Temporal Concept Analysis Explained by Examples

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Abstract. The purpose of this paper is to show by examples the advantages of Temporal Concept Analysis (TCA) - the theory of temporal phenomena described with tools of Formal Concept Analysis (FCA). TCA is developed in three main branches: first the branch of Conceptual Time Systems with actual Objects and a Time relation (CTSOTs) where each temporal object is at each time granule at exactly one place; that gives rise to a first notion of states, transitions and life tracks of an object. The second branch of TCA is centered around the notion of a Temporal Conceptual Semantic System (TCSS) which allows to introduce the notion of a distributed object which may occupy at each time granule a certain volume, called its trace. That leads to a clear mathematical distinction of the notions of particles and waves in physics. The third branch of TCA is based on the notion of a Temporal Relational Semantic System (TRSS); it uses the developments in Temporal Conceptual Semantic Systems for combining the conceptual graphs by J. Sowa and the concept graphs by R. Wille with conceptual scaling.

1 Introduction to Temporal Concept Analysis

Temporal Concept Analysis (TCA) is the theory of temporal phenomena described with tools of Formal Concept Analysis (FCA). While FCA was introduced by R. Wille [9] in 1982, TCA was introduced by the author [11] in 2000 and further developed since then. In the following we assume that the reader is familiar with the basic notions in FCA as explained in [2].

One of the leading ideas in TCA was to represent the notion of a *state* of an object at a certain time in a temporal system. For that purpose a suitable notion of a temporal system including a formal representation of time has to be chosen in such a way that the notion of an object and the notion of a state of an object at a certain time granule (like 'a minute' or 'a day') can be introduced in a natural way.

Looking for a suitable notion of a temporal system it was clear from the beginning of the development of TCA that a temporal system should be described as a data table (interpreted as a result of an observation) as opposed to a system description based on rules. To use the powerful knowledge representation by means of (nested) line diagrams of concept lattices the observations should finally be represented by a formal context. For that purpose the transformation of a data table with arbitrary values (mathematically described as a *many-valued context* (G, M, W, I)) into a *formal context* (G, N, J) is necessary; this transformation is called *conceptual scaling* and can be done in a meaningful way by representing the values of a *many-valued attribute* $m \in M$ as formal concepts of a suitable formal context $S_m := (G_m, M_m, I_m)$ which is called a *conceptual scale* of m, if G_m contains all values of m. The derived context $\mathbb{K} = (G, N, J)$ of a many-valued context (G, M, W, I) with respect to a family $S_m := (G_m, M_m, I_m)$ $(m \in M)$ is defined by $N := \{(m, n) \mid m \in M, n \in M_m\}$ and $gJ(m, n) \iff m(g)I_m n$ for $g \in G$ and $(m, n) \in N$. Any subset $Q \subseteq N$ is called a *view*. The subcontext $\mathbb{K}_Q := (G, Q, J \cap (G \times Q))$ is called the $Q-$part of the derived context. In the following, the concept lattice of a suitable $Q-$part will play the role of a map into which relevant structures like traces of objects, transitions, and life tracks will be embedded (see Fig.1,2,3,6).

The three main branches of TCA are described in the following three sections just by their leading ideas and some examples. Hints to the mathematical definitions in TCA will be given in these sections.

2 Conceptual Time Systems

The first intuitive idea about the notion of a *state*, which was the beginning of TCA, came suddenly to my mind when I was standing alone under the bright sun of Crete on the ruins of the ancient palace of Minos in Knossos, and I vocalized:

The states are just the object concepts of suitable contexts.

That happened in May 1993 just after a conference in Chania (Crete) where I had many fruitful discussions with R.E. Kalman [3] on general systems. In my first paper [11] on temporal conceptual systems the notion of a Conceptual Time System (CTS) and the notion of a state of a CTS was introduced. A simple example of such a temporal system is given in the following subsection.

2.1 A chemical process in a distillation column

In cooperation with a chemical engineer we investigated the temporal behavior of a distillation column. At each of 20 days 13 variables had been measured once. For 4 of these 13 variables the corresponding data table is indicated in Tab.1 by the measurement values at the first and the last day. It is clear that there is a simple notion of a state of this distillation column at some day, namely the tuple of all the measurement values observed at this day, for example the state of the distillation column at day 1 is the quadruple $(129, 616, 616, 119)$ (according to Table 1). As usual, the experts wish to talk a little bit coarser, for example about low, middle and high values of some variables. In close cooperation with the chemical engineers we developed conceptual scales for all of the variables. For reflux and energy1 the resulting state space together with the transitions of the distallation column is shown in Fig.1, which is called a *transition diagram* for this distillation process with respect to the chosen granularity.

