Stable Algorithms for Adaptive Control and Adaptation of Uncertain Dynamic Objects Based on Reference Models

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Abstract. The article proposes procedures for the formation and construction of stable algorithms for adaptive control and adaptation of indefinite dynamic objects that ensure the system's operability when changing coordinate and parametric perturbations within a sufficiently wide range. To solve the problem of synthesis of control algorithms for a priori indeterminate objects, the speed gradient method and signal adaptation algorithms are used. In this case, the differential components of the algorithm monitor the parametric perturbations that are slowly changing over a wide range. When constructing stable control algorithms, algorithms for pseudo-circulation of redefined matrices based on skeletal and singular value decompositions are used to ensure the convergence of the desired solution. The given stable algorithms for adaptive control of indefinite dynamic objects based on the used effective pseudo-circulation algorithms contribute to improving the accuracy of determining the parameters of the control law.

Keywords: undefined dynamic object, adaptive control, algorithms of adaptation, speed gradient method, algorithms for signal adaptation.

1 Introduction

An analysis of the scientific and technical literature [1-5] concerning the questions about the origin of model uncertainty shows that, in general, different versions of the classification of its sources coincide. No mathematical model can accurately and completely describe a real object or process [1-5]. This is due to many reasons, and therefore a nominal model of an object or process cannot be considered complete without quantifying possible errors, hereinafter referred to as model uncertainty. This is usually quite a time-consuming model, and if an incorrect description of the uncertainty is used, then erroneous results can be obtained. Uncertainties can be described in various ways, they may be namely, by bounds on the parameters of a linear model, bounds on non-linearity, bounds in the frequency domain, bounds in the stochastic definition based on entropy, etc.

The need to manage systems with avoidable uncertainties has led to the emergence of a whole class of adaptive control systems. An adaptive controller typically contains a control device and an adaptation device. Various definitions of the term adaptive control [4-7] characterize the complexity of the task-the construction of the laws of control of systems in the conditions of structural or parametric uncertainty [7, 8]. In this regard, there is a need to develop adaptive control systems that allow for high-quality system functioning in conditions where the control object differs from the calculated model or when its mathematical model is not known or is not complete [4, 6, 8, 9]. In [8], the problem of adaptive output control of parametrically and functionally indeterminate objects is considered.

The following methods are used to construct adaptive control systems: the direct Lyapunov method, the stochastic approximation method, the method of recurrent target inequalities, the speed gradient method, the quadratic criterion of absolute stability, methods based on the identification approach [3, 7, 10, 11], and others.

Depending on the amount of a priori information, adaptation algorithms are constructed using gradient methods that are used for undefined object parameters and deterministic external perturbations, as well as methods based on statistical theory [12]. In this case, the rules of the distribution of the object parameters and perturbations are assumed to be known.

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2 Problem definition

The adaptive approach considers nonlinear equations of an object of the form:

$$\dot{x}(t) = \varphi(x, u, \chi, f),\ x(t_0) = x^{(0)}, \tag{1}$$

$$y = w(x, u, \chi, v). \tag{2}$$

Most often, instead of equations (1), (2), linearized equations are used:

$$\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)r(t), \tag{3}$$

where $x(t) \in R^n$, $y(t)$ – vectors of the state and measured variables of the object, respectively; $r(t) \in R^m$, $u(t)$ – vectors of driving and controlling actions, respectively; A and B – matrices of unknown object parameters with corresponding dimensions; ΔA and ΔB – matrices of configurable parameters; $f(t)$, $v(t)$ – vectors of external disturbances and measurement interference; φ, w - known vector functions of their arguments.

The purpose of the adaptation is to ensure that the vector $x(t)$ tends to the state vector of the reference model $x_M(t) \in R^n$, which is the solution of the equation

$$\dot{x}_M(t) = A_M x_M(t) + B_M r(t), \tag{4}$$

where $n \times n$ -matrix A_M and $n \times m$ -matrix B_M (A_M – the hurwitz matrix) set the desired movement dynamics.

To get the adaptation algorithm, select the target functional

$$Q(x) = \frac{1}{2} e^T P e, \tag{5}$$

where $e = e(t) = x(t) - x_M(t)$ – error vector, $P = P^T > 0$ – positive definite $n \times n$ -matrix.

