_{j=1}^{m} A_{l,j} x[I+1,j] + B_l. \tag{7}$$

If we consider sequentially for each value of the array identifier-index two neighboring points I and I+1, then the number of unknown constants will decrease for their difference:

$$\bar{Y}[I+1,l] - \bar{Y}[I,l] = \sum_{j=1}^{m} A_{l,j} \{x[I+1,j] - x[I,j]\}. \tag{8}$$

However, m unknown constants Aij for each l will need to be determined, for example by the elimination method. To ensure the necessary condition for their determination, it is necessary to supplement equation (5) with m-1 more equations. The latter is possible under the conditions that N is greater than m or equal to the number of components of the vector $\bar{X}$:

$$\begin{cases} \{\bar{Y}[I+3,l] - \bar{Y}[I+2,l]\} = \sum_{j=1}^{3} A_{l,j} \{x[I+3,j] - x[I+2,j]\}; \\ \{\bar{Y}[I+2,l] - \bar{Y}[I+1,l]\} = \sum_{j=1}^{3} A_{l,j} \{x[I+2,j] - x[I+1,j]\}; \\ \bar{Y}[I+1,l] - \bar{Y}[I,l] = \sum_{j=1}^{3} A_{l,j} \{x[I+1,j] - x[I,j]\}. \end{cases} \tag{9}$$

Let us introduce the notation of the coefficient values:

$$\left. \begin{aligned} C_{I+3,I+2,j} &= x[I+3,j] - x[I+2,j]; \\ C_{I+2,I+1,j} &= x[I+2,j] - x[I+1,j]; \\ C_{I+1,I,j} &= x[I+1,j] - x[I,j]. \end{aligned} \right\} \tag{10}$$

Then system (9) by brute force I will allow us to compose a system of m equations of type (5):

$$\begin{cases} \{\bar{Y}[I+3,l] - \bar{Y}[I+2,l]\} = \sum_{j=1}^{3} A_{l,j} C_{I+3,I+2,j}; \\ \{\bar{Y}[I+2,l] - \bar{Y}[I+1,l]\} = \sum_{j=1}^{3} A_{l,j} C_{I+2,I+1,j}; \\ \bar{Y}[I+1,l] - \bar{Y}[I,l] = \sum_{j=1}^{3} A_{l,j} C_{I+1,I,j}. \end{cases} \tag{11}$$

System (11) has a unique solution if there is at least one inhomogeneous equation among the equations. Its simplification by the elimination method step by step reduces the number of unknowns. In the case of three components of the vector $\bar{X}$, after the first step, system (11) will take the form:

$$\begin{cases} D_{1,I,l} = \sum_{j=1}^{2} L_{1,I,j} A_{l,j}; \\ D_{2,I,l} = \sum_{j=1}^{2} L_{2,I,j} A_{l,j}. \end{cases} \quad (12)$$

if we apply the notation of constants (10):

$$D_{1,I,l} = C_{I+2,I+1,3}\left\{\overline{Y}\left[I+3,l\right] - \overline{Y}\left[I+2,l\right]\right\} - C_{I+3,I+2,3}\left\{\overline{Y}\left[I+2,l\right] - \overline{Y}\left[I+1,l\right]\right\};$$
$$D_{2,I,l} = C_{I+1,I,3}\left\{\overline{Y}\left[I+2,l\right] - \overline{Y}\left[I+1,l\right]\right\} - C_{I+2,I+1,3}\left\{\overline{Y}\left[I+1,l\right] - \overline{Y}\left[I,l\right]\right\};$$
$$L_{1,I,j} = C_{I+2,I+1,3} C_{I+3,I+2,j} - C_{I+3,I+2,3} C_{I+2,I+1,j};$$
$$L_{2,I,j} = C_{I+1,I,3} C_{I+2,I+1,j} - C_{I+2,I+1,3} C_{I+1,I,j}.$$

