Multidimensional signals interpolation based on NEDI for HGI compression

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Abstract. Adaptive interpolation of multidimensional digital signals is considered. An adaptive algorithm for digital signals interpolation is proposed, intended for hierarchical compression. The prototype of the proposed interpolator is the NEDI (New Edge-Directed Interpolation) algorithm. In this paper, the NEDI interpolation algorithm is modified for use on special hierarchical grids, which are used for hierarchical signal compression. Experimental researches of the proposed interpolator are performed with the hierarchical compression of natural digital signals. Experiments confirm that the proposed adaptive interpolator allows improving the efficiency of hierarchical compression of digital signals.

1. Introduction

Availability of digital information processing devices continues to increase. This entails an increase in the data size of processed digital signals, and this problem can not be solved by increasing capacity of storage devices. Moreover, multidimensional signals, including multi- and hyperspectral [1-3] remote sensing data, as well as results of sensing by quadrocopters and other unmanned aerial vehicles, are also becoming more accessible. This further exacerbates the problem of an excessively large size of digital signal data. The only acceptable solution at the moment is compression of digital signals [4-5].

To date, there are many [4-8] methods of compression of digital signals. The most popular of these methods is the JPEG compression method [11], based on discrete cosine transform (DCT) [9] and subsequent entropy coding [10] of transformants (DCT results). The more efficient [12] compression method JPEG-2000 [13], which uses the discrete Wavelet transform [14], is much less widely used.

These methods of the JPEG group are used very widely, due to wide variety of hardware devices in which they are embedded. However, there are a number of problems that raise requirements for the quality of compressed digital data. First of all, this is polygraphy and processing of remote sensing data. In these areas, one has to deal with digital signals, which are unique. When compressing such signals, strict quality control is necessary. In addition, such signals may also have a high bit capacity. Moreover, such signals can have substantially more than three spectral bands (hyperspectral signals often have hundreds of spectral bands). In other words, when compressing such signals, complexity can arise already at the stage of processing data of specific formats.

Fractal [15] compression methods, according to the author, currently have the largest compression ratio. However, their propagation is difficult due to specific, in most cases unacceptable signal distortions, as well as excessively high computational complexity.

Also, it should be noted an important drawback, corresponding to all the above methods of signal compression. This drawback follows from the need to transform the signal into a corresponding space of transformation coefficients. Accordingly, it is not always possible to control the error in the specified space of coefficients. For the mean-square error, such control is possible in a number of cases due to Parseval's equality. But for more strong quality measures, for example for maximum error, the specified error control for the above compression methods is usually impossible.

In the author's opinion, using specific compression methods that do not require the transformation to spectral (or any other) auxiliary spaces is promising in specific areas that raise high demands to the quality of digital signals. In this paper, the method of hierarchical compression is chosen as such method [16, 17]. This method is based on multiple non-redundant resampling of initial array of signal samples and interpolation of signal samples based on the specified resampled arrays.

Hierarchical compression methods have a number of important advantages, such as fast multiscale access to fragments of compressed data, the ability to control the speed of formation of a compressed data stream, the possibility of increasing noise immunity and the possibility of error control (including the maximum error [18]). The task of research and further increasing the efficiency of hierarchical compression methods of digital signals is certainly topical.

An important step in hierarchical compression methods is an interpolator in which samples of more resampled signal are used to interpolate samples of less resampled signal. The most common algorithm of hierarchical interpolation is simple averaging [19-20] from the nearest signal samples of more resampled hierarchical levels of the signal. However, the averaging interpolator is not effective enough, because it is not adaptive (it performs in the same way, regardless of local signal characteristics).

One of the ways to take into account the local characteristics of a digital signal is context modeling [21-23], which has become widespread, in particular, in statistical coding [7]. In the simplest case, the context for a next encoded symbol is the previous symbol (or several previous symbols), and the context model is the estimation of conditional probability distribution of the encoded symbol. Taking into account the context, that is, using the conditional probability distribution instead of the unconditioned distribution makes it possible to increase the algorithm adaptability to variable statistical properties of the signal, which leads to an increase in the efficiency of the compression method as a whole.