We find the derivative of the target functional by virtue of the system of equations (3)

$$\omega(x,t) = \dot{Q} : \dot{Q} = e^T P((A + \Delta A)x + (B + \Delta B)r(t) - A_M x_M - B_M r(t)).$$

It is required to construct a feedback law $u = U(x)$, which provides for a given set of initial conditions $\Omega \subset R^n$ to achieve the control goal

$$\lim_{t \to \infty} Q(x(t, x_0)) = 0,$$

where $x(t, x_0)$ - solution of system (3), (5) with initial condition $x_0 \in \Omega$.

One of the main methods for synthesizing algorithms for controlling a priori indeterminate objects is the speed gradient method. The speed gradient method, originally formulated for adaptive control problems, has many applications [4, 6, 14]. The method is based on the use of Lyapunov functions and requires setting the control goal. Several types of velocity gradient algorithms are described: algorithms in differential and finite forms, built on a local or integral target functional. The conditions for achieving the goal, convergence, and robustness of the algorithms are established. Some control problems were successfully solved with the help of algorithms constructed using the speed gradient method [13-16].

3 Synthesis of adaptive control and adaptation of undefined dynamic objects

To solve the problem, you can use speed gradient algorithms that have the form

$$d\Delta A / dt = -\Gamma P e x^T,\ d\Delta B / dt = -\Gamma P e r^T, \tag{6}$$

where $\Gamma = \gamma I_n$, $\gamma > 0$ – multiplier step.

When a perturbation is applied to the control object (3), the algorithm (6) should be regularized to prevent an unlimited growth of the configurable parameters. We will choose the regularizing function in the form $\xi(\theta)=\alpha\theta,\ \alpha>0$. Then the algorithms (6) will take the form

$$\begin{aligned} d\Delta A/dt &= -\gamma\left[Pex^T+\alpha(\Delta A-\overline{\Delta A})\right], \\ d\Delta B/dt &= -\gamma\left[Per^T+\alpha(\Delta B-\overline{\Delta B})\right], \end{aligned} \tag{7}$$

where $\overline{\Delta A}$ and $\overline{\Delta B}$ – some a priori estimates ΔA and ΔB.

The above results are also valid in the case when not all the coefficients of the equation of the object (3) are available to the setting [4].

We establish the conditions under which algorithms (6), (7) are identifying, i.e.

$$A+\Delta A(t)\to A_M,\ \ B+\Delta B(t)\to B_M \text{ in case of } t\to\infty.$$

From (3), (4) we get the equation for the error $e(t)=x(t)-x_M(t)$:

$$\dot{e}=A_M e+(A+\Delta A-A_M)(e+x_M)+(B+\Delta B-B_M)r(t). \tag{8}$$

You can show it [4, 6, 13, 14], what if the vector function $col(x_M(t),r(t))$ is integrally non-degenerate, i.e., the reference model (4) is sufficiently fully excited by the input action $r(t)$ (for example, if the model (4) is controllable, and the spectrum of the function $r(t)$ contains at least n frequencies).

Consider a control problem with a reference model for an op-amp

$$\dot{x}(t)=Ax(t)+Bu(t),$$

where $x(t)\in R^n$, $u(t)\in R^m$, $r(t)\in R^m$.

According to the speed gradient scheme, we get

$$\dot{Q}=\omega(x,\theta,t)=e^T P(Ax+Bu-A_M x_M-B_M r(t)).$$

Let for any x, $x_M\in R^n$, $r\in R^m$ the equation

$$Ax+Bu_*-A_M x_M-B_M r(t)=A_M e \tag{9}$$

solvable with respect to $u_*\in R^m$. Then u_* satisfies the relation

$$u_*=K_r^* r(t)+K_x^* x, \tag{10}$$

where $K_r^*=B^+B_M$, $K_x^*=B^+(A_M-A)$.

