Temporal Concept Analysis

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Abstract. This paper introduces Temporal Concept Analysis (TCA) as the theory of temporal phenomena described with tools of Formal Concept Analysis (FCA). The basic notion in TCA is that of a Conceptual Time System introduced by the author [42,45] such that state spaces and phase spaces can be defined as concept lattices which clearly represent the meaning of states with respect to the chosen time description. In this paper two closely related types of conceptual time systems are introduced, namely 'conceptual time systems with parts' and 'conceptual time systems with actual objects'. The latter yields a generalization of classical systems with many similar subsystems or particles. Some industrial and scientific applications of conceptual movies of subsystems (particles) are presented.

1 Introduction

For the investigation of temporal phenomena many theories have been developed. A nice overview over the history of the development of formal representations of 'time' can be found in [3] – ranging from debates about 'time' in ancient Greece over Newton's 'absolute time' that 'flows uniformly without relation to anything external' to Einstein's [12] four-dimensional curved space-time-continuum. Quantum theoretical considerations [19] led to the idea of smallest units in space and time and pragmatically we describe temporal phenomena using discrete time in suitable granularities from years to nano-seconds. The investigation of biological and sociological systems led to general system theory [8], the development of computers initiated automata theory [1,11], Petri nets [29,30], and mathematical system theory [20,22,26,47]. More recently processes [32,33] were investigated using situation calculus [24], event calculus [21], temporal logics [14,15,36], data base theory including Knowledge Discovery in Databases [23], and description logics [2,25,31].

1.1 What is a system, what is a state?

For any two of these theories the system descriptions are different, and Lin [22] states in his book 'General Systems Theory: A Mathematical Approach', p. 347:

There might not exist an ideal definition for general systems, upon which a general systems theory could be developed so that this theory would serve as the theoretical foundation for all approaches of systems analysis, developed in various disciplines.
Though nearly all of the above mentioned theories have some formal or intuitive notion of a 'state' of a system a precise definition of a 'state' seems to be very difficult. Zadeh ([48], p.40) wrote in his paper 'The Concept of State in System Theory':
To define the notion of state in a way which would make it applicable to all systems is a difficult, perhaps impossible, task. In this chapter, our modest objective is to sketch an approach that seems to be more natural as well as more general than those employed heretofore, but still falls short of complete generality.
Clearly, a general definition of 'state' can be given only within a general system description – and that should cover the most simple and useful system descriptions namely protocols of statements or measurements taken at some 'real system'. While arbitrary statements about a 'real system' are formally represented by n-ary relations measurements and their values at a certain 'object' or at a certain 'point of time' are usually represented in data tables, hence by a ternary relation interpreted as 'the measurement taken at a certain point of time yields a certain value'.

1.2 Data Tables, Concept Lattices, and States

The practical relevance of data tables for knowledge representation in general and the theoretical importance of lattices introduced by Birkhoff [10] led Wille [37] to the basic notion of a 'formal concept' of a given 'formal context' and the introduction of Formal Concept Analysis. The standard reference is Ganter, Wille [17,18]. For earlier related work the reader is referred to Barbut [4] and Barbut, Monjardet [5]. The connection to arbitrary data tables, formally described as many-valued contexts, was investigated in the Research Group Concept Analysis at Darmstadt University of Technology and led to the development of Conceptual Scaling Theory [16].
Applications of Formal Concept Analysis to temporal data led the author to the idea that the notion of 'state' should be defined as a formal concept of a suitable formal context which describes the observations at a 'real system'. Therefore, a general system description was needed. It should contain a general time description and a description of events such that the states of the system are related to these descriptions in a simple and meaningful way, for example, that a system is at each 'point of time' in exactly one state. These ideas led the author to the development of a Conceptual System Theory [42,45] where the definition of a Conceptual Time System was introduced. In contrast to many other theories these systems are described from the data side and not from the side of the laws. Then laws can be defined using implications, clauses and, more generally, the object distribution [46].

1.3 Granularity and Conceptual Scaling

Besides that 'data – law polarity' the representation of granularity plays an important role. It seems to be obvious that the choice of a coarser granularity in the data yields

fewer states. Therefore, to define 'states' a granularity tool should be introduced in the formal description of a system. In Conceptual Time Systems the granularity tool is Conceptual Scaling Theory which yields flexible possibilities for the generation of mental frames. These frames are constructed as concept lattices of formal contexts (called scales) which are used to describe the contextual meaning of the values of many-valued contexts ([16,18]). Each many-valued context together with a family of scales generates a formal context, the derived context ([18], p.38). The concept lattice of the derived context can be embedded into the direct product of the concept lattices of the chosen scales. The indiscernability classes of objects in the derived context describe the granularity generated by these scales on the object set of the given many-valued context.
Conceptual Scaling Theory is closely related to other granularity describing theories, as for example Fuzzy Theory (Zadeh [49,50], Wolff [40,41]), Rough Set Theory (Pawlak [27,28], Wolff [43]) and the theory of information channels in the sense of Barwise and Seligman [6] as described by the author [44].

