Fast curvature based registration of MR-mammography images

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Abstract. We introduce a new non-linear registration model based on a curvature type regularizer. We show that affine linear transformations belong to the kernel of this regularizer. Consequently, an additional global registration is superfluous. Furthermore, we present an implementation of the new scheme based on the numerical solution of the underlying Euler-Lagrange equations. The real DCT is the backbone of our implementation and leads to a stable and fast $\mathcal{O}(n \log n)$ algorithm, where n denotes the number of voxels. We demonstrate the advantages of the new technique for synthetic data sets. Moreover, first convincing results for the registration of MR-mammography images are presented.

1 Introduction

Registration of 2D or 3D medical images is necessary in order to study the evolution of a pathology of a patient, or to take full advantage of the complementary information coming from multimodal imagery. In our application, which is related to MR-mammography, the time evolution of an agent injection has to be studied subject to patient motion. Recent examples of the use of deformable models to perform a non-rigid, automatic registration include [1,2,3,4,5,6].

There are several problems with fully automatic registration approaches. If the initial rigid alignment is off by too much, the non-rigid matching procedure may perform poorly. Therefore it is desirable to incorporate the rigid alignment step, also known as global matching, into the non-rigid scheme.

In this note we propose a novel curvature based penalizing term which not only provides smooth solutions but also allows for automatic rigid alignment.

2 Approach

We refer to the template image as T and the reference as R. The purpose of the registration is to determine a transformation of T onto R. Ideally, one wants to determine a displacement field $\mathbf{u} : \Omega \to \Omega$ such that $T(\mathbf{x} - \mathbf{u}(\mathbf{x})) = R(\mathbf{x})$. The question is how to find such a mapping $\mathbf{u} = (u_1, \ldots, u_d)$. A typical approach is the minimization of a measure $\mathcal{D}$, for example

$$\mathcal{D}[\mathbf{u}] = \frac{1}{2}\|R - T(\cdot - \mathbf{u})\|_{L_2}^2 = \frac{1}{2}\int_\Omega \left(T(\mathbf{x} - \mathbf{u}(\mathbf{x})) - R(\mathbf{x})\right)^2 d\mathbf{x}. \tag{1}$$

A regularizing term $\mathcal{S}$ is introduced in order to rule out discontinuous and/or suboptimal solutions. The problem now reads, find a mapping $\mathbf{u}$ which minimizes the joint criterion $\mathcal{J}[\mathbf{u}] = \alpha\mathcal{S}[\mathbf{u}] + \mathcal{D}[\mathbf{u}]$. In this note, we investigate the novel smoothing term

$$\mathcal{S}^{\text{curv}}[\mathbf{u}] = \sum_{\ell=1}^{d} \int_{\Omega} (\Delta u_\ell)^2 \, d\mathbf{x}. \tag{2}$$

The reason for this particular choice is twofold. The integral might be viewed as an approximation to the curvature of the ℓth component of the displacement field and therefore does penalize oscillations. Most interestingly, $\mathcal{S}^{\text{curv}}$ has a non-trivial kernel containing affine linear transformations, i.e., $\mathcal{S}^{\text{curv}}[C\mathbf{x} + \mathbf{b}] = 0, \quad C \in \mathbb{R}^{d\times d}, \ \mathbf{b} \in \mathbb{R}^d$. Thus, in contrast to many other non-linear registration techniques, the new scheme does not require an additional affine linear pre-registration step for being successful.

To illustrate the difference between our new curvature based registration and the elastic registration approach [1] we consider an academic example. As the reference image a gray square on a white background positioned in the top left corner is used. In contrast, the considered template has the very same square in the bottom right corner. In other words, an appropriate affine linear transformation would produce a perfect registration result. It turns out, that both the curvature based and the elastic registration lead to a perfect registration, in the sense that the difference between the reference and deformed template vanishes. However, a tracking of the individual pixel reveals that the path towards the optimal registration is completely different. In Figure 1 the templates as well as the interpolation grid, i.e., the points $\mathbf{x} - \mathbf{u}(\mathbf{x})$, are shown. As it is apparent from this figure, the curvature based registration finds the optimal registration result by computing an almost affine linear transformation, i.e. $\mathbf{u}(\mathbf{x}) \approx \text{const}$. In contrast, the displacement computed by the elastic registration scheme is highly non-linear.

