

3d synthetic aperture radar image

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Abstract. Synthetic aperture radar (SAR) is a coherent active microwave imaging method. In remote sensing it is used for mapping the scattering properties of the Earth's surface in the respective wavelength domain. The algorithms for the formation of 3D radar images in multi-position interferometric systems for remote sensing of the Earth are considered. Examples of reconstruction of the relief map for systems with one and two transmit antenna are presented.

1. Introduction

The application of interferometric data processing to obtain information about the terrain and its changes, implementation of high resolution (1-3 m) regimes have become the main trends in the development of modern radar systems for space observation. Such processing of space-based synthetic aperture radar's data includes the following steps: synthesis a pair of complex radar images of the same surface region, their spatial overlap with the formation of an interferogram; phase noise filtering on the obtained interferogram; deployment of the phase of the interferogram and its full geocoding (recalculation of the values of the expanded phase in the values of the relief heights and the transition from the flight coordinate system to any cartographic projection).

The purpose of this paper is to review the methods of interferometric data processing to obtain 3d synthetic radar image of a surface model.

It is necessary to first describe the basics of 3d SAR imaging and explain its features.

A second antenna is installed on an aircraft at a certain distance from the first antenna in order to enable forming a three-dimensional map of the underlying surface mode in synthetic aperture radar. As a rule, the spacing can be carried out either in height or in a horizontal plane perpendicular to the direction of flight.

There are several options for the operation of SAR in 3D mode [1]:

1. Radiation is performed through both antennas by turns.
2. Radiation is made through one antenna (for example, A_1), and receiving is performed through two antennas A_1 and A_2 .
3. On the radiation, the transmitter operates on one of its own antennas (A_3), the receivers have two own antennas A_1 and A_2 .

3D images of surface are analysed by hypertrace transformation [2].

Let's consider the basic geometrical relations necessary for calculation of objects' height in SAR 3D mode.

2. “Two transmitters, two receivers” system

Figure 1 shows the basic geometric equations for the 3D SAR system, for the case when the aircraft is equipped with two transmitter and two receivers, operating through its own spaced from each other antenna.

Using the basis of the Pythagorean theorem we can write:

$$\begin{cases} (H_1 - h)^2 + r_1^2 = R_1^2 \\ (H_2 - h)^2 + (r_1 + d)^2 = R_2^2 \end{cases} \quad (1)$$

Values of the heights H_1 and H_2 of the antennas and antenna spacing d in the horizontal plane can be considered known while SAR is working. Also known and sloped range R_1 and R_2 . You need to find the horizontal distance r_1 and the height of the object h . Before solving the equation (1) we can first calculate r_1 and then determine h or vice versa. The solution to the system is determined by the ratio between the values d and $|H_1 - H_2|$. If $|H_1 - H_2| > d$ you choose the first method, otherwise the second. Two solutions are obtained when you use each method. A preference in favor of a solution that gives value r_1 least different from the values of the horizontal range, designed for zero height $r_1^{(0)} = (R_1^2 - H_1^2)^{1/2}$, i.e. $|r_1 - r_1^{(0)}| \rightarrow \min$ [3].

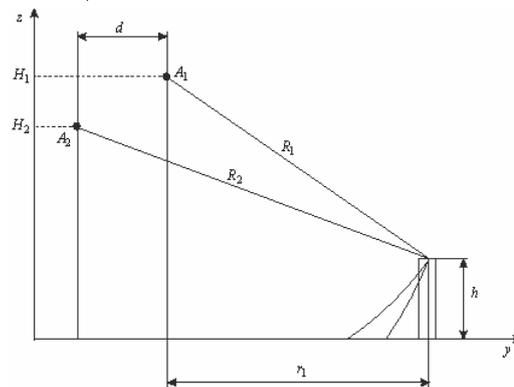


Figure 1. Basic geometric equations in “two transmitters, two receivers” system.