Its solution will be represented by an expression for calculating the characteristic constants:

$$A_{l,1} = \frac{D_{1,I,l} L_{2,I,2} - D_{2,I,l} L_{1,I,2}}{L_{2,I,2} L_{1,I,1} - L_{1,I,2} L_{2,I,1}}. \quad (13)$$

Thus, we consider alternatives of models for suitability for description of connected sets. Suppose that they are reduced to linear form (3), and constants *Aij* are found as functions of identifier *I* for a group of neighboring points. Then the properties of models can be studied, compared and classified between groups before the approximation in comparable conditions by the nature of their changes. This tool for analyzing the suitability of a model for description, based only on the results of the analysis of connected numerical sets, will obviously be a tool of artificial intelligence.

5. 3. Formation of the interval of existence of permissible values, determined by the methodology of direct and indirect measurement

To demonstrate the capabilities of the artificial intelligence tool as the existence of properties of visual analysis, condition (5) was analyzed. Thus, an arbitrary model q from the set of sets of a limited number K, which is represented by the operator $F_q(\overline{x}, a_{qk})$, was considered.

Let us assume that the operator is continuous and differentiable, which acts in the set of definition of the vector $\overline{x}$ and the values of the constants a_{qk} are defined. They are calculated according to

the data from the set (1) for a set of four values of identifiers *I*, that is, for an arbitrary value *I* and three following according to the algorithm (13). For this set, they will be a single and invariable set. However, if this model is ideal, then they will not be variable. If the operator does not give an ideal description, then the constants thus defined for two neighboring values of an arbitrary identifier *I* have deviations of the values Δa_{qk}. Thus, expanding the operator into a series taking into account the deviations Δx_j, the components of the vector $\overline{x}$ we write:

$$F_q(\overline{x}) = F_q(\overline{x})\Big|_{\substack{x_j = x_{ij}, \\ a_{qk} = a_{qki}}} + \sum_{j=1}^{m} \left(\frac{\partial F_q}{\partial x_j}\Bigg|_{\substack{x_j = x_{ij}, \\ a_{qk} = a_{qki}}} \Delta x_j \right). \quad (14)$$

We also present (14) using local absolute $\Delta F_q(\overline{x})\Big|_{\substack{x_j=x_{ij},\\ a_{qk}=a_{qki}}}$ and relative errors $\mathcal{E}$. Such an expansion is the basis for calculating the absolute error:

$$\Delta F_q(\overline{x})\Big|_{\substack{x_j=x_{ij},\\ a_{qk}=a_{qki}}} = \sum_{j=1}^{m}\left(\frac{\partial F_q}{\partial x_j}\Big|_{\substack{x_j=x_{ij},\\ a_{qk}=a_{qki}}}\Delta x_j\right)+\sum_{k=1}^{K}\left(\frac{\partial F_q}{\partial a_{qk}}\Big|_{\substack{x_j=x_{ij},\\ a_{qk}=a_{qki}}}\Delta a_{qk}\right). \tag{15}$$

However, the relative error is more informative:

$$\varepsilon=\frac{1}{F_q(\overline{x})\Big|_{\substack{x_j=x_{ij},\\ a_{qk}=a_{qki}}}}\left[\sum_{j=1}^{m}\left(\frac{\partial F_q}{\partial x_j}\Big|_{\substack{x_j=x_{ij},\\ a_{qk}=a_{qki}}}\Delta x_j\right)+\sum_{k=1}^{K}\left(\frac{\partial F_q}{\partial a_{qk}}\Big|_{\substack{x_j=x_{ij},\\ a_{qk}=a_{qki}}}\Delta a_{qk}\right)\right]. \tag{16}$$

Under these conditions, we write a system of equations for l x N constants for elements of a set of type (2) for N values of the vector $\overline{x}\left[I\right]$. Suppose that its solution is found and represented as a set of characteristic constants a_{qk} for each identifier I. Further analysis of the variational properties of such a set will contain information about the suitability of the model for describing by the chosen form q for the set of values of the connected sets $\overline{x}\left[I\right]$ and $\overline{Y}\left[I\right]$ in comparable conditions (6). At the same time, such variability can be estimated by the dynamics of changes in the function of the dependence constants on the identifier I and by the requirements imposed on the permissible value of the maximum possible relative error.

