Resolving the issue of incorrect use of the averaging formula for calculating the power of conduction current pulses and how this is relevant to robotics⋆

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Abstract

By means of practical examples, the methodological incorrectness of using the formula of the average value of a function in original form to calculate the power of conduction current rectangular pulses in the active resistance is demonstrated, which persists to this day. The cause of this incorrectness is revealed and, in order to eliminate it, an empirically generalized formula is proposed which includes its original form as a special case. The proposed formula will help to improve the accuracy of the evaluation of energy consumption and the efficiency of its conversion in robotic devices, especially in switching voltage converters. This should contribute to increasing the duration of continuous operation and the likelihood of robotic devices completing tasks, as well as to the overall needs of global energy saving.

Keywords

duty cycle, pulsed power, average power, elementary function, composite function, physical quantity

1.Introduction

Modern robotic devices - from household appliances to military and aerospace equipment - are supplied by batteries in conjunction with step-up and step-down switching voltage converters (SVCs) in various combinations of DC and AC power sources. Billions of the most widely used pulse-width modulation (PWM)-based SVCs in the world are produced, and the count is rising.

The continuous operating time of the robotics is limited by the battery capacity, which must be taken into account in the design. However, the probability of the robotic devices completing the tasks is reduced by random and significant increases in the time required to maneuver for target detection, to avoid enemy countermeasures, to overcome obstacles in the landscape, or to increase the strength and change the direction of wind or water surface waves, and so on. And as increasing battery capacity makes robotic devices bigger and heavier, reducing their capabilities, improving the efficiency of SVCs is one of the ways to extend their operating time. In such a context and on such a scale, the relevance of even a small increase in the efficiency of SVCs is obvious, and not only in terms of increasing the continuous operating time and the likelihood of the robotic devices completing tasks. But also in terms of global energy savings, as PWM-based SVCs are very commonly used in the renewable energy industry. In particular, the integration of PWM-based SVCs with photovoltaic panels and batteries, which is the main technical solution for solar power, is also employed in some robotic devices.

Against this background, it is obvious that it is important to address the shortcomings that have been identified both in the theoretical foundations of known and new methods for improving the efficiency of SVCs and in the practice of the use of mathematical tools for the evaluation of the performance of their work. In particular, in [1, p. 85-86], which aims to increase the continuous

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operating time of robotics, identifies the cause of the contradiction between the real efficiency of SVCs and the existing evaluations of the efficiency of energy accumulation in their reactive

components. It lies in the overly idealised theoretical provisions on which these evaluations are based. Primarily these relate to the energy losses in the active resistance of the batteries, without which it is impossible to adequately evaluate the efficiency of the SVCs.

And when applying the conclusions of [1] to the results of previous studies [2], the incorrect practice of using the formula of the average value of the function to calculate the power of rectangular pulses of conduction current was revealed, which can lead to inaccurate efficiency evaluations in designing not only SVCs, but also other pulse devices.

For example, such an incorrectness was found in the Multisim simulator (version: 11.0.278) from National Instruments Corp, which has many users.

This incorrectness is of a methodological nature, and to prevent it, it is necessary to observe the type of function that describes the value being calculated and the scope of the formula for averages, which is derived in its original form for elementary functions.

For this, we propose a formula which is adapted to ensure the correct calculation of average values of composite functions in the form of a product of elementary functions.

The power of the conduction current, in particular in the active resistance losses of the batteries, is precisely such a composite function. Therefore, the use of the proposed formula will help to improve the accuracy of evaluating the efficiency of not only SVCs based on the PWM principle, but also any other composite indicators of various pulse nodes of robotics, etc.

2.A brief overview of the practice of using the average value of a function formula to calculate the parameters of rectangular pulses

In mathematics, a formula exists for calculating the values commonly referred to as the average (or mean) value of a function, which in its original form is as follows, for example in [3, p. 542]:

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx \tag{1}$$

where f(x) is integrated and continuous on the interval [a, b] functions.

In many fields of radio electronics, including robotics, this formula is used in a structurally unchanged form to calculate the average values of the parameters of rectangular pulses of physical quantities of various nature, most commonly voltage, current and power in an active resistance. Only the mathematical notation is replaced by the notation of the parameters of pulses from the field of application in which the formula is used.