Possible solutions of the equation (1) are defined by equations (2) and (3):

$$\begin{aligned} r_1 = & -(H_1^2 \cdot d + H_2^2 \cdot d + R_1^2 \cdot d - R_2^2 \cdot d + d^3 + 2 \cdot H_1 \cdot H_2 \cdot d + \\ & + (H_1^2 - 2 \cdot H_1 \cdot H_2 + H_2^2 - R_1^2 + 2 \cdot R_1 \cdot R_2 - R_2^2 + d^2)^{1/2} \cdot \\ & \cdot (2 \cdot H_1 \cdot H_2 - H_1^2 - H_2^2 + R_1^2 + 2 \cdot R_1 \cdot R_2 + R_2^2 - d^2)^{1/2} \cdot \end{aligned} \quad (2)$$

$$\cdot (H_2 - H_1) \cdot \frac{1}{2 \cdot (H_1^2 - 2 \cdot H_1 \cdot H_2 + H_2^2 + d^2)};$$

$$h = - \frac{H_2^2 - H_1^2 + R_1^2 - R_2^2 + d^2 + 2 \cdot r_1 \cdot d}{2 \cdot H_1 - 2 \cdot H_2}$$

$$\begin{aligned} h = & -(H_1 \cdot d^2 + H_2 \cdot d^2 + H_1^3 + H_2^3 - H_1 \cdot H_2^2 - H_1^2 \cdot H_2 - \\ & - H_1 \cdot R_1^2 + H_1 \cdot R_2^2 + H_2 \cdot R_1^2 - H_2 \cdot R_2^2 + d \cdot \\ & \cdot (H_1^2 - 2 \cdot H_1 \cdot H_2 + H_2^2 - R_1^2 + 2 \cdot R_1 \cdot R_2 - R_2^2 + d^2)^{1/2} \cdot \\ & \cdot (2 \cdot H_1 \cdot H_2 - H_1^2 - H_2^2 + R_1^2 + 2 \cdot R_1 \cdot R_2 + R_2^2 - d^2)^{1/2}) \cdot \end{aligned} \quad (3)$$

$$\cdot \frac{1}{2 \cdot (H_1^2 - 2 \cdot H_1 \cdot H_2 + H_2^2 + d^2)};$$

$$r = - \frac{R_1^2 - R_2^2 + d^2 + (H_2 - h)^2 - (H_1 - h)^2}{2 \cdot d}$$

On the practice ranges R_1 and R_2 are known with an item resolution accuracy, so the calculations of value h and r_1 use value $\Delta R = R_2 - R_1$, calculated on the basis of the measurement result of the phase difference

$$\Delta R = \frac{\Delta\varphi^{(w)}}{4\pi} \lambda = \frac{\varphi_2 - \varphi_1}{4\pi} \lambda, \quad (4)$$

where φ_1 and φ_2 - phases of the signals received from the first and second antennas, respectively; λ - wave's length.

Because the value of the phase shift is in the range $[0, 2\pi)$, as a rule, there is ambiguity of the measurement values ΔR . To fix it uses the "unwrap" phase. Consider possible approaches to its implementation [4].

The first approach is based on the preliminary construction of the dependence of phase shifts to the zero level, depending on slant range to the antenna A_1 [5]:

$$\Delta\varphi^{(0)}(R_1) = 4\pi\Delta R^{(0)}(R_1)/\lambda = 4\pi((H_2^2 + ((R_1^2 - H_1^2)^{1/2} + d)^2)^{1/2} - R_1)/\lambda \quad (5)$$

Then the value of the phase shift $\Delta\varphi^{(u)}(R_1)$ used to calculate the difference between the sloping distances, determined from the relationship:

$$\Delta\varphi^{(u)}(R_1) = \Delta\varphi^{(w)} + \text{trunc}\left(\frac{\Delta\varphi^{(0)}(R_1)}{2\pi}\right)2\pi \quad (6)$$