In this paper, context modeling is used for the development of adaptive interpolation algorithms that are part of a compression method based on hierarchical grid interpolation (HGI). The proposed adaptive interpolators allow increasing the efficiency of the hierarchical compression method.

For the hierarchical method of signal compression, an interpolator based on the NEDI algorithm [24] using context modeling is proposed. When developing this interpolation algorithm, a set of surrounding signal samples is considered as a context for each signal sample.

2. Hierarchical compression of multidimensional signals

Hierarchical grid interpolation (HGI) [16, 25-26] is based on special hierarchical representation of an integer nonnegative multidimensional signal $\mathbf{X}=\left\{x\left(\vec{c}\right)\right\}$ in the form of a set of hierarchical levels $\mathbf{X}_l$:

$$\mathbf{X}=\bigcup_{l=0}^{L-1}\mathbf{X}_l\ ,\quad \mathbf{X}_{L-1}=\left\{x_{L-1}\left(\vec{c}\right)\right\},\quad \mathbf{X}_l\quad=\left\{x_l\left(\vec{c}\right)\right\}\setminus\left\{x_{l+1}\left(\vec{c}\right)\right\},\ l<L-1\,,$$

where L is the number of hierarchical levels $\mathbf{X}_l$, $\{x_l\left(\vec{c}\right)\}$ is the signal resampled with step 2^l, $\vec{c}$ is the vector of multidimensional signal arguments.

With hierarchical compression, the hierarchical levels $\mathbf{X}_l$ are compressed sequentially, from the highest (most resampled) level $\mathbf{X}_{L-1}$ to the lower levels. The proportion of data size of the highest level $\mathbf{X}_{L-1}$ is sufficiently small already for $L\geq 4$, so the compression algorithm of this level does not matter. So, only compression algorithm of any "non highest" hierarchical level $\mathbf{X}_l$, $l<L-1$ is described.

Stage No. 1. Interpolation.

Interpolation of samples of the current signal level $\mathbf{X}_l$ is based on the samples $\{\bar{x}_k\left(m,n\right),\,k>l\}$ of already processed hierarchical levels $\{\,\overline{\mathbf{X}}_k\,,k>l\,\}$:

$$\hat{x}_l(\vec{c}) = P\left(\bigcup_{k=l+1}^{L-1} \overline{X}_k\right), \tag{1}$$

where $\hat{x}_l(\vec{c})$ is interpolating value, $P(..)$ is a function that defines a certain interpolator. Further, interpolators are considered in detail.

Stage No. 2. Calculation of difference signal.

The differences between the initial $x_l(\vec{c})$ and interpolating $\hat{x}_l(\vec{c})$ (1) values of the current level samples are calculated:

$$f_l(\vec{c}) = x_l(\vec{c}) - \hat{x}_l(\vec{c}). \tag{2}$$

Stage No. 3. Quantization.

The difference signal (2) is quantized by the quantifier with a uniform scale ([..] is the integer part of a number):

$$q_l(\vec{c}) = sign\left(f_l(\vec{c})\right)\left[\left(\left|f_l(\vec{c})\right| + \varepsilon_{\max}\right)\Big/\left(2\varepsilon_{\max}+1\right)\right], \tag{3}$$

allowing to control the maximum error [19] $\varepsilon_{\max}$:

$$\left|f_l(\vec{c})\right| = \left|x(\vec{c}) - \overline{x}(\vec{c})\right| \le \varepsilon_{\max} . \tag{4}$$

The quantized signal (3) is then compressed by an entropy encoder and stored in an archive.

Stage No. 4. Recovering.

The restored values of the signal samples are calculated (already during compression):

$$\overline{x}_l(\vec{c}) = \hat{x}_l(\vec{c}) + \left(1+2\varepsilon_{\max}\right) q_l(\vec{c}), \tag{5}$$

which are necessary for interpolation (1) of the following (more resampled) hierarchical levels $\{\mathbf{X}_k, k<l\}$ of signal.