For example, by the equivalent transformation of formula (1), the calculation is performed that combines such parameters of a rectangular pulse sequence as pulse power P_i and average power P_{avg} through the pulse duty cycle D [4], which corresponds to the interval [a, b] in (1):

$$D = \frac{P_{avg}}{P_i} = \frac{\tau}{T}, \tag{2}$$

where τ is the pulse duration time and T is the pulse repetition period.

The graphical representation of the values of pulse power P_i over the pulse duration time and average power P_{avg} over the pulse repetition period is shown in Figure 1. These values are calculated using the familiar methods of integral calculus.

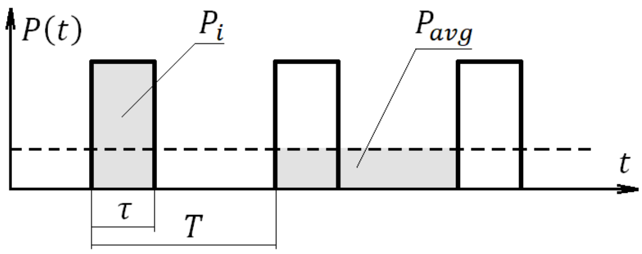

Figure 1: Graphical representation of the parameters of a sequence of rectangular pulses.

Specifically, for a rectangular pulse, the pulse power is:

$$P_i = \frac{1}{\tau}\int_0^{\tau} P(t)\,dt, \tag{3}$$

where P(t) is a function that describes the change in power over time, in this case, during the time of the rectangular pulse τ.

And finally, the average power of rectangular pulses over their repetition period T:

$$P_{avg} = \frac{1}{T}\int_0^{\tau} P(t)dt = P_i \cdot D. \tag{4}$$

Formula (4) is the same as (1), except that it uses the notation of rectangular impulse parameters instead of the notation commonly used in mathematics. In this form, this formula is used in [5, p. 6] to calculate the pulse power of the optical radiation of pulsed lasers, and in [6] - the average voltage value to evaluate the distortion of the pulse signal.

As further will show, in [5] and [6] we have examples where the physical nature of the quantities to be calculated and the mathematical function describing them correspond to the scope of formula (1) in its original form and its form (4).

Instead, the same calculation error that can be observed in the Multisim simulator is given by the incorrect use of formula (4) and its equivalent transformation according to Ohm's law in [7, p. 4] for calculating the average power of rectangular pulses on a linear resistor:

$$P_{avg} = P_i \cdot D = \frac{\tau}{T} \cdot P_i = \frac{\tau}{T} \cdot \frac{U^2}{R}. \tag{5}$$

It should also be noted that formulas similar to (1) are given in the section on the method of digitization and power calculation in the review of digital analyzers and meters manufactured by Yokogawa Corporation, according to which the instantaneous voltage value is multiplied by the instantaneous current value and then integrated over a certain period [8, pp. 13, 14]. However, these devices are designed to measure industrial frequency alternating current, and we had neither the opportunity nor the need to test them, as this is beyond the scope of our work.

The reviewed sources do not cover all the areas of electronics in which the formulae (1) - (5) are applied. However, they are sufficient to discuss the consequences and causes of the identified incorrectness in the use of these formulae, which persists to this day.

3.An example of a discrepancy in the results of calculating the average power of rectangular conduction current pulses using different formulas

Let us compare the results of the average power calculation using formula (4) and a similar formula in [7, p. 4] with the results of the calculation according to Ohm's law.

For this purpose, we will use the voltage and current values measured on the SVC prototype described in [2] according to the scheme in Figure 2, and the voltage, current and power values obtained by simulation in Multisim.

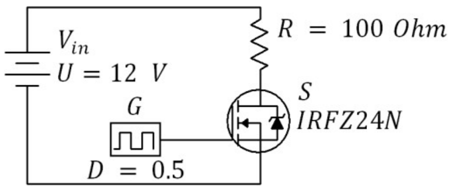

Figure 2: A scheme for the measurement of the average voltage of the conduction current of rectangular pulses in a linear resistor on the basis of the SVC prototype.

In the circuit shown in Figure 2, a DC voltage U, value 12 V, is periodically applied from the source V_{in} to the linear resistor R, value 100 Ohm, via the electronic key S.

To simplify the calculations, the duration τ of the 5 kHz frequency pulses of the generator G is chosen to be equal to half their repetition period T, i.e. the duty cycle D = 0.5.

Figure 3 shows a photo of the oscilloscope screen in the mode of measuring the parameters of a sequence of rectangular pulses with a frequency of ‘Frec = 5.08 kHz’, duty cycle ‘-Duty = 50.20%’ and ‘+Duty = 49.80%’, voltage ‘Vamp = 12.8 V’ and voltage ‘Avg = 6.12 V’, which are marked in red.