5.4. Formation of a tool for quantitatively proving the best fit of the data description model before the approximation begins

Direct application of the discrete quadratic norm (3), to condition (13) taking into account the properties of the norm of the sum of functions gives:

$$\begin{gathered}\frac{1}{F_q(\overline{x})\Big|_{x_{ij},a_{qki}\max}}\left[\left\|\sum_{j=1}^{m}\left(\frac{\partial F_q}{\partial x_j}\Big|_{\substack{x_j=x_{ij},\\ a_{qk}=a_{qki}}}\Delta x_j\right)\right\|+\left\|\sum_{k=1}^{K}\left(\frac{\partial F_q}{\partial a_{qk}}\Big|_{\substack{x_j=x_{ij},\\ a_{qk}=a_{qki}}}\Delta a_{qk}\right)\right\|\right]\le\\ \le\|\varepsilon\|\le\frac{1}{F_q(\overline{x})\Big|_{x_{ij},a_{qki}\min}}\left[\left\|\sum_{j=1}^{m}\left(\frac{\partial F_q}{\partial x_j}\Big|_{\substack{x_j=x_{ij},\\ a_{qk}=a_{qki}}}\Delta x_j\right)\right\|+\left\|\sum_{k=1}^{K}\left(\frac{\partial F_q}{\partial a_{qk}}\Big|_{\substack{x_j=x_{ij},\\ a_{qk}=a_{qki}}}\Delta a_{qk}\right)\right\|\right].\end{gathered} \tag{17}$$

The latter proves that if the permissible value of the relative maximum possible error is determined, then:

- the boundary between two sets of suitable and unsuitable for description from the sets of model forms selected for consideration:

$$\begin{gathered}\frac{1}{F_q(\overline{x})\Big|_{x_{ij},a_{qki}\max}}\left[\left\|\sum_{j=1}^{m}\left(\frac{\partial F_q}{\partial x_j}\Big|_{\substack{x_j=x_{ij},\\ a_{qk}=a_{qki}}}\Delta x_j\right)\right\|+\left\|\sum_{k=1}^{K}\left(\frac{\partial F_q}{\partial a_{qk}}\Big|_{\substack{x_j=x_{ij},\\ a_{qk}=a_{qki}}}\Delta a_{qk}\right)\right\|\right]\le\\ \le\|\varepsilon\|\le\frac{1}{F_q(\overline{x})\Big|_{\substack{x_{ij},\\ a_{qki}\min}}}\left[\left\|\sum_{j=1}^{m}\left(\frac{\partial F_q}{\partial x_j}\Big|_{\substack{x_j=x_{ij},\\ a_{qk}=a_{qki}}}\Delta x_j\right)\right\|+\left\|\sum_{k=1}^{K}\left(\frac{\partial F_q}{\partial a_{qk}}\Big|_{\substack{x_j=x_{ij},\\ a_{qk}=a_{qki}}}\Delta a_{qk}\right)\right\|\right]\end{gathered} \tag{18}$$

- the value of the permissible deviation of the characteristic constants for the description form q is limited by their permissible error and the properties of the model:

$$\left\|\Delta a_{qs}\right\| \leq \frac{\left\|\varepsilon\right\| F_q(\bar{x})\Big|_{x_{ij},a_{qki}\max} - \left\|\sum_{j=1}^{m}\left(\frac{\partial F_q}{\partial x_j}\Big|_{x_{ij},a_{qki}} \Delta x_j\right)\right\| - \left\|\sum_{\substack{k=1,\\ k\neq s}}^{K}\left(\frac{\partial F_q}{\partial a_{qk}}\Big|_{x_{ij},a_{qki}} \Delta a_{qk}\right)\right\|}{\frac{\partial F_q}{\partial a_{qs}}\Big|_{x_{ij},a_{qki}\max}}. \quad (19)$$

