Detection of trends in non-stationary time series: a comparison of moving window smoothing methods [*]

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Abstract

The extraction of time series trends using moving window smoothing methods has been known for a long time and remains widely used. The window-based nature of these methods allows for various applications, including single-pass smoothing with a fixed window size and iterative smoothing of previously smoothed series using the same window or with varying window sizes. This study compares four methods: simple moving average, two weighted moving averages based on formulas by M. Kendall and J. Pollard, and median smoothing. These methods are applied to the same time series—monthly Wolf numbers of the 24th solar activity cycle. The evaluation criteria include the sum of squared deviations, maximum deviation, smoothing coefficient, sums of positive and negative deviations (considering their signs), and shift coefficient. The experimental study indicates that qualitatively, the smoothed series exhibit differences in shape, while quantitatively, the differences in their numerical characteristics are insignificant.

Keywords

moving window smoothing, smoothing formulas, time series, trends, trend shift.

1. Introduction

The representation of work situations in various areas of human activity is very often represented by the values of some determining or effective indicator. The sequence in time of its values has the form of a time series. In this regard, the dynamics of changes in this indicator, i.e. its trend, is of interest. To determine the trend and describe it, many different methods, methodologies, and approaches have been developed. One of these methods is sliding window smoothing of time series level values. In other words, a limited interval of levels slides along the time series – a window within which their average value is calculated. This value replaces the level of the time series that is opposite the middle of the window. Further, the window boundaries move one level in the direction of time and the process repeats. Naturally, the following question arises: is there a significant difference between these methods. The analysis of their algorithms does not provide an answer to this question. Therefore, an empirical answer to this question can only be given by experimental studies, namely, to establish differences in the results of smoothing and to assess the shift of the trend from its real position. In this case, the smoothed series itself is considered a trend. In [1, 2] provides general ideas about the methods and methods of processing time series, particular, identifying and highlighting trends in their development, and briefly describes the methods of smoothing. It is the smoothing methods that are used to highlight trends and build formal models of time series. As shown in [3], smoothing time series levels frequently and fairly accurately detects nonlinear monotonic trends while smoothing out noise and emissions. This study presents the results of using four methods of sliding window smoothing of time series levels. These are the method: a simple moving average, two methods of weighted moving average, proposed by M. Kendel and

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J. S. Schumacher. Pollard, and the method of median smoothing, better known as median filtering. The moving average, as noted in [4, 5], developed in the 1920s, is the oldest data smoothing process and continues to be a useful tool. This method relies on the notion that observations close in time are likely to have similar values, and therefore the averaging procedure for such values eliminates random variation and noise from the data.

As a test for this study, a real time series was used, which describes the lunar dynamics of solar activity, representing the observed number of spots on the Sun's surface. A feature of this time series, which corresponds to the 24th cycle, is its non-stationarity. This non-stationarity is even visually manifested in the monotony and nonlinearity of the trend and the change in the dispersion of its levels. Information about these data, namely: monthly values – values of time series levels of a given cycle of solar activity and time limits of cycles are given in the articles [6, 7, 8]. Identification of trends in the behavior of time series levels by the methods of sliding window smoothing is described in many information sources, in particular: a simple moving average, described in detail in [9, 10], weighted moving averages, their essence and features are presented in [11, 12], and in relation to this study, the algorithms given in the works of M. Kendel [13] and J. Pollard [14] are used. The fourth method, median anti-aliasing, is also a sliding window anti-aliasing method, but it uses a ranking procedure to determine the median of levels in a window. This procedure is non-linear, since instead of calculations, ordering is performed, which is actually non-linear. The popularity of median smoothing explains the fact that among all statistical indicators, the median is the most stable.

The essence of median smoothing as a median filtering of one-dimensional signals is described in [15]. [16, 17] discusses the use of median filtering and the median criterion for application to one-dimensional signals and time series. It is the use of median filtering that contributes to the best elimination of various deviations and noises, and the median filtering also ignores sharp changes in the behavior of signals. This study also considers the features of window smoothing in three variants – separately with windows of different sizes, smoothing of a previously smoothed row, both with a constant window size and with a gradual increase in its size.

The aim of this study is to identify differences in the application of these sliding window algorithms for determining time series trends in different variants of their application - multiple and repeated smoothing.

Experimental study of these methods was carried out in the Microsoft Excel spreadsheet environment. Evaluation of the results of the application of these methods from the qualitative side consists in visual analysis of their graphic image, and from the quantitative side - the ratio of the maximum deviation between the original and smoothed series to the root of the sum of the squares of the differences between them, as well as a comparison of the sums of negative and positive values to determine the trend shift.