According to the data sheet of the DSO2000 oscilloscope from Hantek Technologies Co. Ltd. the voltage 'Avg' represents: ‘The arithmetic mean of the entire waveform or selected area’.

Next, we calculate the average voltage U_{avg} across the resistor R by the value D:

$$U_{avg(calc)} = U \cdot D = 12 \cdot 0.5 = 6\ (V), \tag{6}$$

where U = 12 V is the amplitude of voltage pulses equal to the voltage of the source V_{in}.

As expected, considering the precision of the supply voltage and duty cycle settings and the measurement accuracy, the measured value of Avg = 6.12 V with an error of 2% corresponds to the calculated value of $U_{avg\ (calc)}$ = 6 V, which was used for further calculations.

And then, according to Ohm's law, we calculate the average value of the current pulses $I_{avg\ (calc)}$, which flows through the resistor R:

$$I_{avg(calc)} = \frac{U_{avg(calc)}}{R} = \frac{6}{100} = 0.06(A). \tag{7}$$

And from the values of $U_{avg\ (calc)}$ and $I_{avg\ (calc)}$ calculated in (6) and (7), we find the calculated value of the average pulse power, which is equal to:

$$P_{avg(calc)} = U_{avg(calc)} \cdot I_{avg(calc)} = 6 \cdot 0.06 = 0.36\ (W). \tag{8}$$

Figure 3: A photo of the screen of the DSO2000 oscilloscope from Hantek Technologies Co. Ltd. during the measurement of the average voltage Avg = 6.12 V of a sequence of rectangular pulses.

The calculations of the average values of voltage and current (6) and (7) are confirmed by simulation in Multisim, the results of which are shown in Figure 4.

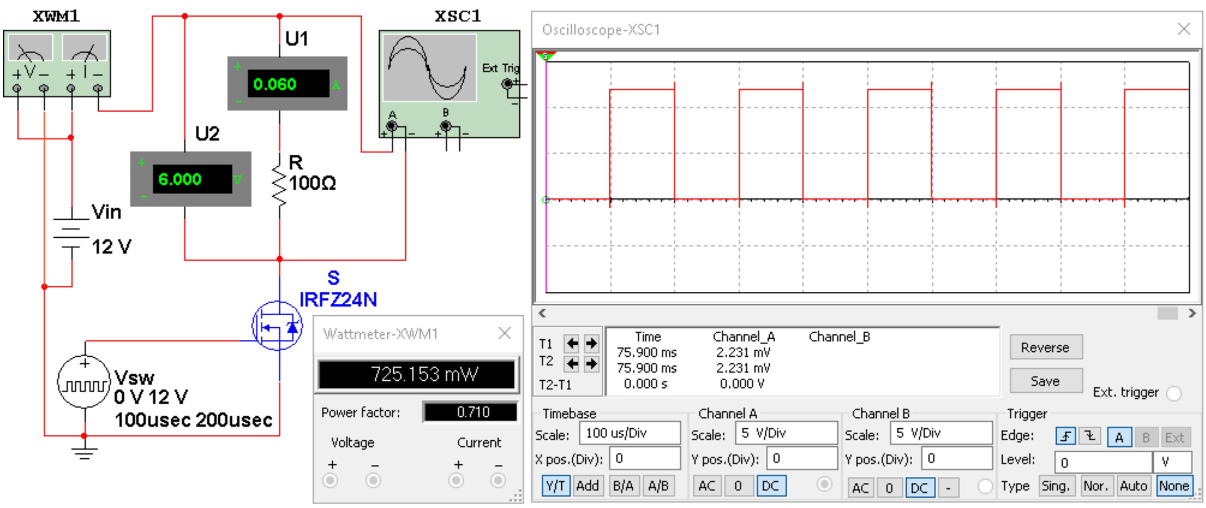

Figure 4: Screenshot of simulation results examining the average power of rectangular conduction current pulses in a linear resistor in the Multisim environment.

Namely, on the voltmeter U2 in Figure 4 we have a voltage reading $U_{avg\,(sim)}$ of 6 V, which corresponds to the average voltage Avg = 6.12 V measured on the SVC prototype, as well as to the average voltage $U_{avg\,(calc)}$ = 6 V calculated in (6).