Table 1: Data table of a Distillation Column

time granule	day	reflux	energy1	input	pressure
1	1	129	616	616	119
...	...	...	...	...	...
20	20	127	556	664	120

It should be intuitively clear what Fig.1 represents. We do not repeat here the mathematical definitions of a Conceptual Time System as introduced in [11]; we just describe it here in a data table language: the values in the first column are denoted as time granules and interpreted as granules of time as for example a minute or a day; the set of the other columns is divided in two parts, the *time part*, which has in Table 1 just a single column, namely the column of *day*, while the other columns form the *event part*, which consists of the 4 columns of *reflux, energy1, input, pressure*. In this example, the time granules just denote the days, in general a time granule is described by its values in the time part, which might have several many-valued attributes, for example *day*, *month*, *year*.

We now focus on the *reflux-energy1-part* of the data table and show the derived context of this part in Tab.2. For example, at time granule 1 the distillation column has all three reflux-attributes in Tab.2, while it has only the first two energy1-attributes, but not the last, since the energy1-value 616 is not <= 570. The two attributes at the top-concept of Fig.1 have been introduced to tell the reader of the diagram the range of observed values for reflux and energy1.

Table 2: The derived context for reflux and energy1 each scaled with an ordinal scale of a 3-chain

time granule	reflux 126-183	reflux<= 140	reflux<= 133	energy1 514-693	energy1<= 660	energy1<= 570
1	×	×	×	×	×	
...	...	...	...	...	...	...
20	×	×	×	×	×	×

Now we discuss the role of the object concepts in a CTS. If g is a formal object (=time granule), then the intent of the object concept $\gamma(g)$ is the set of all attributes which g has in the derived context - and this intent of $\gamma(g)$ is a very nice description of the state of the CTS at the time granule g with respect to the chosen granularity. It is also good to know all time granules which have the attributes of g; they form the extent of $\gamma(g)$. Therefore, the *state* of a CTS

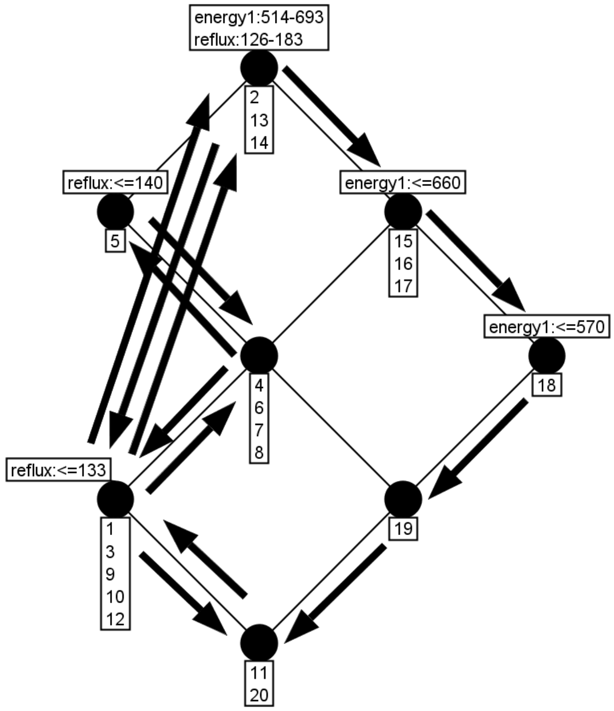

Fig. 1: A chemical process represented in a transition diagram

at time granule g is defined as the object concept $\gamma(g)$ in the derived context of the CTS (with respect to the chosen scales). That approves my intuitive state idea which I had at Knossos.

For example, state $\gamma(1)$ has the intent consisting of all attributes in Table 2 but the last, namely "energy1 <= 570". The extent of $\gamma(1)$ is the set $\{1, 3, 9, 10, 11, 12, 20\}$. The extent of an object concept $\gamma(g)$ should clearly be distinguished from its *contingent* which is defined as the set of all objects h such that $\gamma(h) = \gamma(g)$.The contingent of an object concept $\gamma(g)$ is shown in Fig.1 just below the concept node of $\gamma(g)$. For example, the contingent of $\gamma(1)$ is the set $\{1, 3, 9, 10, 12\}$.