3. Averaging interpolation for hierarchical signal compression

For reasons of computational complexity of interpolation under hierarchical compression, we usually [19-20] use trivial averaging over the nearest already restored signal samples of more resampled hierarchical levels. To simplify the algorithm description, the averaging interpolator is considered for a two-dimensional signal $\mathbf{X} = \left\{x(\vec{c})\right\} = \left\{x(m,n)\right\}$.

Let's consider two types of interpolated samples: "type I" with indices of the form $(2m+1,2n+1)$ and "type II" with indices of the form $x_l(2m+1,2n)$ and $x_l(2m,2n+1)$. Let's consider the simplest way of processing samples of both samples types when using the averaging interpolator. Interpolation is performed based on the restored samples of previous (more resampled) hierarchical level:

$$\hat{x}_l(2m+1,2n) = \left[\left(\overline{x}_{l+1}(m,n) + \overline{x}_{l+1}(m+1,n)\right)\Big/2\right], \tag{6}$$

$$\hat{x}_l(2m+1,2n+1) = \left[\left(1+\overline{x}_{l+1}(m,n)+\overline{x}_{l+1}(m+1,n)+\overline{x}_{l+1}(m,n+1)+\overline{x}_{l+1}(m+1,n+1)\right)\Big/4\right] \tag{7}$$

This averaging interpolation algorithm has low computational complexity. But this algorithm has insufficiently high efficiency, since it does not take into account any local signal characteristics.

4. Context modeling for data compression

Application of context modeling approaches [21-23] for data compression is based on the «modeling and coding» idea proposed by Rissanen and Langdon [21]. In accordance with this idea, the compression procedure consists of two steps: modeling and coding.

By modeling, we mean the construction of model of information source that generates compressed data. By coding, we mean the process of reducing data size based on the results of modeling. So, the "coder" creates a compressed stream, which is a compact form of the processed data, based on the information supplied by the "modeler".

Let the probability of the symbol s_i be $p(s_i)$. From Shannon's theorem [7] on coding the source of messages, it is known that the symbol s_i is best represented by $-\log_2 p(s_i)$ bits. Often the source structure is unknown or complex, so it is necessary to build a source model that allows you to find an estimate of the probability $\overline{p}(s_i)$ of each symbol s_i.

Estimation of the symbol probability during the simulation is performed on the basis of known (or estimated) symbols statistics and, possibly, a priori assumptions. Therefore they speak about statistical modeling. In other words, the "modeler" predicts the probability of each symbol.

At the stage of statistical coding, the symbol s_i is replaced by a code with a length of $-\log_2 p(s_i)$ bits. The more accurate the estimation of the signals probability, the more effective the codes are, and the more effective the compression as a whole.

5. Signal interpolation based on NEDI

In the initial setting, context modeling is used for entropy coding, but we use context modeling for interpolating signals. In particular, in terms of context modeling, the New Edge-Directed Interpolation (NEDI) [24] can be described.

Let's describe the interpolation algorithm NEDI. Let $\mathbf{X} = \{x(m,n)\}$ be the original signal of size *WxH*. We need to calculate a signal $Z = \{x(m,n)\}$ of size 2*Wx*2*H* with twice the best resolution. If we use the NEDI, this signal is calculated as follows. Signal samples with even indices are equal to samples of the original signal:

$$z(2m,2n) = x(m,n)\,. \tag{8}$$

The signal samples with indices $(2m+1, 2n+1)$ are calculated (see Figure 1) as follows:

$$z(2m+1,2n+1) = \sum_{i=0}^{1}\sum_{j=0}^{1}\alpha_{2i+j}\,x(m+i,n+j)\,, \tag{9}$$

where $\alpha_i,\ i = 0..3$ are local interpolation coefficients.

So the missing signal samples are calculated as a weighted sum of the four nearest reference signal samples. In this case, it is necessary to calculate the optimal local weighting coefficients $\alpha_i,\ i = 0..3$. These coefficients are calculated by the optimization of the quadratic error:

$$\varepsilon^2 = \sum_{i,j\in D}(x(i,j)-z(i,j))^2 \to \min_{\alpha_0,\alpha_1,\alpha_2,\alpha_3}\,, \tag{10}$$

where *D* is area of interpolation coefficients estimation. This area includes *N* signal samples.