And on the ammeter U1 we have a current reading $I_{avg\,(sim)}$ of 0.06 A, which is equal to the average current $I_{avg\,(calc)}$ calculated in (7).

Thus, the average value of the power obtained by the simulation is:

$$P_{avg(sim)} = U_{avg(sim)} \cdot I_{avg(sim)} = 6 \cdot 0.06 = 0.36\ (W). \tag{9}$$

The value of $P_{avg\,(sim)}$ in (9) coincides with the result of the power $P_{avg\,(calc)}$ calculated in (8) based on the average voltage Avg = 6.12 V measured on the SVC prototype, which is shown in Figure 3.

Instead, the XWM1 wattmeter reading $P_{avg\,(XWM1)}$ in Figure 4 is twice as high as the $P_{avg\,(calc)}$ result in (8) and the $P_{avg\,(sim)}$ result in (9).

This reading of $P_{avg\,(XWM1)}$ = 725.153 mW from the XWM1 wattmeter coincides with the result of the average power calculation using formula (4) and the formula (5) for the average power of positive rectangular pulses in [7, p. 4], and its equivalent transformation according to Ohm's law based on the amplitude of the voltage U of the pulses:

$$\begin{aligned} P_{avg\,(XWM1)} &= P_{avg\,(4)} = D \cdot P_i = D \cdot U \cdot I = D \cdot U \cdot \frac{U}{R} = P_{avg\,(5)} = \frac{\tau}{T} \cdot P_i = \frac{\tau}{T} \cdot \frac{U^2}{R} = \\ &= 0.5 \cdot \frac{12^2}{100} = 0.5 \cdot \frac{144}{100} = 0.72(W). \end{aligned} \tag{10}$$

Therefore, we have reason to believe that the Multisim software is using formula (1) in its original form, or in a writing similar to (4), or in [7, p. 4] as (5), to calculate the readings from the XWM1 wattmeter, which gives an overestimated result of 0.72 W.

4.Discussion of the cause of the discrepancy in the calculation results and their possible consequences for practice

The formula (1) is derived in mathematics for functions that are integrated and continuous over some interval, but their type is not explicitly defined, as for example in [3, pp. 541-552].

Nevertheless, from the descriptions of the formula in known sources, for example [9], it can be concluded that the scope of its application is limited to elementary functions explicitly defined by a single formula, the argument of which must be an independent variable.

Furthermore, for practical calculations of physical quantities such as current, voltage and power, formula (1) should be used not only according to the rules of mathematics, but also according to the provisions of the International System of Units (SI) and the International System of Quantities (ISQ) [10].

They determine seven basic physical quantities: length, mass, time, current, thermodynamic temperature, amount of matter and light intensity, which cannot be derived from other quantities. All other physical quantities are derived from the basic ones and are determined from them by relationship equations that are independent of the units of measurement.

Obviously, mathematical equivalents of basic physical quantities should be functions that are not derived from functions describing other quantities. In mathematics, such functions are called elementary. And for the derivatives of physical quantities, mathematical equivalents in the form of composite or elementary functions can be used in the equations of relationship.

Voltage is therefore a derivative of a basic physical quantity and is described as the product of an elementary function - current - and a constant, which is the value of the active resistance through which this current flows. In other words, it is described by an elementary function.

Instead, by definition, power is a composite function of the product of separate elementary functions - current and voltage. And only for the ease of calculation, it can be reduced to one of these functions by equivalent transformations using Ohm's law, but not vice versa.

That is, they are related by the logical operation of implication as a basis and a consequence:

$$P(t) = u(t) \cdot i(t) \Rightarrow P(t) = \frac{u(t)^2}{R} = i(t)^2 \cdot R. \tag{11}$$

So, to calculate the average values of current or voltage described by elementary functions, it is legitimate to use formulae (1) - (5), as is the case in [6], as well as in the software of the ammeter and voltmeter in Multisim, and the DSO2000 oscilloscope for the voltage 'Avg'.

Instead, it is incorrect to calculate the average power value, which is a composite function of the product of current and voltage according to formulae (1) - (5).

As an illustration of a similar division in the methodology of mathematics, let us consider the operations of finding the derivatives of elementary and composite functions.

The derivatives of elementary functions of the form f(x) are defined as the limit of the ratio of the increment of the function Δf(x) to the increment of its argument Δx, which tends to zero (if such a limit exists).

And conversely, there is a special rule for finding the derivatives of composite functions of the form f(g(x)). Namely: the derivative of a composite function is equal to the product of the derivative of the external function in the intermediate argument and the derivative of the internal function in the argument x.