We now focus on the arrows in Fig.1; they represent *transitions*. We have drawn an arrow from $\gamma(g)$ to $\gamma(g+1)$ to denote the transition from day g to day $g+1$ for $1 \leq g \leq 19$. But we have to explain how the transitions are defined in general. That was introduced by the author in [13]. It is clear from the example in Fig.1 that transitions should not be defined as pairs of states (as it is done in Automata Theory), since the two arrows from state $\gamma(1)$ to state $\gamma(2)$ denote different transitions, one happens during the time step from the first day to the second day, for short: $1 \to 2$, the other one during $12 \to 13$. Therefore, we describe the first transition by the pair of pairs $((1, 2), (\gamma(1), \gamma(2)))$ and in the same way the others. For the general definition of a transition we extend the structure of a CTS by a given binary relation R on the set G of formal objects, since we interpreted the formal objects as time granules. (Later on, in Temporal Conceptual Semantic Systems we will use the concepts of the time scale as time granules.) The relation R is called the *time relation*. In nearly all practical applications with discrete time the time relation is the set of pairs (t, t+1) for $t \in \{0, ..., n-1\}$. Since we do not like to assume any linearity of time, we just assume that R is a binary relation on the set G of time granules and interpret R as the set of observed time steps. Hence for a CTS with a time relation R any transition is of the form $((g, h), (\gamma(g), \gamma(h)))$ where $(g, h) \in R$. For more details the reader is referred to [13].

In Fig.1 we can follow the *life track* of the CTS with respect to the chosen view, if we start in state $\gamma(1)$, go to state $\gamma(2)$, and so on until we reach $\gamma(20)$.

Fig.2 shows a *nested transition diagram* where the outer diagram represents the two variables *reflux* and *energy1* (as in Fig.1), while the inner diagram represents the two variables *input* and *pressure* in a 5x4-grid.

From such diagrams the process can be understood quite well. A short and coarse description of that process might be:

During the first two weeks the distillation column was mainly in states with low input and middle pressure - with the exceptions of day 10 (high pressure) and day 11 (low pressure) - while it moved during the last week to high input, small energy1, small pressure, and during the last three days also to small reflux.

Typically, in such applications the experts suggest first a coarse granularity by a few "cuts" like "reflux≤140". After having studied the concept lattice with a coarse granularity it is usually refined, depending on the data and on the interest of the experts. That leads in a few steps to valuable visualizations of multidimensional processes.

The following example of the family of an anorectic young woman served as a motivation for me to develop temporal conceptual systems in which many objects may move, each with its own life track. Hence we need a more general notion of the *state of an object of a system* as opposed to the *state of a system* as in the previous example.

2.2 The development of the family of an anorectic young woman

The following example in Fig.3 describes the development of an anorectic young woman (SELF), her father, mother, and her self ideal (IDEAL) during a period of about two years. The underlying formal context was constructed by the psychoanalyst N. Spangenberg [6–8] on the basis of four repertory grid questionings taken about each half year from the beginning (time granule 1) until the end (time granule 4) of the psychoanalytic treatment of his patient. SELF1, the self at the beginning of the treatment, has the attributes “distrust”, “reduced spontaneity”, “pessimistic” and “self-accusation”, SELF2 has only the attributes “pessimistic” and “self-accusation”, SELF3 is in the same state as SELF1, and SELF4 reaches the state of IDEAL2,3,4 having none of the (negative) attributes. Indeed, the patient was healthy again at this time. It is remarkable that the life