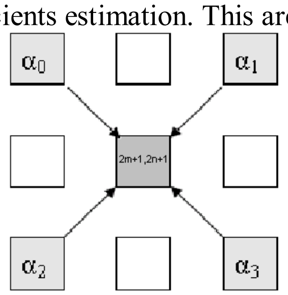

Figure 1. Location of reference signal samples for NEDI interpolation

We place the samples $x(i,j)$ from the estimation area *D* into the array $\vec{Y} = [y_0, y_1, ..., y_{N-1}]^T$. The elements of the array $\vec{Y}$ are assigned to signal samples from the estimation area *D*, starting from the upper left corner and then according to the progressive scan of area (see Figure 2). Also consider the matrix

$$C = \begin{bmatrix} c_{0,0} & \cdots & c_{0,3} \\ \cdots & \cdots & \cdots \\ c_{N-1,0} & \cdots & c_{N-1,3} \end{bmatrix}. \tag{11}$$

Each row of this matrix consists of reference signal samples, which are used to interpolation of the samples of the estimation area (see Figure 2).

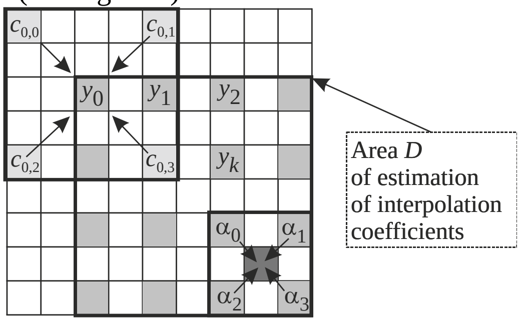

Figure 2. Estimation area *D* of the interpolation coefficients of the NEDI algorithm.

So criterion (10) is represented as:

$$\varepsilon^2 = \sum_{k \in D} (y_k - C_k \vec{\alpha})^2 \to \min_{\vec{\alpha}}, \tag{12}$$

where C_k is the row number k of the matrix C, $\vec{\alpha} = [\alpha_0, \alpha_1, \alpha_2, \alpha_3]^T$ is the vector of unknown interpolation coefficients.

To solve the optimization problem (12), we equate to zero the following partial derivatives:

$$\begin{cases} \dfrac{\partial \varepsilon^2}{\partial \alpha_0} = (y_0 - (\alpha_0 c_{0,0} + \alpha_1 c_{0,1} + \alpha_2 c_{0,2} + \alpha_3 c_{0,3})) \cdot c_{0,0} + \ldots + (y_{N-1} - (\alpha_0 c_{N-1,0} + \alpha_1 c_{N-1,1} + \alpha_2 c_{N-1,2} + \alpha_3 c_{N-1,3})) \cdot c_{N-1,0} = 0 \\ \dfrac{\partial \varepsilon^2}{\partial \alpha_1} = (y_0 - (\alpha_0 c_{0,0} + \alpha_1 c_{0,1} + \alpha_2 c_{0,2} + \alpha_3 c_{0,3})) \cdot c_{0,1} + \ldots + (y_{N-1} - (\alpha_0 c_{N-1,0} + \alpha_1 c_{N-1,1} + \alpha_2 c_{N-1,2} + \alpha_3 c_{N-1,3})) \cdot c_{N-1,1} = 0 \\ \dfrac{\partial \varepsilon^2}{\partial \alpha_2} = (y_0 - (\alpha_0 c_{0,0} + \alpha_1 c_{0,1} + \alpha_2 c_{0,2} + \alpha_3 c_{0,3})) \cdot c_{0,2} + \ldots + (y_{N-1} - (\alpha_0 c_{N-1,0} + \alpha_1 c_{N-1,1} + \alpha_2 c_{N-1,2} + \alpha_3 c_{N-1,3})) \cdot c_{N-1,2} = 0 \\ \dfrac{\partial \varepsilon^2}{\partial \alpha_3} = (y_0 - (\alpha_0 c_{0,0} + \alpha_1 c_{0,1} + \alpha_2 c_{0,2} + \alpha_3 c_{0,3})) \cdot c_{0,3} + \ldots + (y_{N-1} - (\alpha_0 c_{N-1,0} + \alpha_1 c_{N-1,1} + \alpha_2 c_{N-1,2} + \alpha_3 c_{N-1,3})) \cdot c_{N-1,3} = 0 \end{cases}$$

