Dynamic programming method for optimization problem of multi-modal transportation

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Abstract

An optimization problem of multi–modal transportation is investigated. The multi–step decision–making process is constructed. Dynamic programming method for solving the problem is proposed. It should be noted that the dynamic programming is not provided computational advantages, but facilitates changes and modifications of tasks.

Keywords: Optimization Problem, Dynamic Programming, Multi–Step Decision Making Process, Multi–Modal Transportation.

1 Introduction

There are many approaches to the planning of multi–modal transportations. In [Flo11] authors deal with a real–world problem and in particular focus on intermodal freight transport. A system that solves multi–modal transportation problems in the context of a project for a big company is described. It is combined Linear Programming with automated planning techniques in order to obtain good quality solutions. A new hybrid algorithm, combining linear programming and planning to tackle the multi–modal transportation problem, exploiting the benefits of both kinds of techniques is proposed there. The system also integrates an execution component that monitors the execution, keeping track of failures if it is necessary, maintaining most of the plan in execution.

A parameterized model for the optimization of a multi–modal transportation network has been developed and implemented in [Per13] aiming at evaluating the possibilities to increase the absorption rate by a multi–modal transport service. Different software packages have been integrated in order to implement the transport model and an optimization algorithm. A novel formulation of the optimization process has been developed, by combining the differential evolution algorithm and a constructive heuristic.

In [Sun15] a review on the freight routing planning problem in the multi–modal transportation network from the viewpoints of the model formulation and solving approaches is presented. In this study, the formulation characteristics are distinguished and classified into six aspects, and the optimization models in the recent studies are identified based on their respective formulation characteristics. Furthermore, the solution approaches developed to solve the optimization models are discussed, especially the heuristic algorithms.

In this paper, we deal with an optimization problem of multi–modal transportation. Transportation of goods by several types of transport such as road, rail, sea or river are considered. Decision maker can select the

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carrier of the goods for a given initial point of departure and final destination of its receipt. Optimization of the decision–making process is carried out by the dynamic programming method.

2 Problem Statement

Integrated route planning may include:

- Estimation of the cost of services for each carrier;
- The study of the geographic features and limitations in terms of binding to the warehouses, railway stations, sea and river ports;
- Analysis of the cost of transhipment of goods;
- The timetable for delivery.

In the model scheme shown in Fig.1, is planned four segments of the route with three overloads: the supplier – the initial segment AB or AC is serviced by road transport to the station B or C, next segment – by railway from the stations B or C to D or E ports, then by sea transport from port D or E to the ports of F or G, the final section – motor transport from ports F or G to the consumer H.

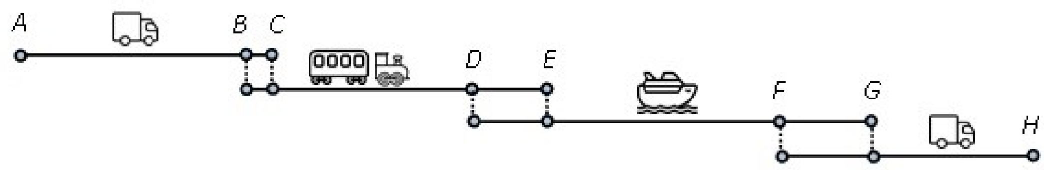

Figure 1: Example of multi–modal transportation graph

Selection of decision maker is to implement a route through nodes B or C, D or E or F and G in such a way that the total transportation cost was minimal. Possible taking into account additional criteria, such as delivery time or the various risks.

Table 1: The cost of transportation segments

Type of transport	Segment	Cost (RUB per container)
road transport	AB	30000
	AC	40000
railway	BD	40000
	BE	50000
	CD	20000
	CE	30000
sea transport	DF	60000
	DG	90000
	EF	70000
	EG	80000
road transport	FH	60000
	GH	50000

Data [Cos16] to construct the optimal route for multi–modal transportation model example listed in Table 1.

3 Formation of Multi–Step Decision–Making Process

To construct a mathematical model of a multi–modal transportation, allowing the use of dynamic programming for finding a solution to the optimal route search task required to describe a multi–step decision–making process, the scheme of which is shown in Fig.2.

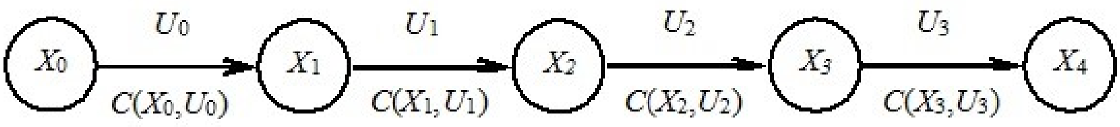

Figure 2: Graph of multi–step decision–making process

Suppose that the system is in the state X_0. As a result of the decision U_0 system goes into a state of X_1 and the price of such decision is $C(X_0, U_0)$. From the standpoint of a model example (see Fig.1), the state X_0 corresponds to locating the goods from the supplier A, solution U_0 is to use the first motor carrier to station B or C. Next, the system will be in the state X_1, i.e. at station B or C. As a result of the decision U_1 system goes into a state of X_2 which corresponds to the carriage by rail to stations of D or E, and the price of the decision will be $C(X_1, U_1)$. This process will continue until the final state of X_4 when the goods will be delivered to the consumer.

Optimization of the above constructed multi–stage decision–making process is carried out by the method of dynamic programming [Wag75].