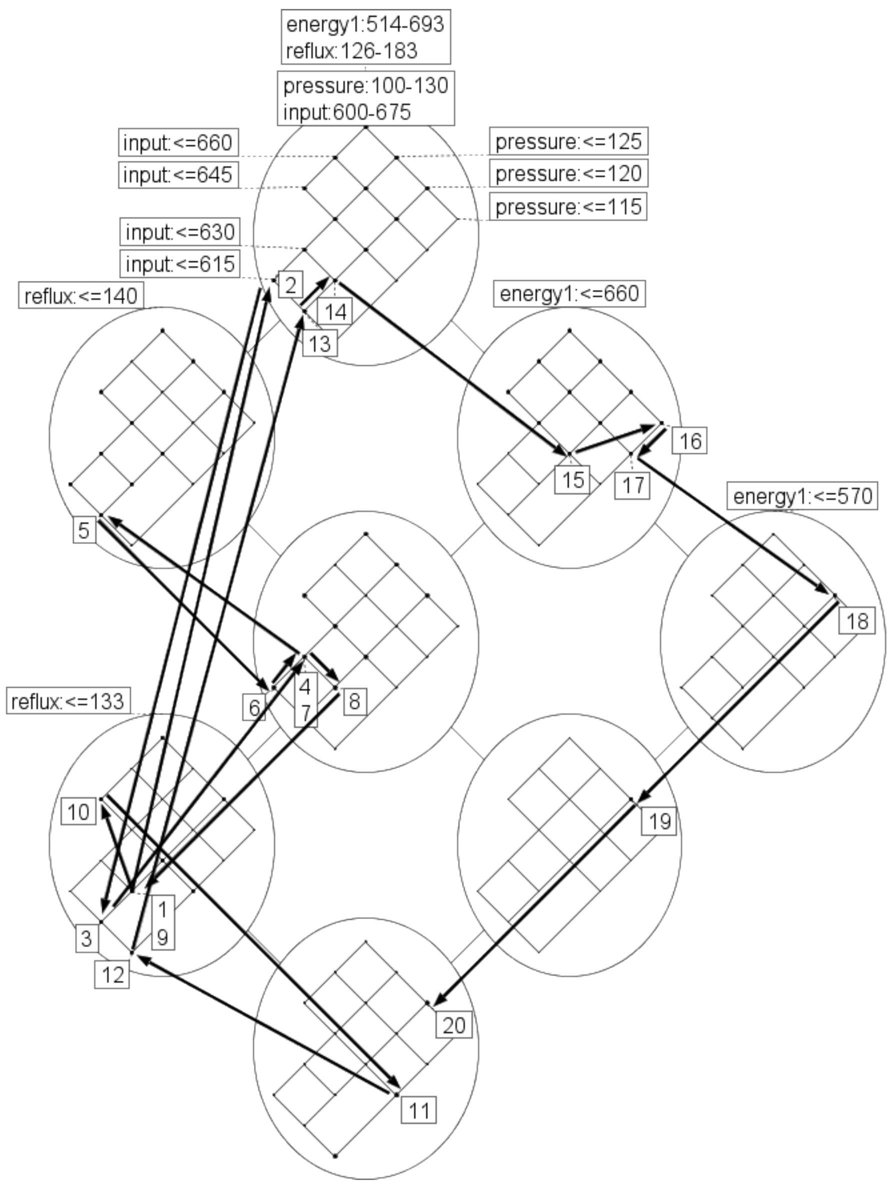

Fig. 2: A chemical process represented in a nested transition diagram

tracks of FATHER and MOTHER start from quite different states and end in similar states, the FATHER having all (negative) attributes of this context. For further information the reader is referred to [6–8].

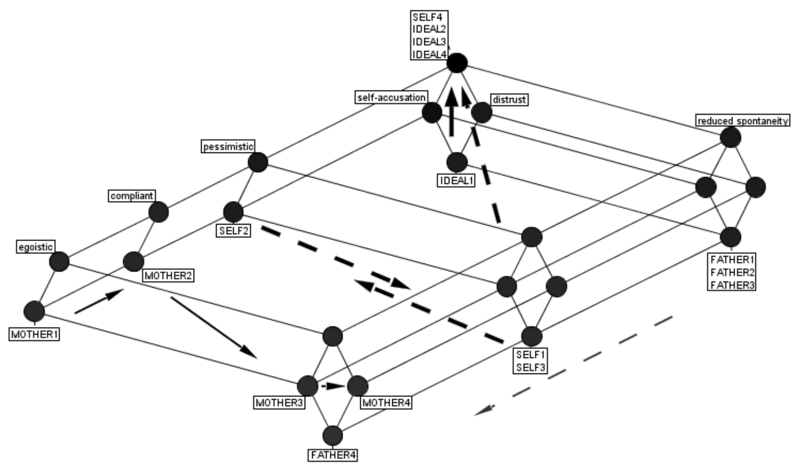

Fig. 3: The development of an anorectic young woman and her family over about two years

2.3 Conceptual Time Systems with actual Objects and a Time relation (CTSOTs)

To have a simple mathematical notation for temporal systems in which many objects (for example persons) are moving the author has introduced in [14] the notion of a Conceptual Time System with actual Objects and a Time relation (CTSOT). The main idea is a straightforward generalization of a CTS by introducing a set P of *persons* or *particles* and a set G of *time granules*, and taking a subset of $P \times G$ as the set of formal objects. For example, the pair (SELF,1) is taken as a formal object denoting the person SELF at time granule 1. Then the notion of a state of a person p at a time granule g can be introduced as the object concept of the formal object (p, g). Transitions and life tracks can be defined in the same way as explained previously [13–16].