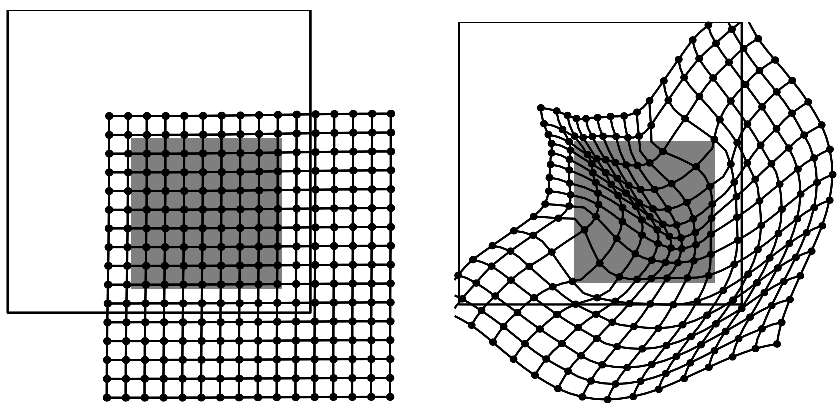

Fig. 1. Template image with interpolation grid; LEFT: after curvature based registration; RIGHT: after elastic registration.

In accordance with the calculus of variations, a function $\mathbf{u}$ which minimizes the joint functional $\mathcal{J}$ for the particular choice $\mathcal{S} = \mathcal{S}^{\mathrm{curv}}$ has to satisfy the Euler-Lagrange equation

$$\mathbf{f}(\mathbf{x}, \mathbf{u}(\mathbf{x})) + \alpha \Delta^2 \mathbf{u}(\mathbf{x}) = 0 \text{ for } \mathbf{x} \in \Omega, \tag{3}$$

subject to appropriate boundary conditions. The so-called force field $\mathbf{f}$ is the Gateaux derivative of the distance measure $\mathcal{D}$, i.e., $\mathbf{f}(\mathbf{x}, \mathbf{u}(\mathbf{x})) = (T(\mathbf{x} - \mathbf{u}(\mathbf{x})) - S(\mathbf{x}))\ \nabla(T(\mathbf{x} - \mathbf{u}(\mathbf{x})))$. The above fourth-order non-linear PDE is known as bipotential or biharmonic equation and well studied, see, e.g., [7]. To solve (3) numerically, we apply a finite difference discretization adapted to the particular simple geometry of the domain Ω in conjunction with a time marching scheme. This approach results in a system of linear equations $\mathcal{A}\mathbf{u}^{k+1} = \mathbf{f}^{\,k}$. Consequently, the main work in the overall scheme is the repeated solution of this linear system. It can be shown that $\mathcal{A}$ is diagonalizable by cosine-transform matrices. Thus, a proper, real DCT-type technique leads to a fast and stable $\mathcal{O}(n \log n)$ implementation, where n denotes the number of voxel.

3 Experiments

To illustrate the performance of the new approach we present the registration of two clinical 2D magnet-resonance (MR) images of a female breast. We are indebted to Bruce L. Daniel (Department of Radiology, Stanford University) for providing the medical data. The task is to register low resolution MR-scans (256×256) of the wash-in and wash-out phase to a high resolution MR-scan (512×512), which are viewed as a gold-standard by the radiologist, taken just between the wash-in and wash-out phase. The overall goal is to study the dynamic behavior of the contrast agent in detail. Figure 2 displays the arbitrarily chosen section 24 of the high resolution MR-scan and the difference to section 24 of a wash-in phase MR-scan before and after registration. Note that the difference has been reduced by about 30%.