The solution of this system of equations:

$$\vec{\alpha} = (C^T C)^{-1} \cdot (C^T \vec{Y}), \tag{13}$$

This expression allows us to calculate the optimal interpolation coefficients $\alpha_i, i = 0..3$. The advantage of considered context NEDI algorithm is the adaptability to local signal characteristics, the disadvantage is high computational complexity. By context, in this case, we mean the set of surrounding reference samples that constitute the area of parameters estimation. By context modeling, in this case, we mean the local estimation of interpolation coefficients.

6. NEDI-based interpolation algorithm for hierarchical signal compression

In this paper, we propose a modification of the NEDI interpolation algorithm for non-redundant hierarchical sample grids, which are used for hierarchical signal compression. At each "non-highest" hierarchical level $\mathbf{X}_l$, $0 \le l \le L-2$, an estimation area D is chosen for interpolation of each sample of hierarchical level. This area D consists of already restored samples of previous (more resampled) hierarchical levels $\mathbf{X}_k$, $l+1 \le k \le L-1$, which are stored into array $\vec{Y}_l$. The "own" matrix C_l (see the previous section) corresponds to each element of this array.

In Figure 3, for two hierarchical levels, the array's $\vec{Y}_l$ samples are shown in light color, the matrix elements belonging to the hierarchical level $\mathbf{X}_1$ are shown in dark color (the interpolated sample belongs to the hierarchical level $\mathbf{X}_0$).

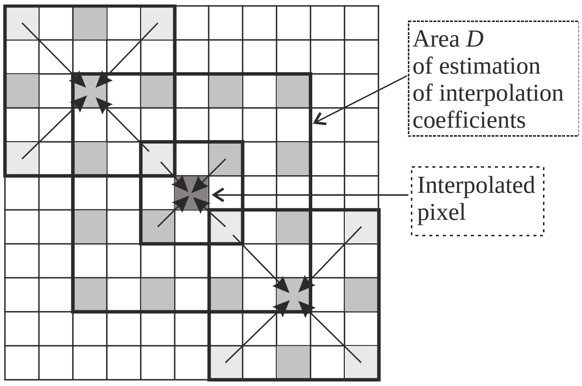

Figure 3. Estimation area D of the NEDI interpolation coefficients for hierarchical compression.

Estimation of local interpolation coefficients is performed in the same way as expression (13), taking into account the described specificity of estimation areas:

$$\vec{\alpha}_l = (C_l{}^T C_l)^{-1} \cdot (C_l{}^T \vec{Y}_l) . \qquad (14)$$

First, the algorithm described above is applied to interpolation of samples with indices of the form ($2i$+1, $2j$+1), then the interpolation coefficients for the remaining samples are computed (similarly, the entire situation is rotated by 45 degrees).

The advantage of the proposed interpolation algorithm is its adaptability to a context (local characteristics of the signal in a small neighborhood of current sample). This increases the accuracy of interpolation of the signal sample. The disadvantage of the algorithm is high computational complexity of calculating the interpolation coefficients.

It should be noted that it is not always possible to compute an inverse matrix to the matrix $C_l{}^T C_l$, since matrix $C_l{}^T C_l$ can be ill-conditioned. A feature of ill-conditioning is the situation when the number of conditionality of the matrix $C_l{}^T C_l$ exceeds a certain "conditioning threshold" T (parameter of proposed interpolation algorithm). In this situation, instead of NEDI interpolation, averaging interpolation (6-7) is used. The "conditionality threshold" T affects the efficiency of compression, because this threshold affects the interpolation algorithm. However, the proportion of signal samples for which the situation of ill-conditionality arises is quite small.

7. Experimental research of adaptive interpolation algorithm

We developed a software implementation of the proposed adaptive context interpolator based on NEDI. We built this interpolation algorithm into the hierarchical compression method. To research the effectiveness of the proposed interpolator, computational experiments were performed in natural test signals. Some of these test signals are shown in Figure 4.