While the CTSOTs cover the wide range from particle systems in physics to discrete systems in Computer Science they are not general enough to cover also waves and other distributed objects and their states as for example the distributed state of an electron as represented by some cloud around the center of the electron. Such distributed objects also occur very often in workaday life, for example as a moving high pressure zone on a weather map. Such distributed objects, including waves, can be represented in Temporal Conceptual Semantic Systems which will be explained in the next section.

3 Temporal Conceptual Semantic Systems (TCSSs)

As opposed to CTSOTs where each object p is at each time granule g at exactly one place, namely at the object concept $\gamma(p, g)$ (where γ is the object concept mapping of the chosen view) we now wish to represent also objects which are 'distributed'. For that purpose, we should use neither the time granules (as in CTSs) nor the actual objects (p, g) (as in CTSOTs) as formal objects of the many-valued context of the temporal system. Instead, we take the row numbers of the data table as the formal objects; they can be interpreted as names of the *statements* represented by the information given in that row.

3.1 Basic Notions in TCSSs

Conceptual Semantic Systems have been introduced by the author in [15] for the purpose to represent distributed objects, like waves, in a natural way. The main idea was to represent the concepts in an application domain as formal concepts of formal contexts, called *semantic scales*. Since statements are often shortly described by a tuple of concepts of some application domain we represent each such statement as a tuple of formal concepts. These formal concepts (usually represented by their names) are then chosen as values in a many-valued context in such a way that each row, labeled by g, represents a statement which is denoted by the tuple $(m(g))_{m \in M}$ where $m(g)$ is a formal concept of the semantic scale of m. Since the formal objects of the many-valued context of a CSS are interpreted now as names of statements and not as time granules as in CTSs we need a new representation of time granules in TCSSs. As formal representations of time granules we take the formal concepts of the time scale of a specified many-valued time attribute. The time relation is defined as a binary relation on the set of formal concepts of the time scale. Objects in a temporal system are mathematically represented in TCSSs as tuples of concepts of the semantic scales, as for example the tuple $(\mathbf{High}, \mathbf{Monday})$ denoting a high pressure zone at Monday in the next example. Such an object (which is not a formal object) may be *distributed* in the sense, that it has been observed at more than one places (=object concepts in the concept lattice of the $Q-$part $\mathbb{K}_Q$ of the chosen view Q). This set of object concepts (of the formal objects denoting row labels or statements) where an object has been observed is mathematically defined as the *trace* of an object in $\mathbb{K}_Q$. The *state* of an object $\mathbf{o}$ at a time granule $\mathbf{t}$ is then defined as the trace of the tuple $(\mathbf{o}, \mathbf{t})$. *Transitions* of an object $\mathbf{o}$ from time granule $\mathbf{s}$ to time granule $\mathbf{t}$ are defined similarly as the transitions in CTSs. The technique of embedding many traces into the concept lattice of some suitable context $\mathbb{K}_Q$ is the conceptual generalization of drawing a usual geographic map (see Fig.6). For the mathematical definitions the reader is referred to [15, 19, 24].

Remark: In the definition of a Conceptual Semantic System we do not explicitly represent the relational aspect of a statement as it is done in Conceptual Graphs [4, 5], in Power Context Families and Concept Graphs [10]. These relational structures have been combined with temporal CSSs by the author in Temporal Relational Semantic Systems [21–23] which will be discussed in section 4.

In the following section we use a moving high pressure zone as a typical example of a *distributed* temporal object in a TCSS. We explain this special TCSS starting with a data table and interpreting the values of the data table as formal concepts of suitable formal contexts.