The value of the slant range R_2 used for the calculation h and r_1 is determined from the relation:

$$R_2 = R_1 + \frac{\Delta\varphi^{(u)}}{4\pi} \lambda \pm 2\pi m \quad (7)$$

To determine the value m , you can use the following approach. Typically, the height difference between adjacent pixels is relatively small and we can assume that the height of the object in some neighborhood is constant. Presented according to m estimates of the altitude differences in the neighboring pixels have a pronounced minimum, and calculations show that this minimum is achieved when the value of the parameter \hat{m} corresponding to the true values of the altitude and slant range. The algorithm of the calculation value \hat{m} is the following:

1. Around the current image point with coordinates x_0, y_0 , set the gate;
2. Sets the range of values m : $m = m_{\min} \dots m_{\max}$
3. For each point in the gate with coordinates x, y , a values $r_{x,y}$ and $h_{x,y}$ are calculated with current value m ;
4. Calculate total measurement error of the heights and horizontal distances. Terms $(y_1 - y_0)$ and $(y_2 - y_0)$ take into account the current offset of pixels in the horizontal range:

$$\Delta h_m = \sum_{x_1=x_0-dx}^{x_0+dx} \sum_{y_1=y_0-dy}^{y_0+dy} \sum_{x_2=x_0-dx}^{x_0+dx} \sum_{y_2=y_0-dy}^{y_0+dy} |h_{x_1,y_1} - h_{x_2,y_2}|$$

$$\Delta r_m = \sum_{x_1=x_0-dx}^{x_0+dx} \sum_{y_1=y_0-dy}^{y_0+dy} \sum_{x_2=x_0-dx}^{x_0+dx} \sum_{y_2=y_0-dy}^{y_0+dy} \Delta y_{x_1,x_2,y_1,y_2} \quad (8)$$

$$\Delta y_{x_1,x_2,y_1,y_2} = |(r_{x_1,y_1} - (y_1 - y_0)) - (r_{x_2,y_2} - (y_2 - y_0))|$$

5. As a result, selects the value \hat{m} at which

$$\Delta h_{\hat{m}} \rightarrow \min, \Delta r_{\hat{m}} \rightarrow \min \quad (9)$$

Because of spacing antennas, the resulting images have some mutual shift, and its magnitude will depend on the range. In this regard, before calculating the elevation, you must perform the mutual correction of the shifts of image elements. For this calculate values of distances R_2 corresponding to

the range R_1 on the zero level and overwrite the elements of the second array with the ranges R_2 in cells that correspond to values in the range R_1 [6]:

$$J_{R_1,x}^{(kop)} = J_{R_2,x}^{(2)} \quad (10)$$

$$R_2 = (H_2^2 + ((R_1^2 - H_1^2)^{1/2} + d)^2)^{1/2}$$

where $J_{R_2,x}^{(2)}$ - image from the second antenna.

Figure 2 shows an example of the recovery bump maps for the considered case when the following system parameters: $H_1 = H_2 = 10m$, $d = 5m$ the maximum altitude of the relief 5m.

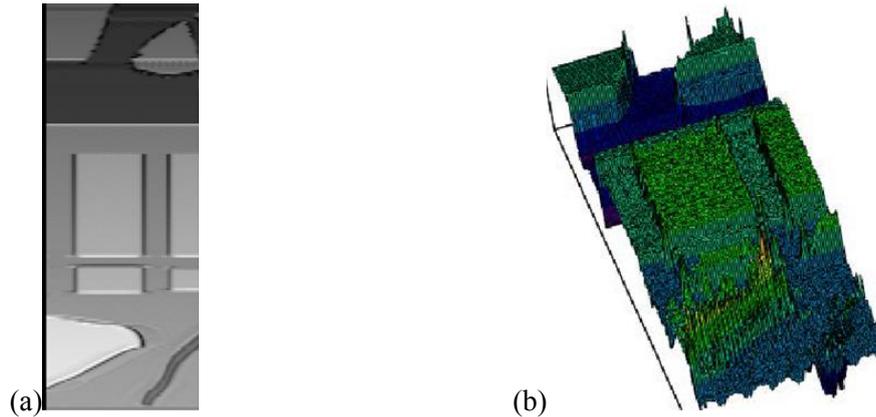


Figure 2. An example of the recovery of the terrain surface. a – synthesized image, b – 3D relief of the surface.