In other words, $A_M-A\in L(B)$, $B_M-B\in L(B)$, where $L(B)$ is a linear subspace generated by the columns of the matrix *B*. The latter conditions, in turn, are equivalent to the relations

$$rankB=rank\{B,B_M\}=rank\{B,A_M-A\}. \tag{11}$$

Conditions (11) are called adaptability conditions [4]. As a vector of configurable parameters, we choose vector $\theta=col(K_x,K_r)$ and proceed to the synthesis of the adaptation algorithm. In this case, the speed gradient has the form:

$$\nabla_{K_x}\omega(x,\theta,t)=B^T Pex^T,\ \nabla_{K_r}\omega(x,\theta,t)=B^T Per^T.$$

Choosing the speed gradient algorithm in the form [6] we get

$$u=K_r r(t)+K_x x, \tag{12}$$

$$dK_r dt = -\gamma B^T Per(t), \;\; dK_x / dt = -\gamma B^T Pex^T .$$

The algorithm (12) is operable when the coordinate and parametric perturbations change within a sufficiently wide range. If the parametric perturbation changes at a high rate, the adaptation process deteriorates. In this case, it is advisable to use signal adaptation algorithms.

Signal adaptation algorithms are faster and less cumbersome, but they work in a narrower range of changes in object parameters than a parametric algorithm. If the operation of the main loop or the adaptation loop requires the measurement of all variables of the control object state, then an observer can be introduced into the system [17].

In systems with signal-parametric adaptation, the signal tuning law is usually chosen as a relay law in order to ensure high performance in the system and to compensate for rapidly changing parametric disturbances. Parametric tuning laws include integral components that compensate for parametric and coordinate perturbations that change over wide ranges, but slowly.

In [17], the static and dynamic properties of robust structures of linear and nonlinear self-oscillating stabilization systems with complete invariance of static errors under constant influences without the use of integrating elements in the main contour of the system are studied. It is shown that the systems have a low sensitivity to variations in the parameters of the object and control, have a greater "survivability".

Figure 1 shows the structure of adaptive control with a reference model, with signal and parametric settings [6].

It can be shown [4, 13] that equation (9), when the conditions (11) are met, is uniquely solvable with respect to $u_* \in R^m$ for any $x, x_M \in R^n$, $r \in R^m$. To find u_*, we use the relations

$$u_* = K_{xM}^* x_M + K_r^* r + u_s^* , \tag{13}$$

where $K_{xM}^* = B^+(A_M - A)$, $K_r^* = B^+ B_M$, $u_s^* = B^+(A_M - A)$.

Thus, under the conditions of adaptability, there is a matrix $P = P^T > 0$ and a vector function u_* (13), such that $\omega(x, \theta_*, t) \le -e^T G e \le -\alpha_0 Q$. P is found from the Lyapunov equation $PA_M + A_M^T P = -G$ at some $G = G^T > 0$. In the main contour, similarly to (12), we take a controller of the form

$$u = K_{xM} x + K_r r + u_s ,$$

where K_{xM}, K_r and u_s – configurable parameters that form a vector $\theta = col(K_{xM}, K_r, u_s)$.

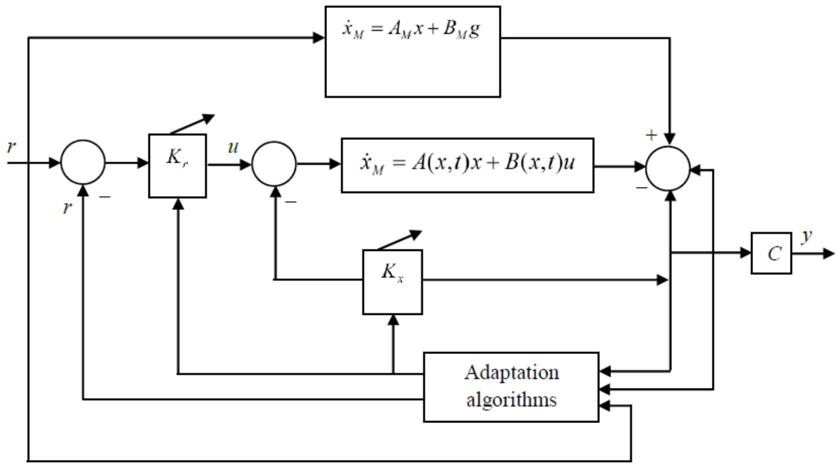

Figure 1: Block diagram with reference model with parametric and signal configuration

4 Stable efficient algorithms for pseudo-inversion of redefined matrices

When calculating according to equations (10) and (13), the pseudo-inverse matrix is represented as follows:

$$B^{+} = (B^{T}B)^{-1}B^{T} . \tag{14}$$

It is known [18-20] that the problem of calculating a pseudo-inverse matrix is generally unstable with respect to the errors in setting the original matrix. In this case, the errors of the initial data naturally depend on the accuracy of the experimental studies, and the characteristics of the calculated process depend on the degree of adequacy of the model to the real process. The influence of rounding errors produced during the implementation of the computational procedure on the accuracy of the desired solution can be analyzed on the basis of known methods of analysis and accuracy balance [19, 21]. In view of this circumstance, there is a need to use efficient algorithms for pseudo-conversion of redefined matrices.