2. Introduction data characteristics

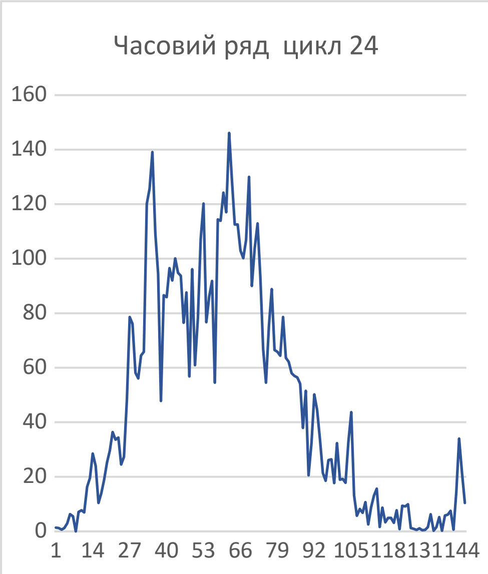
In the real world, time series are usually non-stationary. The behavior of objects and phenomena is characterized not only by nonlinear changes during observations, but also by trends, variances, autocorrelations and other indicators. Therefore, there is no problem in choosing one or another non-stationary series for its use as a test in experimental studies. In our case, the time series of the 24th cycle of solar activity, represented by Wolf numbers, is used. From the point of view of the authors, this time series has the following features:

1. The trend has a clear, visually pronounced bell-shaped shape, with a slight but noticeable left-sided asymmetry;

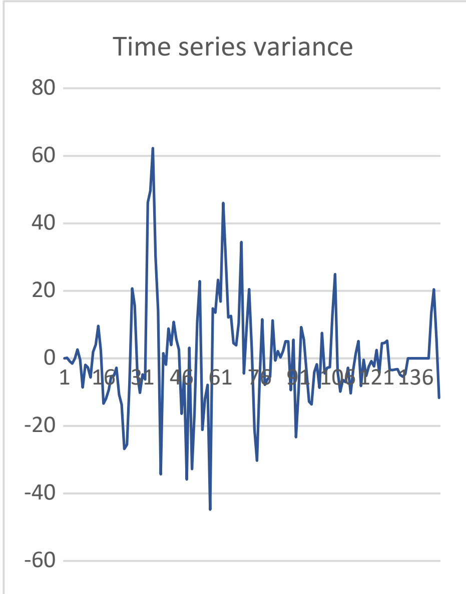
2. The variance of the levels in the middle of the series is quite significant and gradually decreases towards the beginning and end of the series;

3. The graph of residuals (deviations between the time series and its trend) shows their significant variation.

4. The volume of the series is quite convenient for processing and modeling by means of the Excel spreadsheet processor, since it includes 145 levels.



a



b

Figure. 1. Initial data: a – the magnitude of the time series levels corresponds to the values of the Wolf numbers for each month of the 24th cycle of solar activity; b – the value of deviations in this cycle.

Visual analysis of the image of the time series used indicates the nonlinearity of its trend, and the non-stationarity of the variance of its levels, i.e. the values of variance at the beginning, inside and at the end of the time series differ significantly. From the graph of deviations in Fig. 1b it can be concluded that the greatest concentration of deviations is near zero, that is, their distribution is unimodal. It can also be assumed that the shape of the distribution density is close to the shape of the normal distribution law with a slight left-sided asymmetry.

3. Methods and indicators used

In most of the various methodological materials devoted to smoothing of time series levels using sliding window algorithms, simple or weighted formulas of these algorithms are given. They also indicate their main drawback - the loss of smoothed values at the beginning and end of the smoothed series, and the size of the losses (corresponding levels) depends on the window size. However, for some methods there are special formulas for calculating the values lost at the beginning and end of the smoothed series. The issue of repeated smoothing also remains problematic, that is, smoothing of a previously smoothed series with the same window or a window of a different, larger or smaller size. Unfortunately, we also did not find appropriate criteria for limiting repetitions and choosing the optimal window size.

Therefore, the way out of this situation – the loss of levels of the smoothed series, is this approach. Starting from the minimum size of the window, smooth out only the parts of the levels at the beginning and at the end of the time series and fill in the second and penultimate levels with the calculated value. Then, gradually increasing the size of this window to the size of the working window, determine the average and fill in the lost values of the smoothed series with these values. Here the first and last values coincide with the values of the first and last levels of the smoothed original time series. As a rule, for window anti-aliasing, the window size is a multiple of an odd number of levels 3, 5, ... , that is, the window size is defined as:

$$w = 2k + 1$$

where

$k = 1, 2, \dots, N$,

where N is the maximum number of levels in the window.

Methods and procedures investigated. This study considers four methods of sliding window smoothing, namely:

a) simple moving average;
b) weighted moving average according to the formulas of M. Kendel;
c) weighted moving average according to the formulas of J. Pollard;
d) median smoothing (filtering).

The term ***window smoothing*** is used here to distinguish these methods from other anti-aliasing methods, since it is the influence of the window, i.e. its size and method of application, that is of interest. Therefore, the main attention in this study is focused on the methods of their application and changes in the size of the window. Therefore, the essence of this study is to conduct the following experiments:

a) separate smoothing of the time series with windows of different sizes;
b) repeated smoothing with a constant window size;
c) repeated smoothing with a gradual increase in the window size.