According to this rule, the derivative of the product of functions (u · v) is found by the formula, in which each function is differentiated separately:

$$(u + v)' = u' \cdot v + u \cdot v' \tag{12}$$

Whereas the direct differentiation of the product (u · v)′ after performing the operation of multiplication of the functions (u · v) is incorrect.

Therefore, similar to the rule for differentiating composite functions, the calculation of the average pulse power, which is a composite function in the form of the product of elementary voltage and current functions, should not be performed by formula (1) in the original form or its form (4) and (5), but by a special formula.

In fact, the average power of rectangular pulses should be calculated not by the first power of the duty cycle D, as in formulae (4) and (5), but by its square, namely:

$$P_{avg} = P_i \cdot D^2. \tag{13}$$

This is exactly what corresponds to the equivalent transformations of formula (11) for calculating power based on Ohm's law, where D^2 is implicitly included through $u(t)^2$ or $i(t)^2$.

Because the D^2 is included in the average power P_{avg} as a result of the operations with the average values of current I_{avg} and voltage U_{avg}, already calculated by the duty cycle D:

$$P_{avg} = I_{avg} \cdot U_{avg} = \frac{{U_{avg}}^2}{R} = {I_{avg}}^2 \cdot R. \tag{14}$$

And when we perform calculations for D^2 and the amplitude values of the current I and voltage U using formula (13) and its equivalent transformations based on Ohm's law according to (11), the result will coincide with the calculations for the average values of the current I_{avg} and voltage U_{avg}.

For example, according to formula (13):

$$P_{avg} = 12 \cdot 0.12 \cdot 0.5^2 = 12 \cdot \frac{12}{100} \cdot 0.5^2 = 0.12^2 \cdot 100 \cdot 0.5^2 = 0.36\ (W). \quad (15)$$

And by formula (14):

$$P_{avg} = 6 \cdot 0.6 = \frac{6^2}{100} = 0.6^2 \cdot 100 = 0.36\ (W). \quad (16)$$

Thus, the discrepancy in the results of (8), (9) and (10) is caused by a methodological error, namely that the pulse power P_i in (1) - (5) is considered as a basic physical quantity described by an elementary function, whereas it is a derived quantity described by a composite function.

Hence, the square of the duty cycle D in (13) restores the relationship lost in (4) and (5) to Ohm's law and the equivalent transformations of the equations for calculating power (11).

For example, according to (11), halving the current and/or voltage reduces the power by a factor of four. And according to (13), halving D also reduces the average power by a factor of four, whereas according to (4) and (5) it only reduces it by a factor of two.

So the relative error E_{calc} in the calculation power due to the incorrect use of (1) - (5) is:

$$E_{calc} = \left(\frac{D}{D^2} - 1\right) \cdot 100\% = \left(\frac{1}{D} - 1\right) \cdot 100\%. \quad (17)$$

For example, for D = 0.9, the error is:

$$E_{calc} = \left(\frac{1}{0.9} - 1\right) \cdot 100\% = 11\%. \quad (18)$$

And for D = 0.5, which is the middle of the range of values most commonly used in practice for building PWM principle SVCs, it is:

$$E_{calc} = \left(\frac{1}{0.5} - 1\right) \cdot 100\% = 100\%. \quad (19)$$

The value error E_{calc} in (19) corresponds exactly to the discrepancy between the results of calculations (8) and (9) and calculation (10) and the readings of the XWM1 wattmeter in Figure 4.

To illustrate one aspect of the practical consequences of the above calculation error, we will use the preface in [7]. They state: “The power and thermal behavior of fixed linear resistors are mostly based on DC or RMS loads, but pulse loads, like single energy pulse or a continuous flow of pulses, become more and more an important factor in professional electronics”.

And an overestimated result of calculating the power and thermal behavior of resistors will lead to the choice of a higher resistor power than would otherwise be possible.

This facilitates the mode of operation of resistors and increases the reliability of electronic devices.

That is, in this case, the consequence of an overestimated calculation result can be seen as 'positive'. This is most likely the reason why it has not been paid attention to so far.

However, in other practical applications, the consequences of this error can be negative. This is mainly concerned with energy losses, especially in the active resistance of primary power sources and other SVCs components, as it makes it impossible to accurately evaluate their efficiency. Consequently, this can slow down their development.