3.2 A Moving High Pressure Zone

As a small example we construct a TCSS which yields a weather map with a moving high pressure zone over Germany. To keep the data table small we construct a map of Germany using a coarse grid of longitude and latitude coordinates into which we embed the (one-element-) traces of 15 towns. To represent a moving high pressure zone we assume that the pressure has been measured in some weather stations (WS) at two consecutive days, say Monday and Tuesday. The data are (partially) shown in Table 3 where the rows 1,...,15 show the latitude and longitude values of 15 German towns, the rows 16,...,25 show the pressure values (in hectopascal (hPa)) measured at certain days at weather stations located in some of the previously mentioned towns.

Table 3: Data table of a Moving High Pressure Zone

instance	place	latitude	longitude	time	pressure
1	Berlin	52.5	13.4	/	/
...	...	...	...	/	/
15	Wilhelmshaven	53.5	8.1	/	/
16	WS Dortmund	51.5	7.5	Monday	1020
...	...	...	...	...	...
25	WS München	48.1	11.6	Tuesday	980

In Table 3 the row labels are called instances, and instance 1 denotes the statement that the *place Berlin* has *latitude* 52.5 and *longitude* 13.4; no time and no pressure is recorded in this line, shown by the sign "/" in the column for *time* and *pressure*. *Instance* 16 tells that *Weather Station Dortmund* has *latitude* 51.5 and *longitude* 7.5 and has reported at *Monday* a *pressure* of 1020 hectopascal.

The semantic scale for *time* is shown in Fig. 4 by a line diagram of its concept lattice. The values “Monday”, “Tuesday”, and “/” in Tab. 3 are interpreted as the object concepts of the corresponding formal objects. The single arrow in Fig. 4 represents the time relation as a binary relation on the set of all concepts of the specified time attribute.

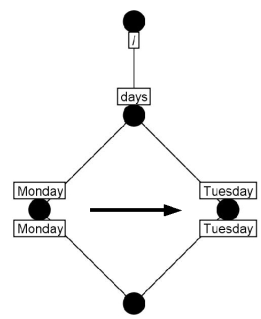

Fig. 4: The semantic scale for *time* with time relation

The semantic scale $\mathbb{S}_p$ for *pressure* is defined as a modified interordinal scale on the multiples of 10 in the interval $[970, 1030]$. Its concept lattice is shown in Fig.5.

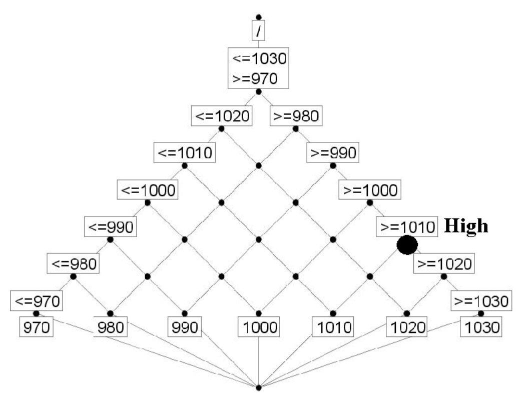

Fig. 5: The semantic scale for *pressure*

In this interordinal scale for pressure we choose the attribute concept $\mu(\geq 1010)$ for the representation of the notion of “high pressure”, and call this formal concept **High**. Combined with the formal concept **Monday** in the time scale we like to form the tuple (**High, Monday**) as our formal representation of such an abstract “object” like “**the High at Monday**”.