Here, the words "separate smoothing" mean that the original time series is first smoothed using the smallest window size, for example 5, then 7, etc., forming the following sequence of window sizes: 5, 7, 9, 11, 13, 15. In other words, the influence of the window size on the smoothing result is investigated. The words "repeated smoothing" mean that the original time series: is first smoothed using the smallest window size, then this already smoothed series will be smoothed again with the same window size, and so on - this is option 2. If for option 2 the window size is constant, then for option 3 the window size increases at each step. In each option and with each method, six repetitions were performed. In these experiments, the main result is the result of the sixth repetition.

An important point regarding the result of this work is the choice of the smoothing option, which will provide the most acceptable form of the trend, that is, its monotonicity, smoothness, simplicity in the sense of associations with known functions and mathematical expressions for its further approximation.

4. Evaluation of smoothing results

To compare the results of the experimental procedures of sliding window smoothing, a visual analysis of graphs was used for different variants of application of these methods, and the following indicators were used as evaluation criteria.

The sum of squares of deviations, to establish the magnitude of the difference between the variants and methods of smoothing in the sense of a quantitative indicator of the smoothing effect.

Maximum deviation. This is the value of the largest deviation between the original and smoothed time series.

The smoothing coefficient, introduced by us (the authors), to compare the smoothing effect itself. This is an empirical coefficient, which is defined as the ratio of the value of the maximum deviation between the levels of the original and smoothed series to the square root of the sum of the squares of the values of these deviations. Its values for different methods and variants of their application may be close or differ from each other. The comparison between the variants indicates the quality of the fit, namely: the larger this value, the smoother and more monotonous the envelope of the smoothed series. Small values of this indicator indicate that the smoothed series repeats the sharp changes of the original series.

Sum of deviations. Here we mean the sum of positive and negative deviations between the original and smoothed time series. The fact is that it is quite natural to say: the arithmetic sum of deviations from the mean value is zero. In this case, the sums of positive and negative deviations are considered.

The trend shift coefficient introduced by us (the authors) is based on the ratio of the difference in absolute values of the sums of added and negative deviations (the smaller absolute value is subtracted from the larger absolute value) to the value of the larger value with preservation of its sign, the value of which is given in percentage. Obviously, the smaller the difference between these two amounts, the smaller the magnitude of the trend shift. This indicator is an indicator of a possible trend shift. So: the magnitude of the difference in the sums indicates the magnitude of the trend shift (offset), and its sign indicates the direction of the shift. If the sum of negative deviations prevails, then the position of the actual trend will be higher than its real position (i.e., an upward shift), if vice versa, then the position of the trend will be lower than the real position. Regarding this criterion for assessing the trend shift, the following can be noted: if for a given set of elements (numbers) of the dynamic series, the sum of deviations exceeding the value of the smoothed series is equal to the sum of the absolute values of deviations smaller than it. Then the smoothed series exactly corresponds to the real trend. Therefore, this method provides grounds to detect the presence of a shift in the envelope of the smoothed series, i.e., its empirical trend.

The effectiveness of smoothing the levels of the time series in this study is presented as follows. It is natural to require the smoothing method to represent the existing trend in the form of a monotonic curve, which can be considered as the trend of the time series. However, the smoothing procedure makes its own adjustments, since the replacements of the values of the levels of the original time series are replaced by a calculated or determined ranking of values, that is, by the values of some sliding subset of values - the window. The power of this subset is determined by the boundaries of a specific window size.

The last two methods given for assessing the quality of smoothing the sliding window are empirical, since they do not take into account the probability distribution of the deviation values. However, as follows from the analysis of the graph in Fig. 1b, the deviations have a unimodal, somewhat asymmetric distribution, close to normal, which in principle gives grounds for their use.

5. Experimental study of sliding window smoothing methods

The same time series was used to conduct the study. The result of smoothing is taken as its trend, i.e. trend. In fact, differences in the results of three options for the application of these methods are experimentally investigated.

Simple moving average method. This is one of the simplest and most common methods that calculates the average value of levels in a moving window. Then this value is replaced by the corresponding level of the original time series, the position of which (time point, level number) corresponds to the middle of the window. Sliding means shifting the window by one level so that the levels of the original series remain in their places, and the boundaries of the window are shifted by one level. The algorithm for calculating a simple moving average has the following form

$$\tilde{y}_i = y_1^* + y_2^* + \ldots + y_k^* + \sum_{j=k+1}^{N-2k} \left[\frac{1}{w} \sum_{i=j}^{j+2k+1} y_i \right] + y_{N-k}^* + \ldots + y_{N-1}^* + y_N^*,$$

where $\tilde{y}_i$ – the smoothed value corresponding to the middle of the window; y^* – the values lost at the beginning and end of the smoothed series; w – the size of the window.

