# Interpolation of Magnetic Fields and Their Gradients for MEG Data with 3D Spline Functions

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*Spline functions are used to interpolate the magnetic fields and their gradients over a surface. The surface itself and the local normals are interpolated in the same way. For all interpolations the local values are obtained from a set of points which correspond to the locations, local normals or magnetic field values of a set of biomagnetic sensors.*

**Keywords:** MEG, Magnetoencephalography, 3D surface, Interpolation, Spline, Mesh

## 1 Introduction

Magnetoencephlography (MEG) is concerned with the measurement and interpretation of the weak magnetic fields generated by the brain[2]. The goal of MEG and EEG is to extract information about neural activities. It is well known that the inverse problem in MEG and/or EEG[7] [1] [13] has no unique solution[3]. For any given distribution of magnetic fields or potentials over the scalp surface, an infinite number of distributions of neural current sources can explain the data. This is true even in the idealized case where the EEG and MEG signals are known everywhere outside the head. In practice, the data are only measured on points or over partial surfaces and noise is present. Of course not all solutions are physiologically reasonable, and even weak constraints can suffice to eliminate much of non-uniqueness in practice, particularly for superficial sources[5]. The direct display of magnetic field on the sensor surface is always useful, but to achieve this it is necessary to use interpolation methods. The use of the interpolated magnetic fields and specific gradient combinations to highlight nearby sources has been proposed in ref.[4] and applied to the analysis of single trial data from planar sensor arrays[6]. The sensors are not always located on a regular grid and sometimes one or more sensors are defective. Any interpolation method should be able to handle such data. Spline functions can meet these requirements. In ref.[8] [9] [10] the electrical potential and its surface Laplacian were interpolated over a plane. Spline functions on a half sphere represent a partial surface of a head and can be easily projected onto a plane. Only the subset of sensors which can be seen from one side of the head have been utilized.

In this paper, the magnetic fields and their gradients are interpolated with the spline functions on a closed surface. In section 2 we introduce the set of 3D spline functions[11] which will be used. In section 3 we describe the mesh generation for the helmet surface and in section 4 the interpolation of the magnetic fields and their gradients. Finally, the magnetic fields and their gradients are displayed on the helmet mesh surface developed in section 3. We have used a method for displaying the 3D data[12] which allowes us to see the magnetic field and its gradient from any direction around the helmet. This method utilizes all the sensors available on the helmet, offering an interpolation with high quality.

## 2 Interpolation with 3D spline function

A 3-dim function $f(x,y,z)$ is measured at $N$ random points $(x_i, y_i, z_i)$ on the 3D space, $i = 1, 2 \cdots\cdots N$, The values are $f(x_i, y_i, z_i)$. Our goal is to recover the function $f(x,y,z)$ from the data $f(x_i, y_i, z_i)$ by interpolation. We express $f(x,y,z)$ as a linear combination of the basis functions $B_k(x,y,z)$

$$f(x,y,z) = \sum_{k=1}^{N} a_k B_k(x,y,z), \tag{1}$$

where $a_k$ are constants to be found. Given N measurements, $a_k$ can be found by inverting the linear system of equations,

$$\sum_{k=1}^{N} a_k B_k(x_i, y_i, z_i) = f(x_i, y_i, z_i) \quad i = 1, 2 \cdots N. \tag{2}$$

We have chosen the spline function $\Psi^{(l)}$ of ref.[11] as basis functions $B_k(x,y,z)$,

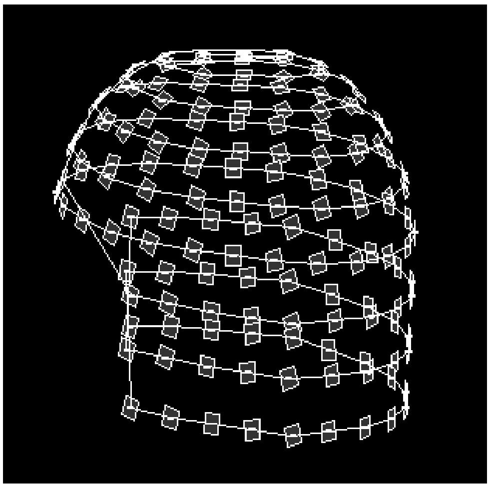
$$B_k(x,y,z) = \Psi^{(l)}((x-x_k)^2 + (y-y_k)^2 + (z-z_k)^2 + \rho^2), \tag{3}$$

where $\rho$ is a parameter, $l = 1, 2$ or 3, $\Psi^{(1)}(x) = x^q$, $\Psi^{(2)}(x) = \ln(x)$, $\Psi^{(3)}(x) = x^m \ln(x)$. Which basis functions should be chosen depends on the smoothness of the interpolated function $f(x,y,z)$. In general, the functions created with choice $\Psi^{(2)}$ will be smoother than those created with choice $\Psi^{(1)}$, and the functions created with choice $\Psi^{(3)}$ will be smoother than those with choice $\Psi^{(2)}$. For choice $\Psi^{(3)}$, the larger $m$, the smoother the interpolation.