In many typical situations, for example, when analyzing inverse problems, mathematical models lead to irregular equations. With the rejection of regularity, the problem of constructing numerical methods is obviously complicated, and this complication is fundamental [18, 22, 23].

Let's consider the most effective ways to pseudo-transform redefined matrices. Let be a matrix composed of m practically linearly independent columns of the matrix, and let be a matrix calculated from the matrix equation:

$$MC = N ,$$

where is a matrix composed of some r practically linearly independent rows of the matrix, and the matrix is composed of elements lying at the intersection of the matrices and [19, 21]. Then let's put

$$B = DC .$$

Thus, we assume that in formula (5), instead of B^{+}, we take the matrix

$$B^{+} = C^{+}D^{+} = C^{T}(CC^{T})^{-1}(D^{T}D)D^{T} .$$

The method of effective pseudo-circulation, based on the singular value decomposition of the matrix B [18-22], i.e. on its representation in the form of

$$B = USV^{T} ,$$

where U – orthogonal $(n\times 2p)$ – matrix; V – orthogonal $(m\times 2p)$ – matrix; S – diagonal $(2p\times 2p)$ – matrix.

Columns u_i and v_i of the U and V matrices are the eigenvectors of the $B^{T}B$ matrix, and the diagonal elements μ_i of the S matrix are the positive roots of the eigenvalues λ_i of the $B^{T}B$ matrix.

The Moore-Penrose pseudo-inverse matrix B^{+} allows us to obtain an estimate of

$$B^{+} = VS^{+}U^{T} = \sum_{i=1}^{r}\frac{1}{\mu_i}v_i u_i^{T} ,$$

where $S^{+} = diag(s_1^{+},\dots,s_i^{+})$ – pseudo-inverse matrix for a matrix S; r – rank matrix B, that is, the number of non-zero singular numbers $\mu_i (i=\overline{1,p})$; $s_i^{+} = 1/\mu_i$, if $\mu_i \neq 0$, and $s_i^{+} = 0$, if $\mu_i = 0$.

The algorithm (14) cannot be used directly in the case where $B^{T}B$ is irreversible [18]. If $rangB \neq m$, then the calculation of B^{+} is almost impossible [19]. Until recently, researchers have tried to overcome the difficulties that arise by relaxing the requirement of regularity. In particular, the case when the image $B^{T}B$ is closed and has finite codimension is studied in sufficient detail.

There are many works of a theoretical and applied nature that study this case [18, 23]. Studies on generalized regularity cover mainly only the finite-dimensional case. These two circumstances do not allow us to use these results in many inverse problems, optimization problems, etc.

According to [19, 21], we represent the symmetric matrix in the diagonal form:

$$B^T B = TUT^T ,$$

where $T = (t_1 \vdots t_2 \vdots \dots \vdots t_{m+n})$ – is a block orthogonal, and U is a diagonal matrix.

Then, according to [19], we can write:

$$(B^T B)^+ = \sum_{i=1}^{m+n} \lambda_i^{-1} t_i t_i^T ,$$

where $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_{m+n} > 0$ – eigenvalues of the matrix $B^T B$.

5 Conclusion

Thus, the use of speed gradient algorithms in adaptive control problems allows us to conclude that the differential components of the algorithm track the parametric perturbations that are slowly changing over a wide range. On the basis of the speed gradient method, recommendations can be made on the choice of the structure of the main contour, i.e., in the end, the method provides a single approach to the synthesis of the main contour and the contour of the adaptation of the system. The given stable algorithms for adaptive control of indefinite dynamic objects based on the use of effective pseudo-circulation algorithms contribute to improving the accuracy of determining the parameters of the control rule.

6 References

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