As a measure of interpolator efficiency, the relative gain in the archive file size was used, that is achieved through the use of the proposed interpolator (9-14) instead of the averaging (6-7) interpolator within the framework of the hierarchical compression:

$$\Delta = \frac{S^{\text{standart}} - S^{\text{new}}}{S^{\text{standart}}} \cdot 100\% ,$$

where S^{standart}, S^{new} are archive files sizes when signal compressing by the hierarchical method using the averaging and proposed interpolators, respectively.

In each computational experiment, we selected the best (by the compression ratio) threshold values T of the condition number of the matrix $C^T C$ for a fixed number L of hierarchical levels and fixed size N of the estimation area. Some typical results are shown in Figure 5.

From the received experimental results it is clear, that the proposed algorithm provides the gain (up to 2%) on the archive size.

(a) (b)

Figure 4. Examples of natural test signals.

From the results of the computational experiments it can be seen that the proposed adaptive interpolator provides a noticeable (up to 8%) gain on the archive size. This gain increases when the maximum compression error and the size of the interpolation coefficient estimation area are increased.

8. Conclusion

An approach based on context modelling was used to develop an adaptive interpolator for hierarchical signal compression. Based on this approach, a context interpolator based on the NEDI algorithm is proposed. The proposed interpolator is implemented as software and is built into the hierarchical method of signal compression. Computational experiments were conducted to research the effectiveness of the proposed contextual interpolator in natural test signals. It is shown that use of the proposed context interpolator instead of the average interpolator is noticeable (up to 8%) increases the efficiency of hierarchical signal compression.

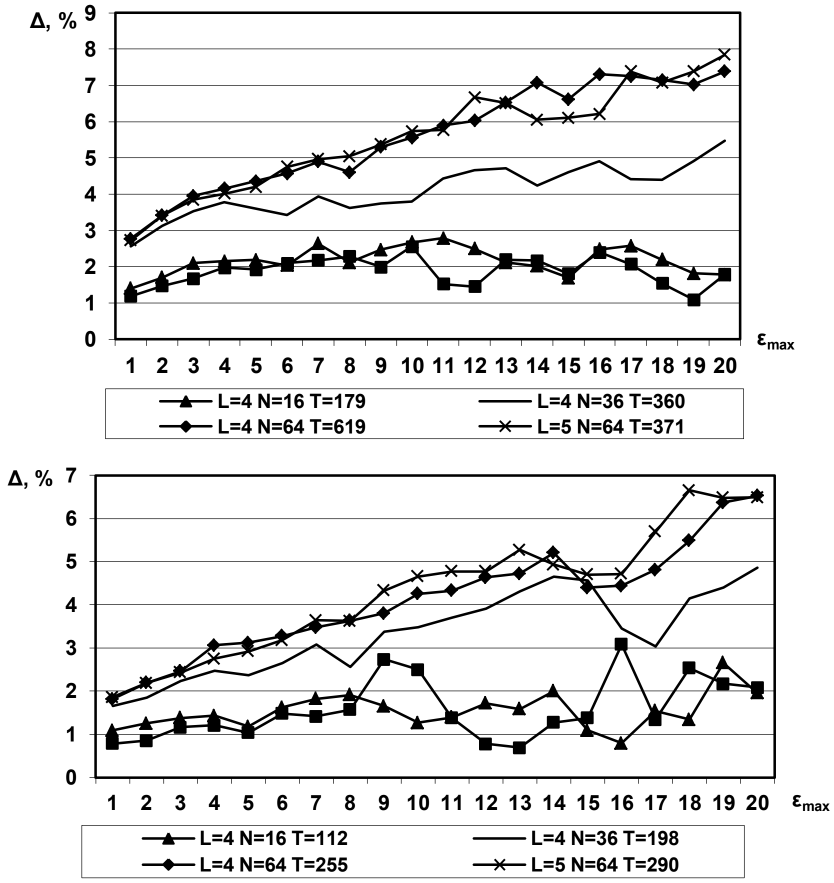

Figure 5. Gain Δ of the proposed context interpolator over the averaging interpolator as a function of the maximum error ε_{max}.

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