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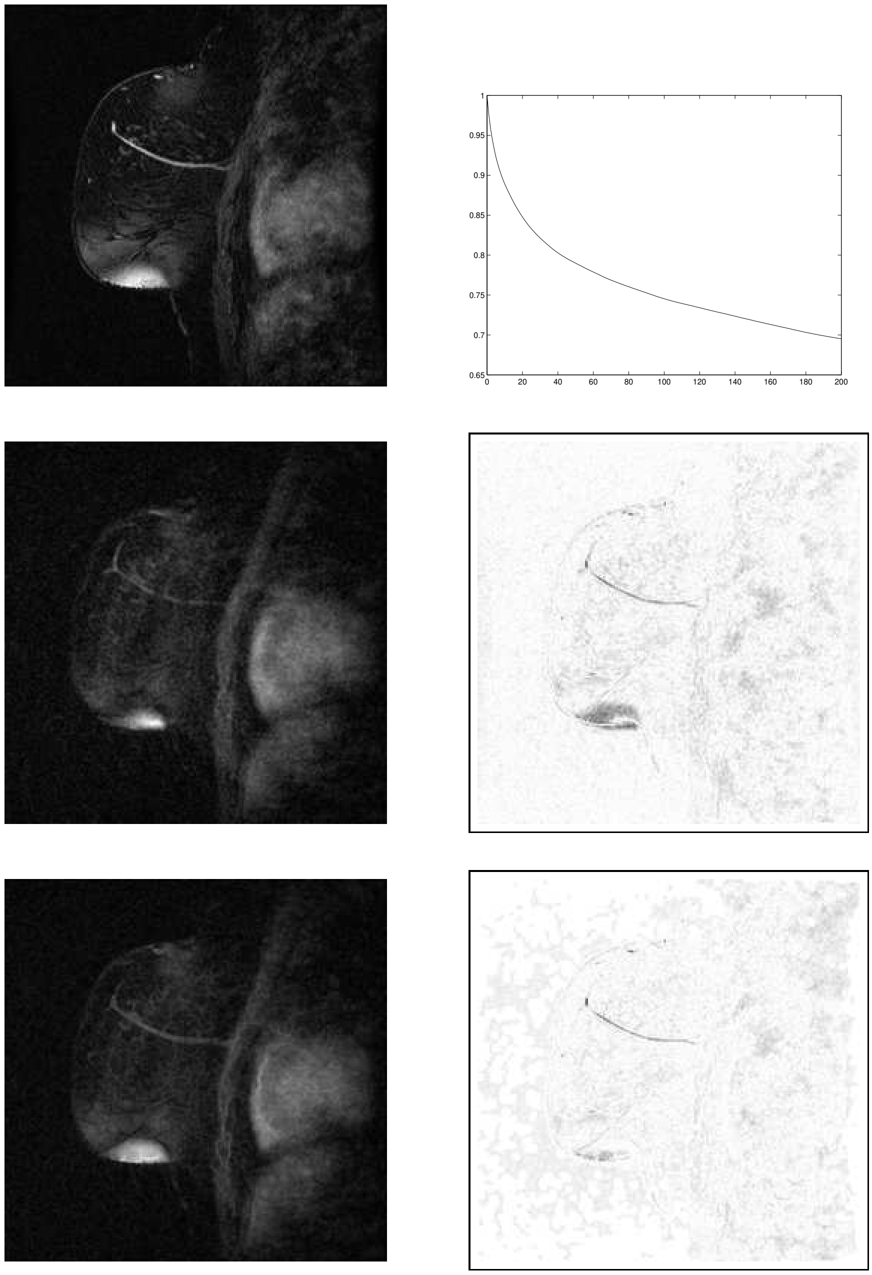

Fig. 2. TOP: Reference R (preprocessed section #24 from a high resolution image taken at optimal time-point); MIDDLE LEFT template T (preprocessed section #24 from the wash-in phase 20); MIDDLE RIGHT difference before registration, $|R - T| = 100\%$; BOTTOM LEFT template $\hat{T}$ after registration; BOTTOM RIGHT difference after registration, $|R - \hat{T}| = 69.5\%$; TOP RIGHT relative distance $|R - T_k|/|R - T|$ versus iteration.