4 Optimization

Using the method of dynamic programming and Bellman's principle of optimality [Bel53] will provide greater flexibility in terms of possible inclusion in the model of various modifications, for example in case emergency situations. The minimum of decision-making price in i steps from the j state will be described by the Bellman's recurrence relation

$$f_i(X_j) = \min_{U_j}\{C(U_j, X_j) + f_{i-1}(X_{j-1})\}. \tag{1}$$

Here U_j is control on step j, $C(U_j, X_j)$ is the price of making a decision on step j. As U_j implemented to set the minimum price of j–th freight. Algorithm transitions from one state to another (see Fig.2) is described below.

1–st step. Suppose that the system was in the penultimate state X_3. Then the price for the decision of the final step according to (1) will be

$$f_1(X_3) = \min_{U_3}\{C(U_3, X_3)\}. \tag{2}$$

2–nd step. Suppose that the system was in the state X_2. Then the price decision of the last two steps will be

$$f_2(X_2) = \min_{U_2}\{C(U_2, X_2) + f_1(X_3)\}. \tag{3}$$

3–rd step. If the system was in a state X_1 then the price of the decision in the last three steps can make one of the alternatives

$$f_3(X_1) = \min_{U_1}\{C(U_1, X_1) + f_2(X_2)\}. \tag{4}$$

4–th step. Finally, let the system is in the initial state X_0. Then the price decision for all four steps will be

$$f_4(X_0) = \min_{U_0}\{C(U_0, X_0) + f_3(X_1)\}. \tag{5}$$

This price corresponds to the minimum value of the entire carriage.

Now, using the data shown in Table 1, it is possible to construct the optimal route for multi–modal transportation for model example (see. Fig.1) in accordance with the above algorithm.

1–st step. Price decision U_3 at a final step in the transition of the system from the penultimate state X_3 will be

$$f_1(X_3 \supset F) = \min\{60\} = 60\,,$$

$$f_1(X_3 \supset G) = \min\{50\} = 50\,.$$

2–nd step. Being in a state X_2 and making decision U_2 the price of the last two steps will be

$$f_2(X_2 \supset D) = \min\left\{\begin{array}{c} 60 + f_1(X_3) \\ 90 + f_1(X_3) \end{array}\right\} = \min\left\{\begin{array}{c} 60 + 60 \\ 90 + 50 \end{array}\right\} = \min\left\{\begin{array}{c} 120 \\ 140 \end{array}\right\} = 120\,,$$

$$f_2(X_2 \supset E) = \min\left\{\begin{array}{c} 70 + f_1(X_3) \\ 80 + f_1(X_3) \end{array}\right\} = \min\left\{\begin{array}{c} 70 + 60 \\ 80 + 50 \end{array}\right\} = \min\left\{\begin{array}{c} 130 \\ 130 \end{array}\right\} = 130\,.$$

3–rd step. Suppose that the system was in a state X_1. Then the price decision of the last three steps can make one of the alternatives

$$f_3(X_1 \supset B) = \min\left\{\begin{array}{c} 40 + f_2(X_2) \\ 50 + f_2(X_2) \end{array}\right\} = \min\left\{\begin{array}{c} 40 + 120 \\ 50 + 130 \end{array}\right\} = \min\left\{\begin{array}{c} 160 \\ 180 \end{array}\right\} = 160\,,$$

$$f_3(X_1 \supset C) = \min\left\{\begin{array}{c} 20 + f_2(X_2) \\ 30 + f_2(X_2) \end{array}\right\} = \min\left\{\begin{array}{c} 20 + 120 \\ 30 + 130 \end{array}\right\} = \min\left\{\begin{array}{c} 140 \\ 160 \end{array}\right\} = 140\,.$$

4–th step. Finally, let the system is in the initial state X_0. Then the price decision for all four steps will be

$$f_4(X_0 \supset A) = \min\left\{\begin{array}{c} 30 + f_3(X_1) \\ 40 + f_3(X_1) \end{array}\right\} = \min\left\{\begin{array}{c} 30 + 160 \\ 40 + 140 \end{array}\right\} = \min\left\{\begin{array}{c} 190 \\ 180 \end{array}\right\} = 180\,.$$

This price corresponds to the minimum value of the entire carriage.

To find a proper sequence of the optimal route with a minimum value of 180 enough to write a minimum prices decision-making at each step.

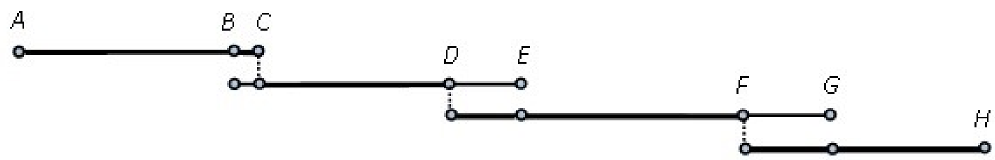

Figure 3: The optimal scheme of multi–modal transportation

The optimal scheme (see. Fig.3) for the considered model example of multi–modal transportation following

$$A \to C \to D \to F \to H\,.$$

5 Conclusion

The model example of multi–modal transportation is studied. The Multi–Step Decision–Making Process is constructed. Optimization of the decision–making process is carried out by the dynamic programming method. Using the method of dynamic programming and Bellman's principle of optimality will provide greater flexibility in terms of possible inclusion in the model of various modifications, for example in case emergency situations [Zav15].

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