At the same time, these equations (18), (19) divide the sets of deviations of constants into two sets of levels. The norm of relative error in one is less, and in the other is more than the given $\mathcal{E}$. This fact of the presence of a quantitative unambiguous division for the norm of error $\|\mathcal{E}\|$ (17) and for the norm of constants $\|\Delta a_{qk}\|$ (19) testifies to the peculiarity and new possibility of artificial quantitative-qualitative analysis. In this regard, it is an AI tool and will be called a tool for analyzing the capabilities and suitability of a model of related data sets.

To quantitatively confirm that changes in characteristic constants from point to point of the definition set as well as relative error and adequacy are indicators of model quality, we will use the data of numerical experiments [4]. During the experiment, five values were taken as a sample, which was experimentally obtained for each of the eight angles of rotation of the rotor ***Y***. As a result, for the mathematical expectation of each angle, which was found after statistical processing of the sample with a volume of five elements each, we have eight values as a function of the discrete system time factor ***X***. Table 1 shows the experimental data of the mathematical expectation of the shaft rotation angle ***Y*** and quantitative characteristics of the quality of approximation by the exponential function, which is calculated using the found approximation constant. Columns 1 and 2 show the discrete time coefficient ***X*** and the value of the mathematical expectation Y, respectively, as the output set according to (1). The function, as a result of the exponential approximation Y_1, the absolute Y_1-Y and the relative $\mathcal{E}$ of its error, as quality characteristics are presented in columns 3-5, respectively. Column 6 shows the value of the constant a in the space of straightened coordinates X, Y, according to (13). Column 7 shows $\mathcal{E}a$ - the relative deviation from the value determined by the approximation in the aligned coordinates: a=0.272099; b=0.699144. Column 8 shows the local values of the adequacy of the approximation E_1 according to the model quality indicator, which is calculated using three of the seven criteria [1].

Table 1
Approximate parameters for the exponential function y=2.012029*1.312717^X

X	Y, degrees	Y_1, deg.	Y_1-Y, deg.	$\mathcal{E}$, %	a	$\mathcal{E}a$ %	E_1
1	1,8795	2,641225	0,777573	41,72311	0,66915	145,9226	12,49884
2	3,6484	3,467181	−0,17175	−4,71985	0,40559	49,05925	976,7113
3	5,4737	4551426	−0,90765	−16,6264	0,28238	3,777749	78,70947
4	7,2505	5,974734	−1,26553	−17,479	0,22524	−17,2203	71,21787
5	9,0879	7,843134	−1,22621	−13,5204	0,17819	−34,5139	119,027
6	10,8478	10,29581	−0,54249	−5,00533	0,15545	−42,8712	868,4757
7	12,6765	13,51549	0,854397	6,748205	0,13249	−51,3068	477,8005

8	14,4687	17,74201	3,287198	22,7412	0,33388	22,70479	42,07232

Table 2 presents quantitative data on the power approximation of the same initial data as in Table 1 columns 1 and 2. Columns 3 - 5 respectively present the approximation by the power function Y_2, the absolute $Y_2 - Y$ and the relative $\mathcal{E}$ errors for different values of the system time X. Column 6 presents the value of the constant a in the space of the directed coordinates X, Y, which is defined between two neighboring points by (13). Column 7 presents its relative deviation $\mathcal{E}a$ from the value a, which is defined as the solution of the approximation problem in the directed coordinates: a=0.986919; b=0.615354. For the approximation by the power function, the approximation constants are calculated: A_2=0.986919; B_2=1.850311. Column 8 shows the local values of the adequacy index E2 of the power function approximation, calculated using three of the seven criteria [19].