At the same time, the existing approach of idealizing the conditions in order to evaluate the energy accumulation efficiency, for example in capacitors, which is reviewed in [1], leads to the fact that the shown error has not been detected so far.

This has also been facilitated by the fact that the use of formulae (1) - (5) is legitimate for voltage and current, masking the fact that their use is incorrect for power.

5.An example of the correct use of the average value formula according to the type of function describing the physical quantity being calculated

As an example of how the physical nature of the calculated parameter and the function by which it is described should correspond to the scope of formula (1) in its original form and its writing (4), (5) for applied calculations, we can have a look at the formulas given in [5, p. 6] for calculating the power and energy of light pulses emitted by a laser.

These formulae are similar to formula (4), (5) and involve calculations using the first power of the duty cycle D. But in this case, there is no incorrectness in the use of formulas.

This is because, unlike the impulse power of the conduction current, which is a composite function of voltage and current product, the impulse power of the main physical quantity SI and ISQ, the intensity of light, is described by an elementary function of the type f(x).

The power of light pulses depends on the intensity of the photon flux in a region of space, just as the power of a current depends on the intensity of the electron flux in an electric circuit.

But there is a difference between them, namely that the intensity of the flow of electrons depends on the voltage applied to the same part of the circuit for which the power is calculated. Therefore, both voltage and current are included in formulae (11) and (14) to calculate the power.

Instead, the intensity of the photon flux is proportional to the current under the influence of the voltage applied to the laser working body only during its excitation. And once the photons have escaped into space outside the laser working body, the effect of this voltage ends. Therefore, this voltage is not taken into account when calculating the power and energy of laser light pulses, which are determined solely by the intensity of the photon flux outside the laser working body.

Hence, the average power of light pulses is described by an elementary function whose argument is the intensity of the photon flux in space, and its calculation should be performed using formula (1) in its original form, or its writings for applied calculations, as (4) and (5).

6.Proposed formula for calculating the average power of rectangular conduction current pulses in a linear active resistance

To correctly apply formula (1) for composite functions of the form f(g(x)) on some interval [a, b], it is necessary to consider that they are formed by separate elementary functions of the form f(x). Therefore, each of these elementary functions must be separately averaged over the interval [a, b].

Consequently, for composite functions in the form of a product of elementary functions, an empirical generalization of the original form of formula (1) can be proposed as follows:

$$f_{avg} = \int_a^b \left(\frac{f_1(x)dx}{b-a} \cdot \frac{f_2(x)dx}{b-a} \cdot \ldots \cdot \frac{f_n(x)dx}{b-a} \right) = \\ = \left(\frac{1}{b-a}\right)^n \int_a^b (f_1(x)dx \cdot f_2(x)dx \cdot \ldots \cdot f_n(x)dx), \tag{20}$$

where n - serial number of the elementary function.

Therefore, formula (1) in its original form, or as it is written for applied calculations in the form of (4) and (5), is a particular case of (20) for n = 1.

Formula (20) is a generalization of the empirically obtained formula (13). It is applicable to composite functions which are the product of elementary functions or their higher powers and rectangular impulses. The work on the justification of the analytically rigorous derivation of formula (20) is ongoing. It is also likely that other formulae will need to be derived for other operations on elementary functions or for pulses of other shapes, but that is beyond the scope of this article.

And to calculate the average power of rectangular pulses of conduction current, in particular in the active resistance of batteries, to evaluate the efficiency of SVCs designed according to the PWM principle, we propose formula (20) in written by (13).

7.Conclusion

To this day, practical calculations of the average power of rectangular pulses of conduction current in the active resistance, which is described by a composite function in the form of the product of voltage and current, are performed using the formula, which in its original form was derived in mathematics by default for elementary functions.

As a consequence of such a methodologically incorrect use of this formula, the results of the average power calculation are overestimated. In some cases, this overestimation does not have critically negative consequences, as it leads to more cautious than necessary decisions. However, it may hinder the search for ways to further develop, for example, SVCs based on the PWM principle or other pulse nodes of robotics, renewable and solar energy sources, etc.

In order to eliminate the existing methodological incorrectness, a formula is proposed for calculating the average power of rectangular conduction current pulses, which can also be used for other parameters described as a product of elementary functions.

At the same time, the issue of establishing the need for further generalization of the formula for calculating average values for parameters described by composite functions of all types, or for deriving separate formulae for specific composite functions and impulses of any shape, not just rectangular, remains unresolved.

Declaration on Generative AI

The authors have not employed any Generative AI tools.

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