A simple moving average is the usual arithmetic average of the values of the levels that are calculated within the window. This average value is some indicator that always reflects the behavior of the main trend, smoothing out minor deviations. The effect of window size is that with small window sizes, the trend differs little from the behavior of the levels of the original time series, but with large window sizes, the resulting trend is more monotonous, smooth, and more accurately reproduces the main trend.

Smoothing by M. Kendel's formulas. The basis for deriving these formulas is the selection within the window size of the values of the coefficients of polynomials. Such selection is a linear combination of observations with these coefficients. These coefficients are defined in [13] and are invariant and suitable for any time series. These formulas are in fact weighted moving averages. The algorithm for calculating the weighted moving average with a specific "window" $w = 2k+1$, size sequentially shifts along the levels of the time series and averages the levels covered by this window. Its form is as follows:

$$\widetilde{y}_i = y_1^* + y_2^* + \ldots + y_k^* + \sum_{j=k+1}^{N-2k}\left[\frac{1}{w}\sum_{i=j}^{j+2k+1}\alpha_i y_i\right] + y_{N-k}^* + \ldots + y_{N-1}^* + y_N^*$$

The content y^* and other quantities are the same as in the previous formula, and the weights are subject to the condition $\sum_{i=1}^{2k+1}\alpha_i = 1$. The averaging operation in the window is presented in square brackets. The monograph [13] provides recommendations for calculating missing values of levels at the beginning and end of the time series.

The main formulas for this smoothing method are given in table 1.

Table 1.
The main formulas for the smoothing method

Window dimensions for *M. Kendel's formulas.*						
Level number in the window	**5**	**7**	**9**	**11**	**13**	**15**
K_0	0,49	0,33	0,26	0,21	0,17	0,15
K_1	0,34	0,29	0,23	0,2	0,17	0,15
K_2	-0,09	0,14	0,17	0,16	0,15	0,13
K_3		-0,1	0,06	0,1	0,11	0,11
K_4			-0,09	0,02	0,06	0,08
K_5				-0,08	0	0,04
K_6					-0,08	-0,01
K_7						-0,07

The values of the Ki weights given in Table 1 are arranged in the window in the following way, for example, for a window size of 7 it looks like this

[K3*xj + K2*xj+1 + K1*xj+2 + K0*xj+3 + K1*xj+4 + K2*xj+5 + K3*xj+6]

where xj are the levels of the time series. In [13], formulas for window sizes up to the 21st are given and methods for eliminating edge effects are given. The choice of the maximum window size in this work was made under the following condition: the maximum window size does not exceed 10% of the volume of the time series elements. The volume of the time series levels used as the original N = 145.

Smoothing according to J. Pollard's formulas. These formulas, like the previous ones, are also applied to the levels of the dynamic series, and their values, given in Table 2, are somewhat different.

Table 2.
Smoothing according to J. Pollard's formulas

Window dimensions for ***J. Pollard's formulas.***						
Level number in the window	**5**	**7**	**9**	**11**	**13**	**15**
K_0	0,56	0,41	0,33	0,28	0,24	0,21
K_1	0,29	0,29	0,27	0,24	0,21	0,19
K_2	-0,07	0,06	0,12	0,14	0,15	0,15
K_3		-0,06	-0,01	0,04	0,07	0,08
K_4			-0,04	-0,03	0	0,02
K_5				-0,03	-0,03	-0,01
K_6					-0,02	-0,02
K_7						-0,01

Formulas for a window up to 23 levels are given in [14]. References to literature sources on recalculation of smoothed values at the beginning and end of the dynamics series are also given.

Median smoothing. This smoothing method is a nonlinear non-computational procedure, quite stable in a statistical sense. It reacts poorly to anomalous values, outliers, etc.

Median smoothing is performed by determining the median by the levels in the window. For this, these levels are ranked, and the median is the value that lies inside the ranked series in the middle. So, the difference is that inside the window the median is determined by its position in the window, and in the previous methods their average values are calculated. A feature of median smoothing is a clearly expressed, for the levels of smoothed time series, non-monotonicity and lack of smoothness of the trend, the presence of horizontal sections and sharp transitions. This function, with an optimal choice of window sizes, preserves sharp transitions without distortion, and also reduces uncorrelated or weakly correlated noise, outliers.

Median smoothing is a sliding window procedure that replaces the level values of the original time series with the median values of the levels in the window. Its algorithm has the following form:

$$\tilde{y}_i = y_1^* + y_2^* + \ldots + y_k^* + \left[MEDIAN(y_{k-3};\ y_{k-2};\ y_{k-1}; y_k; y_{k+1}; y_{k+2}; y_{k+3}) \right] + y_{N-3}^* + y_{N-2}^* + \ldots + y_N^*$$

where y^* – are the original and pre-smoothed values of the time series, and the word MEDIAN is a function of the Microsoft Excel spreadsheet processor for determining the median of the sample. Thus, median smoothing replaces the values of the levels corresponding to the middle of the window with the values of the median of the levels limited by the window.