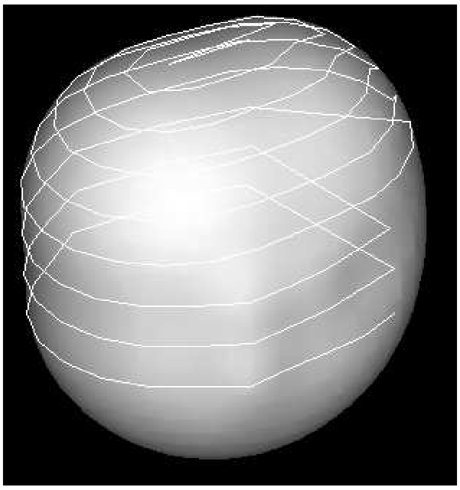
## 3 The Mesh of the helmet

In this section the interpolation on the surface of a sphere is discussed. The examples involved are the creation of the surface mesh of the helmet from all sensor positions $\mathbf{x}_i$ of the BTI system. The surface of the helmet is a closed surface which passes through all sensor positions $\mathbf{x}_i$. This surface is described by a mesh with much more vertices $\mathbf{x}$ than the number of sensors. The vertices $\mathbf{x}$ on the mesh are obtained by interpolation which utilizes the sensor position $\mathbf{x}_i$.

It is known that a surface mesh can be created on the surface of sphere. In order to transform this mesh to the mesh of the helmet, the mesh on a sphere and the mesh on the helmet are compared. The difference between two meshes concerns their radii which are defined by the distances from the vertices of the meshes to the origin point $\mathbf{o}$. The radius of the helmet $r(\mathbf{y})$ is a function on the surface of a unit sphere, where $\mathbf{y}$ is a point



(a) MEG sensor configuration of the helmet.



(b) The surface of the helmet projected from one direction.

Figure 1: BTI MEG helmet system

on the surface of the unit sphere. The positions of sensors are given as $\mathbf{x}_i$. $r(\mathbf{y})$ can be interpolated from the sensor positions $\mathbf{x}_i$

$$r(\mathbf{y}) = \sum_{k=1}^{N} a_k B_k(\mathbf{y}). \tag{4}$$

The interpolation method used is similar to the method described in section 2. The only difference is that here the interpolation is performed on the surface of the sphere instead of the 3D space. The basis function $B_k(\mathbf{y})$ are obtained from Eq.(3) by changing $x, y, z$ to $\mathbf{y}$. The constant of the interpolation $a_k$ are obtained in the same way as in section 2 by solving the linear equations

$$\sum_{k=1}^{N} a_k B_k(y_i) = r_i, \tag{5}$$

where $r_i = \|\mathbf{x}_i\| = \sqrt{x_i^2 + y_i^2 + z_i^2}$ is the radius of the sensor positions. At the end, the mesh vertexes of the helmet are given as follows

$$\mathbf{x} = r(\mathbf{y})\mathbf{y}. \tag{6}$$

The mesh of the surface is created by the mesh transform Eq.(6). Comparing the results of different basis functions we found that the basis functions $\Psi^{(2)}$ have a better performance than the basis functions $\Psi^{(1)}$. Fig.1(a) shows the sensor configuration of the helmet system. Fig.1(b) shows the surface of the helmet projected from one direction using the basis functions $\Psi^{(2)}$.

It is worth mentioning that the interpolation method used here is originally suitable for the application of a three dimensional interpolation, however we have found that it is satisfactory for the case of interpolation on a surface of a sphere.

# 4 The interpolation of the MEG data

Our goal is to derive an analytic expression for the measurements $M$ and for the cross product of the gradient of $M$ with the local mesh normal; the second quantity corresponds to the $\mathbf{V}_3$ transform of ref.[4]. To achieve this goal we first use the measurements $M(\mathbf{x})$ at the sensor positions $\mathbf{x}_i$ to interpolate $M$. We then use the coordinates of the sensors and their local normals (which are known a priority as the normals to the magnetometer coil) to obtain an analytic expression for the local normal $\mathbf{n}$. As before we use $\mathbf{x}_i$ to denote the location of the $i$th vector:

$$M(\mathbf{x}) = \sum_{k=1}^{N} a_k B_k(\mathbf{x}) \text{ and } \mathbf{n}(\mathbf{x}) = \sum_{k=1}^{N} \mathbf{a}_k B_k(\mathbf{x}). \tag{7}$$

the scalar constants $a_k$ and the vector constant $\mathbf{a}_k$ are obtained in the usual way, by solving the linear system of equations

$$M_i = \sum_{k=1}^{N} a_k B_k(\mathbf{x}_i) \text{ and } \mathbf{n}_i = \sum_{k=1}^{N} \mathbf{a}_k B_k(\mathbf{x}_i). \tag{8}$$