3. “Two transmitters, one receiver” system

Figure 3 shows the basic geometric equations for the 3D SAR system, for the case when an aircraft equipped with one transmitter operating, for example, via an antenna A_1 , and two receivers, operating through its own separated antennas A_1 and A_2 .

In this system a signal between antennas A_1 and A_2 the point on the object surface takes place in two ways $S_1 = 2R_1$ and $S_2 = R_1 + R_2$ respectively. In the simulation of the hologram samples of the signal in the first image is recorded with a pixel corresponding to the distance R_1 , and a second hologram pixel $\frac{R_1 + R_2}{2}$.

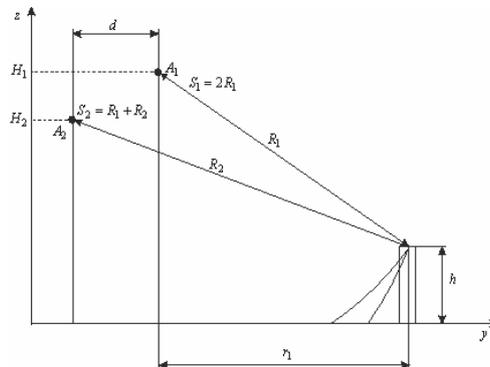


Figure 3. Basic geometric equations in “two transmitters, one receiver” system.

The value of the path S_2 is determined based on the known path S_1 and phase difference due to antenna spacing:

$$S_2 = S_1 + \frac{\Delta\varphi}{2\pi} \lambda m, m = \dots, -2, -1, 0, 1, 2, \dots \quad (11)$$

Knowing the values of the parameters S_1 , S_2 , H_1 , H_2 , d , you can find value and a horizontal range r_1 :

$$\begin{aligned} D2 = & (-d^2 \cdot (-d^2 \cdot S1^2 - 2 \cdot H1^2 \cdot S2^2 - 2 \cdot H2^2 \cdot S2^2 + 2 \cdot S1 \cdot d^2 \cdot S2 + S2^4 - H2^2 \cdot S1^2 - \\ & - 4 \cdot H1^3 \cdot H2 - 2 \cdot d^2 \cdot S2^2 - H1^2 \cdot S1^2 + d^4 + 2 \cdot H2^2 \cdot d^2 + H1^4 + H2^4 + 2 \cdot H1^2 \cdot d^2 + \\ & + 6 \cdot H1^2 \cdot H2^2 + 2 \cdot S1 \cdot S2 \cdot H1^2 - 2 \cdot S1 \cdot S2^3 - 4 \cdot H1 \cdot H2 \cdot d^2 - 4 \cdot H1 \cdot H2^3 + 4 \cdot H1 \cdot H2 \cdot S1^2 - \\ & - 4 \cdot H1 \cdot H2 \cdot S1 \cdot S2 + 2 \cdot H2^2 \cdot S1 \cdot S2 + 4 \cdot H1 \cdot H2 \cdot S2^2 + S1^2 \cdot S2^2))^{0.5} \\ h = & \frac{-H1 \cdot S1 \cdot S2 + H2 \cdot d^2 - H1^2 \cdot H2 + H1^3 + H2^3 + H1 \cdot d^2 - H2 \cdot S2^2}{2 \cdot (H1^2 + H2^2 + d^2 - 2 \cdot H1 \cdot H2)} + \\ & + \frac{H1 \cdot S2^2 + H2 \cdot S1 \cdot S2 - H1 \cdot H2^2 - D2}{2 \cdot (H1^2 + H2^2 + d^2 - 2 \cdot H1 \cdot H2)} \end{aligned} \quad (12)$$