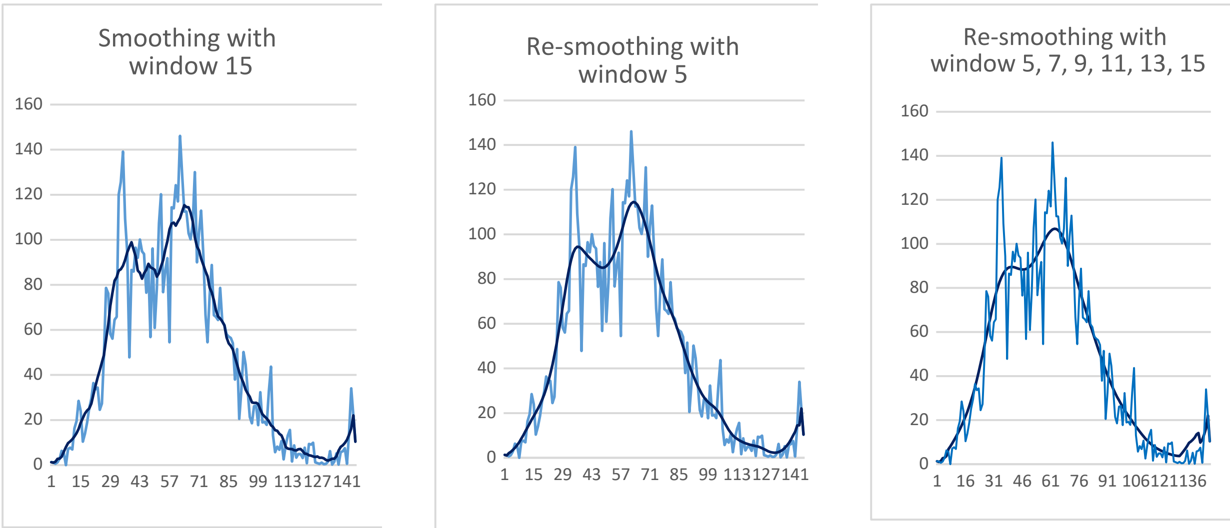
In practice, the window sizes for sliding smoothing methods are chosen by odd numbers, which in turn greatly simplifies the processing processes and interpretation of the results.

Edge effects. These include the loss of levels at the beginning and end of the time series. In this study, this situation is eliminated by the sequential application of different (in the direction of gradual increase) window sizes.

Experimental studies of the application of methods.

This study presents the results of three experiments with the following window sizes: 5, 7, 9, 11, 13 and 15, i.e. with such a number of levels in the window. For preliminary visual assessment, graphs of smoothing options for each method were used. The quantitative characteristics of the results are given in Table 3. The results are presented according to the above indicators. the values of the indicators given in the table correspond to the results at maximum window sizes

Simple moving average. The results of using this smoothing method are given in Fig. 2. Depending on the option of use, the contours of the smoothed rows differ from each other. In Fig. 2a, a cut (jaw) is quite clearly visible, which indicates an insufficient degree of smoothing. In Fig. 2b and Fig. 2c, the teeth are practically absent.



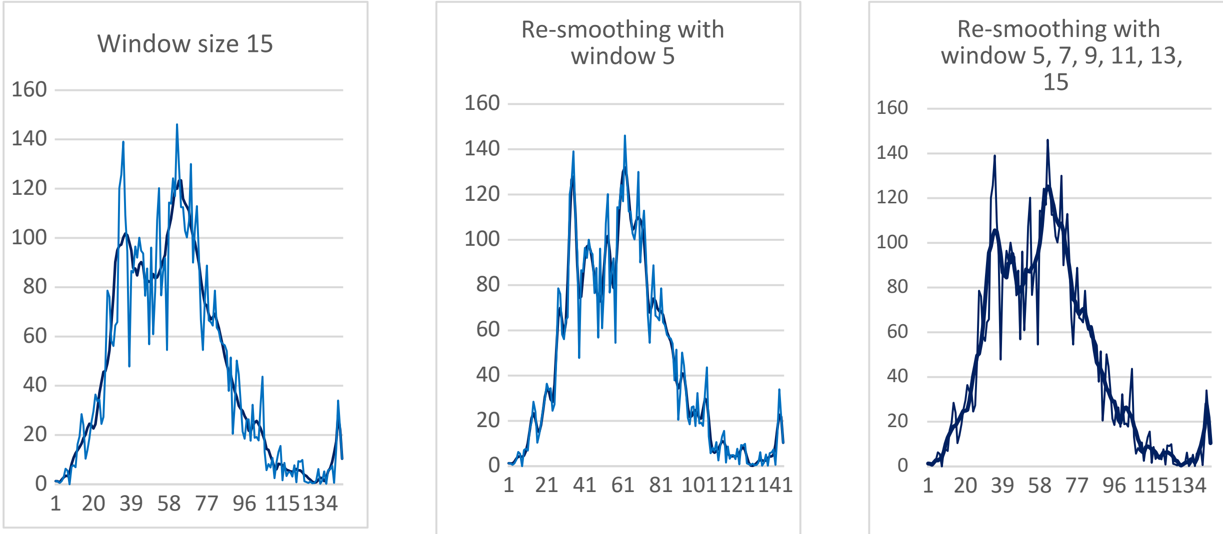
a *b* *c*

Figure 2. Results of smoothing with a simple moving average.