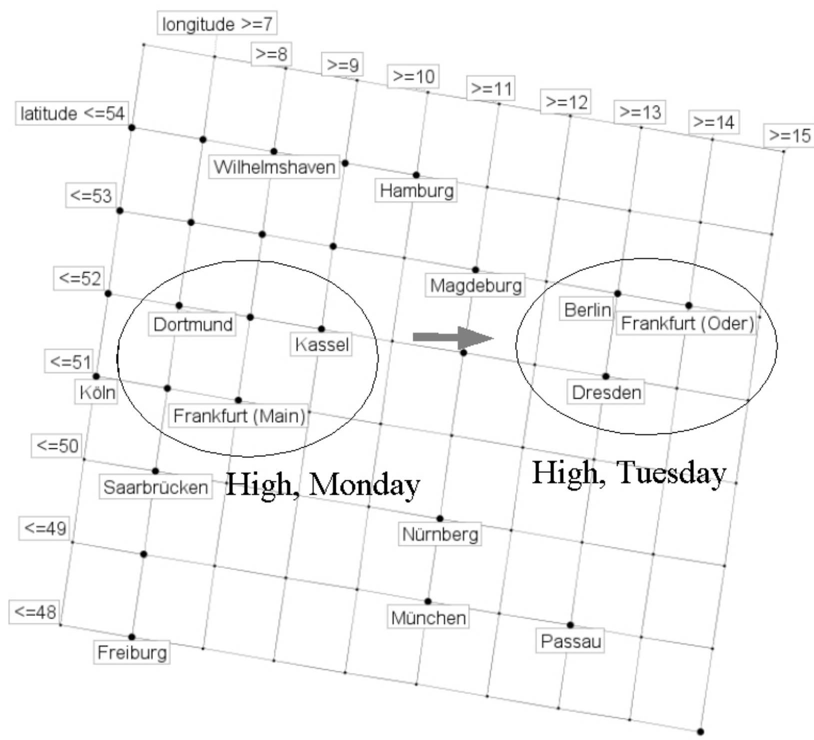

Fig. 6: A Weather Map with a Moving High Pressure Zone

The movement of the selected high pressure zone is visualized in the weather map in Fig.6. Concerning the construction of this weather map we just mention the main steps: 1. Construction of a grid of latitude-longitude-values as a semiproduct of two ordinal scales for latitude and longitude. 2. Embedding the (one-element) traces of towns using their latitude-longitude-values. 3. Embedding of the traces of the tuples (**High**, **Monday**) and (**High**, **Tuesday**) as visualized by ellipses in Fig.6. 4. Visualizing a (short) life track from the trace of (**High**, **Monday**) to the trace of (**High**, **Tuesday**).

The details of the construction of the weather map in Fig.6 can be seen in [19]. For another useful application of TCSSs in a biomedical study of disease processes in arthritic patients the reader is referred to [25].

4 Temporal Relational Semantic Systems

A Temporal Relational Semantic System (TRSS) [21–23] is a relational extension of a Temporal Conceptual Semantic System (TCSS). The investigations of TRSSs are based on the theory of Conceptual Graphs as developed by J. Sowa [4, 5] and its FCA-version of Concept Graphs of a Power Context Family as introduced by R. Wille [10]. The main idea for the definition of a TRSS is to write each relational statement (with a single relation term) into a line of a many-valued context of a CSS and protocol the sequence of speaking this statement. A TRSS has a specified set of time attributes, a specified set of temporal objects, and for each temporal object its time relation [23]. Then states, transitions, and traces can be introduced as in a TCSS.

4.1 A tabular representation of a Temporal Relational Semantic System

In the rows of Table 4 we represent some temporal and non-temporal statements: ‘from 2008 to 2009 Bob lived in England’, ‘in May 2008 Bob lived in London’, ‘in spring 2009 Alice lived in Berlin’, ‘in spring 2009 Bob met Alice in Paris’, and ‘Paris is the native town of Alice’.

Table 4: A data table for temporal relational information

statement	r^*	$TIME_1$	$TIME_2$	$PERSON_1$	$PERSON_2$	LOCATION
1	from.to..lived in.	2008	2009	BOB		ENGLAND
2	in...lived in.	May	2008	BOB		LONDON
3	in...lived in.	spring	2009	ALICE		BERLIN
4	in...met. in.	spring	2009	BOB	ALICE	PARIS
5	.is the native town of.			ALICE		PARIS

The main information for reading the statements in the intended sequence is represented in Table 5, called the *position table*. For example, the first position of *.is the native town of.* is the attribute LOCATION, its second position is the attribute PERSON_1.

Table 5: The position table

r^*	TIME_1	TIME_2	PERSON_1	PERSON_2	LOCATION
from.to..lived in.	1	2	3		4
in...lived in.	1	2	3		4
in...met. in.	1	2	3	4	5
.is the native town of.			2		1

For RSSs the notion of a trace of an object (as a tuple of concepts of the semantic scales) can be used to visualize information concerning several many-valued attributes in the concept lattice of a suitable view. That is shown by some examples of *relational trace diagrams* in [22]. For TRSSs each relation, as for example the relation *in...lived in.* is formally represented as a formal concept of the scale for the relations; therefore the *state* of such a relation at a certain time granule can be defined. That is clearly a powerful tool for the representation of temporal relational knowledge.

5 Conclusions

This paper has shown the leading ideas and some typical examples of several temporal conceptual systems: first the CTSs, second the CTSOTs, third the TCCSs, and fourth the TRSSs. Future research should improve the actual computer programs [1] for the generation of powerful visualizations of temporal relational structures.

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