As in the first case, measuring the phase shift may be ambiguous. The dependence of phase shifts on the zero level from the slant range to the antenna A_1 is described by the expression:

$$\Delta\varphi^{(0)}(R_1) = 2\pi \frac{(H_2^2 + ((R_1^2 - H_1^2)^{1/2} + d)^2)^{1/2} - R_1}{\lambda} \quad (13)$$

Then the value of the phase shift $\Delta\varphi^{(u)}(R_1)$ is used to calculate the difference between the sloping distances is determined from the equation (6). Path S_2 is used for the calculation h and r_1 is determined from the expression:

$$S_2 = 2R_1 + \frac{\Delta\varphi^{(u)} \pm 2\pi m}{2\pi} \lambda \quad (14)$$

The value m is chosen by the equation (9).

Still from the spacing antennas, the elements of the resulting images can have some mutual shift. To compensate for this shift is necessary to perform the image correction according to the relation [7]:

$$J_{R_1, x}^{(\kappa\sigma p)} = J_{\frac{R_1 + R_2}{2}, x}^{(2)} \quad (15)$$

where R_2 is computed as in equation (10).

Figure 4 shows an example of the recovery bump maps for the considered case when the following system parameters $H_1 = H_2 = 10m$, $d = 5m$ the maximum altitude of the relief 5m.

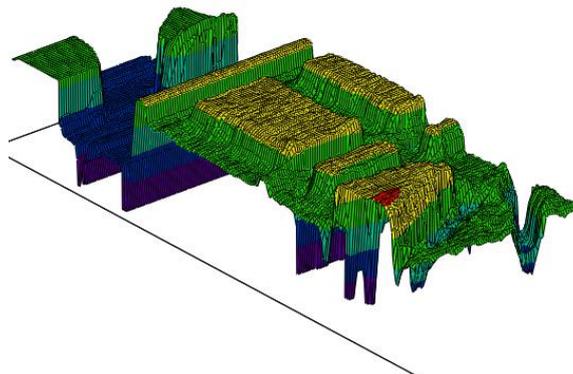


Figure 4. Synthesized 3D image of the surface.

4. Conclusions

Solution of a problem of restoration of a landscape's in a synthetic aperture radar at various configuration of a reception-transmitting path is considered in this work. An original phase unwrapping algorithm based on joint minimization of the estimating error of object's height and

sloped range in neighboring pixels of the image is proposed. Examples of the restored 3D images are presented.

5. References

- [1] Richard B, Philipp H 1998 Synthetic aperture radar interferometry *Inverse Problems* **14(4)** R1-R54
- [2] Fedotov N, Syemov A and Moiseev A 2016 Analysis of conditions that influence the properties of the constructed 3d-image *Computer Optics* **40(6)** 887-894 DOI: 10.18287/2412-6179-2016-40-6-887-894
- [3] Kobernichenko V and Sosnovsky A 2012 Interferometer data processing space radar imagery of high resolution *Physics of Wave Processes and Radio Engineering Systems* **15(3)** 75-83
- [4] Vidal-Pantaleoni A, Rafael O and Miguel F 1999 A Comparison of Phase Unwrapping Techniques in Synthetic Aperture Radar Interferometry *IEEE International Geoscience and Remote Sensing Symposium* 1354-1356
- [5] Osmanoglu B and Dixon T 2011 On the importance of path for phase unwrapping in synthetic aperture radar interferometry *Applied Optics* **50(19)** 3205-3220
- [6] Richards M 2007 A Beginner's Guide to Interferometric SAR Concepts and Signal Processing *IEEE A & E Systems Magazine* **22(9)** 5-29
- [7] Sosnovsky A and Kobernichenko V 2012 Phase unwrapping algorithms investigation in digital elevation maps generation using space-based InSAR *Izvestiya of the Higher Educational Establishments of Russia. Radio electronics* **7** 84-92

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