The original time series is shown as a thin light line, and the smoothed one as a thick dark line; the smoothing parameters are indicated as follows: ***a*** – single simple moving average with a window size of 15; ***b*** – six-fold repeated smoothing with a window size of 5; ***c*** – six-fold repeated smoothing with a change in the window size, i.e. each repeated smoothing is performed with an increase in the window size by one step.

The option with the use of multiple smoothing gives a smoother and more monotonous trend line. It can be assumed, Fig. 2c, that with further repeated smoothing with increasingly larger windows, the depression between levels 40 and 66 may disappear. In addition, the trend line itself looks quite smooth.

Smoothing of time series levels according to the formulas of M. Kendel. In the monograph [13] a set of formulas is given, namely the formulas of weighted recalculation of time series levels. The formulas given in this work provide the size of the smoothing windows from 5 to 21 and represent odd numbers 5, 7, … , 21. Recommendations are also given for eliminating edge effects. In this experimental study, the following window sizes were used: 5, 7, 9, 11, 13 and 15. In Fig. 2. the results of three smoothing options according to the formulas given in table. 1 are shown.



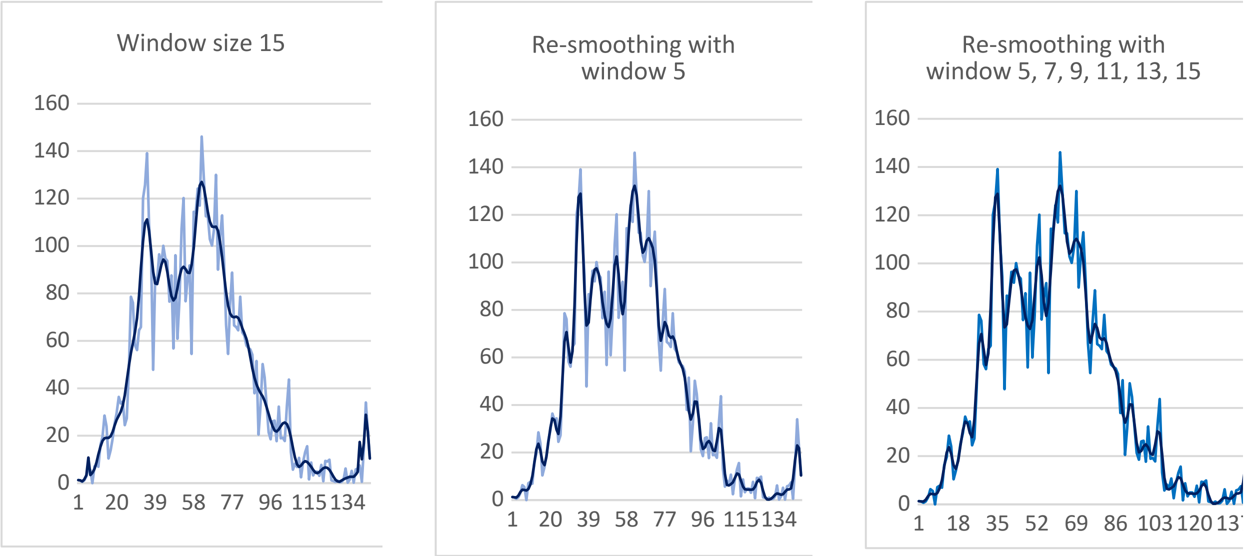
a ***b*** ***c***

Figure 3. Results of time series smoothing according to M. Kendel's formulas

As in the previous description in Fig. 3, the original time series is depicted by a thin light line, and the result, the trend, is depicted by a thick dark line. The three graphs presented correspond to the applied smoothing options. Here, the option: ***a*** – single simple moving average with a window size of 15; ***b*** – six-fold repeated smoothing with a window size of 5; ***c*** – six-fold repeated smoothing with a change in the window size up to and including 15, i.e. each repeated smoothing is performed with an increase in the window size by one step.

This method of weighted smoothing of the sliding window in all three options tries to repeat the nature of the level position, while reducing only the amplitude of the levels and slightly removing sharp changes. Obviously, using this method to smooth a time series with significant dispersion and fluctuation in its levels becomes problematic.