Table 2

Approximate parameters for the power function $y=1.850311*X^{0.986919}$

X	Y, degrees	Y_2	$Y_2 - Y$	$\mathcal{E}\%$	a	$\mathcal{E}a\%$	E_2
1	1,8795	1,850311	−0,01334	−0,71582	0,965383	−2,1821	42463,31
2	3,6484	3,66722	0,028287	0,777341	1,000305	1,356325	26004,73
3	5,4737	5,471731	0,012657	0,231855	0,981563	−0,54265	153890,8
4	7,2505	7,268237	0,027976	0,386401	1,009407	2,278645	101915,5
5	9,0879	9,058815	−0,01053	−0,11609	0,977322	−0,97236	1614494
6	10,8478	10,84468	0,006375	0,058823	1,008409	2,177467	2184052
7	12,6765	12,62664	0,03445	−0,27209	0,992229	0,53806	293896,8
8	14,4687	14,40526	−0,04955	−0,34278	0,966695	-2,04914	185178,1

Analysis of the data in Tables 1 and 2 shows that the relative error of the approximation constant deviation in the aligned coordinates (column 7) changes synchronously, as does the relative error of the model (column 5) and local adequacy (column 8). The error modulus increases, while the adequacy decreases. Thus, for the approximation by the exponential function, the value of the constant a changed almost twice (Table 1), and for the power approximation it practically did not change ((4 - 5) % Table 2). The latter clearly demonstrates that the limits obtained from expressions (17)-(18) are indicators of the quality of the model, as are the relative error and adequacy, but which were calculated without approximation only according to the data of the connected sets.

Thus, the stability feature of the characteristic constant corresponds to a small relative error or high adequacy and in itself reflects the qualitative correspondence of the function type to the one best suited for approximation. High resolution of 10^{-3} and the range of changes in its value is more than 40-60 times greater than the max/min ratio. High sensitivity makes the conclusion based on it applicable for use, and the property itself is useful as an indicator-tool for artificial inference.

6. Discussion of the results of the development of an AI tool for evidential selection of the model type before approximation

The problem of choosing the best model, which arises as a result of the need to reproduce curves, surfaces of bodies, is an urgent task for describing the functions of one and many variables in the course of building empirical models. The areas it covers are not limited to cutting machines or 3D printers, when operating computerized systems or monitoring systems integrated into IoT and cloud services [21-25].

In this regard, an AI tool has been built that opens up the possibility of determining the boundaries of the model quality indicator using expressions (17), (18) without approximation, but only using the data of connected sets.

Such results are explained by the existence of a fundamental connection between the sequence of actions for estimating the interval of admissible values of a vector function for determining integral indicators of the theory of cognition. The latter is proposed to be evaluated together with the methods of direct and indirect measurement, the accuracy class of the instruments and the randomness of the measurement process.

Another explanation of the result is the comparison in co-dimensional conditions, which is provided by reduction to linear forms. Also, the fact that the correction of coordinates according to the similarity theory reduces all alternative forms of models to a single vector function, and the characteristic moments are analytically represented for an arbitrary identifier I and three surrounding points. Also, the intervals of existence of permissible values, which are determined by the methodology of direct and indirect measurement using the Taylor series expansion, quadratic norms and Bunyakovsky-Koshchy inequalities.

The last reason that explains the obtained result is that the created tool (19) uniquely determines the boundaries of the division of the set of deviations of characteristic constants and separates two sets of levels. This fact of the presence of a quantitative unique division for the error rate $\|\mathcal{E}\|$ (17) and for the constant rate $\|\Delta a_{qk}.\|$ (19) testifies to the peculiarity and new possibility of artificial quantitative-qualitative analysis. In this regard, it is an AI tool and will be called a tool for analyzing the capabilities and suitability of the model of related data sets. All of the above are causal and explain the results achieved as a solution to the four tasks set, which are given in the order of their formulation.