Where $M_i = M(\mathbf{x}_i)$ is the measurement value at the $i$th sensor and $n_i = \mathbf{n}(\mathbf{x}_i)$ is the unit normal at the location of the $i$th sensor i.e. the direction perpendicular to the plane of the magnetometer coil of the $i$th sensor. We note that the process of interpolation does not necessarily require a unit normal and hence we need to impose this

$$\mathbf{n}(\mathbf{x}) \Leftarrow \frac{\mathbf{n}(\mathbf{x})}{\|\mathbf{n}(\mathbf{x})\|}, \tag{9}$$

The equations Eq.(8) can be written in matrix form,

$$M = BA \text{ and } \mathbf{N} = B\mathbf{A} \tag{10}$$

where the elements of the matrix are $A_k = a_k$, $B_{ik} = B_k(\mathbf{x}_i)$, $M_i = M(\mathbf{x}_i)$, $\mathbf{N}_i = \mathbf{n}(\mathbf{x}_i)$. The constants matrix can be obtained by inversing the matrix

$$A = B^{-1}M \;\; and \;\; \mathbf{A} = B^{-1}\mathbf{N}. \tag{11}$$

Once the coefficients are determined the vector transformation $\mathbf{V}_3$ of ref.[4] is simply obtained

$$\mathbf{V}_3 M(\mathbf{x}) = \nabla M(\mathbf{x}) \times \mathbf{n}(\mathbf{x}) \text{ or } \mathbf{V}_3 M(\mathbf{x}) = \sum_{k,k'=1}^{N} a_k [\nabla B_k(\mathbf{x}) \times \mathbf{a}_{k'}] B_{k'}(\mathbf{x}) \tag{12}$$

Fig.2(a) and 2(b) shows an example of the interpolation of $M$ and $\mathbf{V}_3 M(\mathbf{x})$ for computer generated data on the helmet mesh with a single current dipole in a conducting sphere.

# 5 Conclusion and acknowledgments

3D spline functions are suitable for the interpolation of the magnetic field and its gradient on a closed surface.

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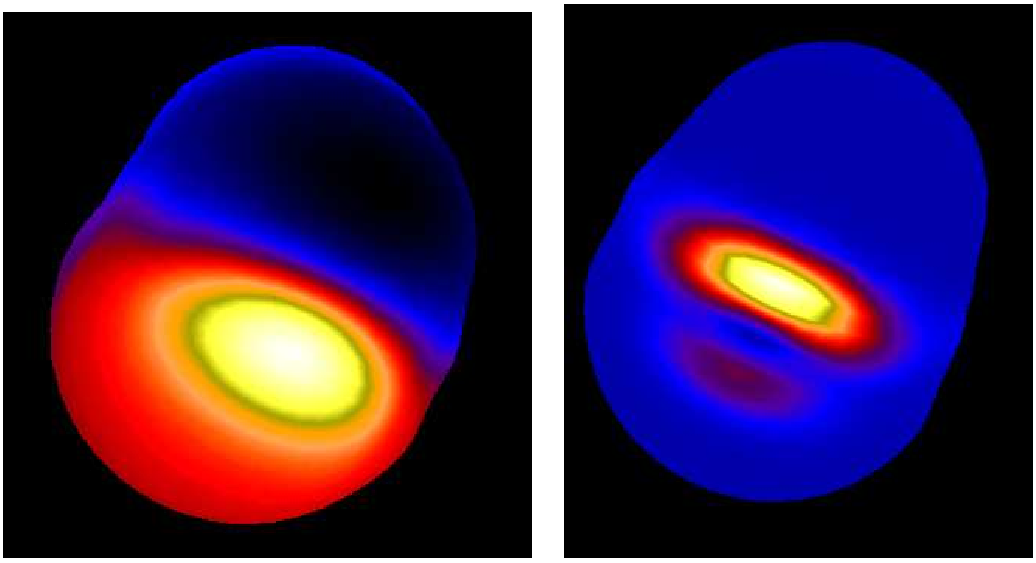

(a) The magnetic field$M(\mathbf{x})$.

(b) A contourplot for the modules of $\mathbf{V}_3 M(\mathbf{x})$.

Figure 2: MEG data are interpolated from values on the sensors of the BTI MEG helmet system

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