Smoothing of time series levels according to J. Pollard's formulas. In [14], a set of formulas is given, which also provides a weighted moving average by recalculating the levels of the smoothed time series. The results of smoothing by this method, in three variants of application, are shown in the form of graphs in Fig. 4. Visual analysis of the results of applying this method indicates that the smoothed series practically reproduces the behavior of the levels of the original series. In Fig. 4b, the smoothed series practically preserves the oscillations of the trend and reacts poorly to large deviations. When smoothing according to the third variant, Fig. 4c, new fluctuations of the trend arise, in particular in the region of the initial and final levels. In general, it can be stated that the use of these formulas is justified only for the first variant, i.e., one-time use with a previously checked window size.



a *b* *c*

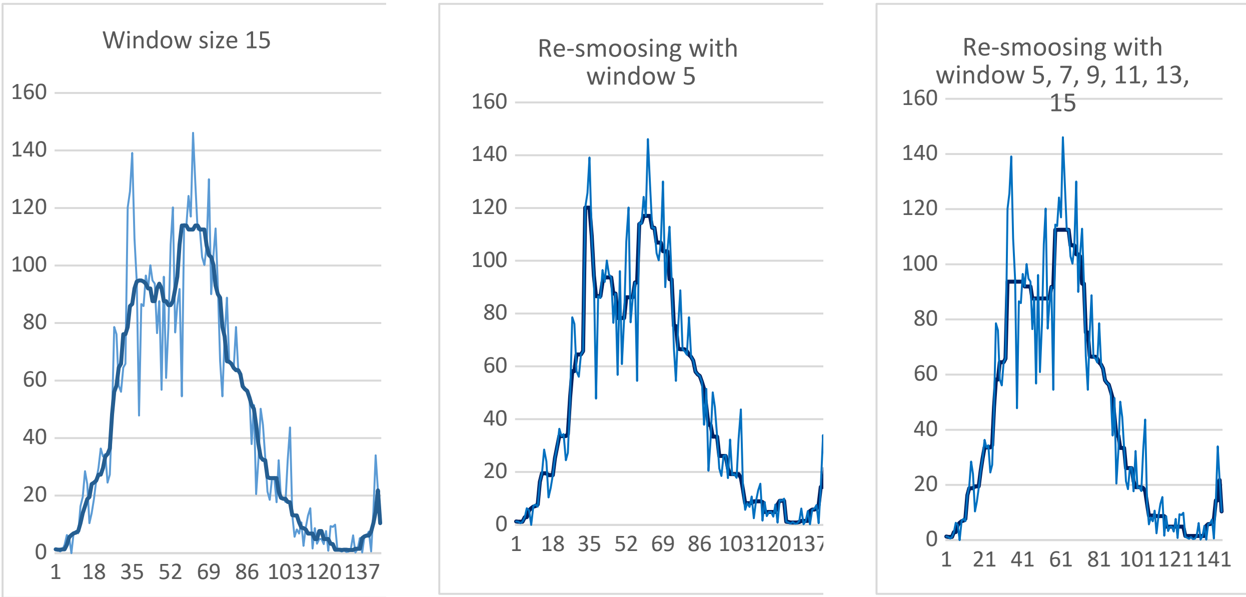
Figure 4. Results of smoothing according to J. Pollard's formulas

The original time series is depicted by a thin light line, and its trend, i.e. the smoothed series, by a thick dark line. The application of this method is also considered in three variants: a – single simple moving average with a window size of 15; b – six-fold repeated smoothing with a window size of 5; c – six-fold repeated smoothing with a change in the window size up to and including 15.

Significant fluctuations after smoothing indicate that the method is possible only with insignificant dispersion and slow smooth fluctuations with small amplitude.

Median smoothing. This type of smoothing, unlike the previous methods, is a nonlinear procedure. Smoothing a time series by determining the median in a sliding window is widely used not only to highlight trends in time series, but also in processing various signals and images to eliminate various interference, noise, etc.

In this experiment, the same conditions were used regarding the parameters of the application program and the sizes of the windows, that is, these three options, as in the previous ones. Median smoothing has several important features, including: eliminating outliers and anomalies in the data, it preserves sharp changes and is resistant to changes in the distribution. A visual representation of the application of median smoothing is shown in Fig. 5. The fact that sharp transitions are preserved, especially at the beginning and end of the series, is obvious.



a ***b*** ***c***

Figure 5. Results of median smoothing

Fig. 5 shows how the appearance (shape) of the trend changes as a result of applying median smoothing (thick dark line). Here, the original time series is depicted by a thin light line, and the options for its application are as follows: a – single simple moving average with a window size of 15; b – six-fold repeated smoothing with a window size of 5; c – six-fold repeated smoothing with a change in the window size up to and including 15, i.e. each repeated smoothing is performed with an increase in the window size by one step. The graph clearly shows horizontal sections of the smoothed series, which indicate that within their boundaries, when the window was sliding, the median value remained constant. In addition, sharp transitions also remained unchanged.

An important point of median smoothing is that, provided that the distribution of levels is symmetrical, the position of the trend corresponds to its real position. In other words, for symmetric distributions, the median and mean coincide.