A distinctive feature of the research and the obtained results is the analytical algorithm for calculating the characteristic constants (13), the deviation interval as a function of the identifier I (19), which takes into account instrumental, methodological and a given relative error. Going beyond the upper limit of the deviation interval of one of the characteristic constants is a criterion-indicator of poor suitability of the function for describing the sets (1) and (2). The stability of the values of the characteristic constants defined on the entire set is the opposite criterion-indicator, which will determine the correspondence of the approximation form. Belonging to the best type from the set considered will determine the smallest value of the estimate of the maximum possible error (expression (18)). Such simplicity, analyticity and suitability for simple automatic inference make the expressions (13), (18) and (19) obtained in the work a tool for artificial unambiguous analysis and inference, that is, an AI tool. Unlike existing well-known criteria such as: relative error and adequacy, they are calculated before approximation [1]. As shown in Table. 1, 2 the dynamics of changes in known indicators relative error and adequacy are synchronously reflected by the dynamics of changes in characteristic constants, but they are calculated before approximation.

In addition, unlike existing ones, such analysis and selection are carried out before the start of approximation, which reduces the total complexity of the work to the complexity of approximating only the best model for the data of related sets.

However, despite the axiomatic obviousness of such a statement, there are no examples described in the literature when the task of quantitative comparison and selection of the type of approximation form, which is based only on the analysis of numerical data sets, was posed. There are also no known sources where the selection of the best of the models before the start of the approximation process is given. The formulation and solution of such a problem significantly reduces the total complexity of the approximation process as a whole, since it is performed only for one form, and the algorithm of such analysis is an AI tool. However, if earlier, after finding the approximation constants, it was necessary to once again verify the correctness of the conclusions for each model, now there is no need for this.

Thus, the methodology for constructing and evaluating a model based on connected sets is supplemented by a selection stage, the basis of which is a quantitative comparison of the stability of constants calculated using connected sets (13), (18) and (19). The complexity of this stage is estimated by the total time t_1 - calculation of characteristic constants using the analytical solution

(13) and time t_2 - calculation using the analytical expression (19), multiplied by the number of models K. Each of the traditional stages is now performed only for one pre-selected approximation form.

Such addition and unification into a single methodology allows, based on the assessment of a given relative error, to prove why one of the proposed forms is better using the automatic inference algorithm (19). Thus, after finding the approximation constants and calculating the adequacy of the model, it is necessary to compare the quality of only one, and not all K models, by assessing the adequacy by several of the seven criteria [1]. Let us denote this time t_3. It consists of the time of determining the characteristic constants and the other (7), the time of assessing the adequacy by m criteria t_am. The total number of approximation constants is twice more then the number of characteristic constants. Thus, if we assume that the time of determining one constant is the same, then the time saving Δt will be:

$$\Delta t = K\left(t_3 + 2t_1\right) - t_3 - Kt_1 - Kt_2 - 2t_1 = \left(K-2\right)t_1 + m\left(K-1\right)t_a - Kt_2.$$

Thus, the time savings become obvious, starting with two models K=2. The reason for this is that the time duration of operations t_1 is less than t_a, but greater than t_2, which is a direct calculation of the analytical expression (19). However, even if they are equalized, and the number of adequacy criteria is reduced to three: reliability, accuracy and depth (m=3), then starting with the number of models K=2, the savings become $3t_a$ and only increase in further. The same will be observed when using the calculation of floating-point operations for running all models and for the proposed method, since time is a positive multiple for calculating time savings.

The analysis of the studies performed by the article demonstrated the main limitation, which is the need to bring them to the co-dimensional conditions of the types of models being analyzed. It is obvious that its elimination lies in the plane of searches and selection of groups of functions, which are inherent in separate classes of problems or research of applicability of vector indicators, calibration algorithms and recurrent networks [6] for application of formal developments of a set of models in a single form. In this regard, it should be emphasized that appeal to the fundamental principle of the theory of cognition and similarity: commensurability of conditions during comparison, eliminated this methodological error and will remove restrictions in its application. of express approximation and quantitative assessment of its results [1, 5] is and will be relevant. Approximation as a complex process of building a neural network with the use of a vector indicator is presented in the work [6]. The coefficients of synaptic weights are analytically determined as a convergent sequence of solutions of a system of nonlinear algebraic equations, which is represented through the initial data for training [6]. The presented practical successes of simultaneous approximation are confirmed by the estimation of the maximum value of the rejected higher-order derivative and the solution of the problem of quantitative assessment is proposed as an application of calibration and database [6]. Other examples of the development of practical quantitative assessment of model quality are given in [1]. In it, the process of building a model is presented as a technology for its creation, and the assessment of effectiveness is presented quantitatively as a generalized assessment through a set of well-known factors. In essence, these factors determine the criteria as components of the adequacy of the model in relation to the object that needs to be described based on data about the function and its derivatives [1]. Simultaneous multifunctional application of approximation as a description of sets of points, functions, their derivatives in the formation of a network for its calibration and training gives its positive results. As a result of such application, the possibilities for creating and simplifying models and quantitatively determining the level and degree of adequacy are expanded [1, 6]. Also in [6] it was demonstrated that the use of the indicator vector expands its functional capabilities.

Also, disadvantages that can limit further application are the need to simultaneously satisfy exact and approximate approximation conditions. Under these conditions, the choice of characteristic constants will require an analytical solution to the optimization problem with equality constraints and inequalities, which will require its own research. It is obvious that further search for ways to improve the form of models and the tool will require changes in the least squares approximation paradigm and group accounting of arguments.

Of course, the choice of content and algorithms for calculating each of them is subject to standardization, which is also a direction for further research into the structure of empirical models and algorithmization of automated systems using AI tools, which will include intellectualized express model builders. An equally important development of further research are cases when the forms of the initial models satisfy exact requirements on surfaces and the number of such constraints is an integral part, which significantly increases the number of constants. Such forecasts, taking into account the results of this article, will obviously form new needs for reviewing both new types of AI tools and new areas of their application.

Conclusions

1. The formed sequence of actions brings the process of evaluating interconnected data sets to co-measurable conditions, which are determined by the accuracy class of the device and the measurement method. The generalized *m*-component vector model of the definition set and *l*-component vector-function model of the value set determines the permissible interval. This allows to study as integral indicators the features of the dynamics of assessing reliability and adequacy. The range of changes of reliability is limited to the interval from 0 to 1 inclusive, and adequacy changes in range from 0 to a magnitude of third or fived orders as un-dimensional and un-limited numbers.

2. The reduction of alternative models of choice set to a single form, including the method straightening of coordinates, made it possible to establish analytical expressions of characteristic constants as functions of only the identifier and 3 neighboring points. The dynamics of changes in the values of characteristic constants reflect the properties of the models and allow them to be compared between groups and classified before the start of approximation in comparable conditions. This tool for analyzing the suitability of a model for description, based only on the analysis of the results of processing related numerical sets, is an artificial intelligence tool.

3. A quantitative assessment of the interval of existence of the permissible value of characteristic constants, which is determined by the accuracy class of the device and the measurement method, has been formed. The values of the limits of the measured value for a model suitable for description within the permissible values of the relative error and not suitable for description have been established. The existence and analytical relationship of the limits of deviations of model constants based on the properties of quadratic norms, which will provide automatic selection for the established permissible relative error, have been established and determined.

4. It has been demonstrated on numerical examples that the formed toolkit for quantitative proof of the best fit of the model type for describing experimental data according to a set of definitions, behaves similarly with the relative error and adequacy. High resolution of 10^{-3} and the range of its value is more than 40-60 times higher than the max/min ratio, which will be useful for the program as an indicator. The fact of deviation of the model constants for different points is an indicator that will be considered as a tool of artificial intelligence since it automatically selects the best form of the model and has similar properties as the relative error and adequacy of the model.

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Declaration on Generative AI

During the preparation of this work, the author utilised ChatGPT and LanguageTool to identify and rectify grammatical, typographical, and spelling errors. Following the use of these tools, the author conducted a thorough review and made necessary revisions, and accepted full responsibility for the final content of this publication.

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