Interpretation of smoothing results. The values of the resulting indicators obtained in the experiments regarding the smoothing features for these four methods are given in Table 3. The conducted studies of these sliding window smoothing methods showed that there are no significant differences between them.

The two indicators proposed by the authors can be considered as estimates of the applications of these methods: the trend smoothing coefficient – V and the trend shift coefficient – D. The calculation of the coefficient V is determined by the following formula:

$$\mathbf{V} = \frac{\Delta y_{\max}}{\sqrt{skv}},$$

where $\Delta y_{\max}$ is the maximum deviation between the original and smoothed series, and is the sum of the squares of these deviations.

The data presented in Table 3 indicate that the differences in the components of this indicator apply only to both methods of weighted sliding window smoothing according to the formulas of M. Kendel and J. Pollard and median smoothing, only when using the second option - six-fold repeated smoothing with a constant window size equal to 5 levels. However, the value of the smoothing coefficient itself, and it is within the set of all values in this study, namely . From this point of view, all these methods differ little from each other.

Table 3.
Comparison of Indicator Components under Repeated Smoothing Methods with Fixed Window Size (5 Levels)

Methods	Application variant	Sum of squares of residues	Maximum deviation $\tilde{y}_{max}$	V	$\sum_{i=1}^{N} \tilde{y}_i < 0$	$\sum_{i=1}^{N} \tilde{y}_i > 0$	D
Simple moving average	**1**	26722	50.64	0.3098	– 660	656	– 0.57
	2	22743	45.82	0.3038	– 604	639	5.38
	3	27112	54.18	0.3290	– 700	659	– 5.81
M. Kendall's formulas	**1**	22530	50.39	0.3357	– 611	600	– 1.76
	2	9062	29.34	0.3082	– 391	394	0.83
	3	20567	48.34	0.3371	– 586	575	– 1.86
J. Pollard's formulas	**1**	15584	41.41	0.3317	– 526	505	– 3.98
	2	8647	28.96	0.3114	– 380	383	0.93
	3	18300	45.85	0.3389	– 558	544	– 2.50
Median smoosing	**1**	26635	58.00	0.3554	– 538	643	16.32
	2	13978	38.80	0.3252	– 343	448	23.48
	3	19569	45.90	0.3281	– 397	571	30.48

To estimate the shift of the smoothed trend, the authors used the D indicator in the following form

$$\mathbf{D} = \frac{\left|\, \left| S_{(+)} \right| - \left| S_{(-)} \right| \,\right|}{Sign(S_{\max}) \cdot S_{\max}},$$

where $S_{(+)}$ – the sum of positive deviations; $S_{(-)}$ – the sum of negative deviations; $S_{\max}$ – є is one of the sums that has a larger modulus value.

The expression in the denominator can be explained as follows: the «Sing» function determines the sign of the sum, i.e.: if $S_{\max}$ = $S_{(+)}$ > 0, then Sing($S_{\max}$) = +1, and if $S_{\max}$ = $S_{(-)}$ < 0, then Sing($S_{\max}$) = −1.

In other words, the sign of the denominator indicates the direction of the shift, and the value of this indicator «shows» the relative magnitude of the shift itself, which for the trends obtained in this study lies in the interval $D \in \left[\left[-5.81,\ 5.38 \right] \right]$.

However, the value of this indicator, like the previous one, is only the relative values of the empirical assessment of the results obtained and requires separate studies for their practical use.

An interesting fact is that for the first and third options for using moving averages, the trend shifts up, and for the second - down. As for median smoothing, there is a significant downward trend shift.

6. Conclusion

According to the results of the experimental study in the methodological plan, the following conclusion can be made.

First, no significant differences were found between the considered methods in the three variants of their application. In a certain sense, the differences exist, but for categorical conclusions they are insignificant.

Secondly, the smoothing coefficient introduced by the authors also does not indicate a significant difference between them, although when comparing these methods, the values of this indicator can be taken into account in practice.

Thirdly, the trend shift coefficient introduced by the authors clearly and quantitatively indicates the presence of a shift and its direction. The trend shift is important for automatic control of systems and objects.

Fourthly, median smoothing is characterized, in particular, by a significant downward trend shift and insufficient smoothness of the trend curve. However, if the level values have a distribution, the median trend is more realistic.

The results of this study show the feasibility of using all four methods in the three considered variants, obviously with arbitrary window sizes. It can be assumed that at least these three ways of using them, namely: single with arbitrary or justified choice of window size, multiple sequential application to a pre-smoothed series of levels with the same window size and multiple sequential application to a pre-smoothed series with increasing window size, have practical application. Actually, one of the aspects of the purpose of this study is to identify the features of using these methods to highlight the trend of a non-stationary time series and build its mathematical model.

Declaration on Generative AI

During the preparation of this work, the authors utilised ChatGPT and LanguageTool to identify and rectify grammatical, typographical, and spelling errors. Following the use of these tools, the authors conducted a thorough review and made necessary revisions, and accept full responsibility for the final content of this publication.

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