1.4 Time in Conceptual Time Systems

Now, we shortly discuss the formal representation of 'time' in Conceptual Time Systems. There are many possibilities to represent 'time' in many-valued contexts. A simple and often used way is to collect all the data for a given 'time point' in a row of a data table. But what is a 'time point'? Should we start with a set of 'time points' used as formal objects of a many-valued context? Should we introduce a linear ordering on this set? Should we introduce intervals of a linearly ordered set of time points? How to represent the non-linear ordering of the intervals? Should we introduce time measurements as many-valued attributes of a many-valued context? What is the role of other measurements in space or other domains?
To give a short answer: we introduce a set G whose elements are called 'time granules' or 'time objects'. They play the role of 'elementary pieces of time' or 'time instants' like 'morning' and they are contextually described by two many-valued contexts **T** and **C** on the same set G of time granules. The first many-valued context **T**, called the 'time part' describes the time granules by time measurements, for example, the time granule 'morning' begins at 9 and ends at 12 o'clock. The second many-valued context **C**, called the 'event part' describes all other measurements taken at the 'real system'. Each many-valued attribute in the time part and the event part has a conceptual scale describing the granularity we wish to use for its values.
That simple and general framework of a Conceptual Time System together with Conceptual Scaling Theory [16,18] allows for introducing states as object concepts of the event part. The idea of a space-time description of a system leads to the definition of the 'general phase space' as a concept lattice from which the state space can be obtained by a simple projection omitting the time part. That includes the classical state spaces in physics as well a discrete state spaces in psychology, biology, linguistics, and computer sciences.

2 Example: Temporal Concept Analysis of Some Observations of Stars

The following example serves for several purposes: At first we use it to demonstrate (for readers not familiar with Formal Concept Analysis) a formal context, its concept lattice, object concepts, attribute concepts and line diagrams. Then we use it as the derived context of a conceptual time system, for the introduction of 'conceptual time systems with parts', 'object construction', and 'conceptual time systems with actual objects'.

2.1 Observing Stars

The following Table 1 is a cross-table of a formal context **K** = (G,M,I) with 4 objects and 7 attributes. It is understood as a short protocol of some observations of stars.

observation	Monday	Tuesday	morning	evening	east	west	luminous
1	×		×		×		×
2	×			×		×	×
3		×	×		×		×
4		×		×		×	×

Table 1: A cross-table of a formal context

Clearly, the first observation tells us that at Monday morning in the east a luminous star was observed. The concept lattice **B**(**K**) is graphically represented by the following line diagram.

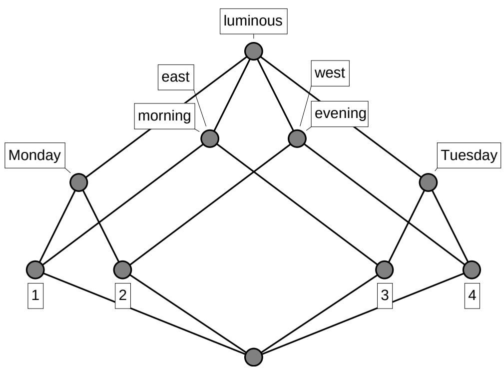

Figure 1: A line diagram of the concept lattice of **K**

Reading example: Observation 1 has the attributes 'Monday', 'morning', 'east' and 'luminous'. The attributes 'east' and 'morning' have the same attribute concept, namely the concept (A,B) with the extent A = {1,3} and the intent B = {east, morning, luminous}. The two object concepts of 1 and 3 are sub-concepts of (A,B). For an introduction to FCA the reader is referred to [38,39].

2.2 A Conceptual Time System

The following Table 2 describes in some sense the 'same' situation as Table 1. Indeed, Table 1 is the derived context of the conceptual time system given by Table 2 and the scales in Table3. There are four 'time objects' or 'time granules', labeled 1,2,3,4, interpreted as 'pieces of time' during which some measurements took place. For each time object g there is a time description of g (in the second and third column) and an event description (in the fourth and fifth column).

observation	day	day time	space	brightness
1	Monday	morning	east	luminous
2	Monday	evening	west	luminous
3	Tuesday	morning	east	luminous
4	Tuesday	evening	west	luminous

Table 2: A many-valued context of four observations of stars

In the general definition of a conceptual time system such tables are described by two many-valued contexts on the same set G of 'time objects'. By definition a conceptual time system contains also for each of its many-valued attributes a scale. In this example the scales are given by the following Table 3 (using some abbreviations):

	Mo	Tu
Mo	×	
Tu		×

	mo	ev
mo	×	
ev		×

	east	west
east	×	
west		×

	lum
lum	×

Table 3: The nominal scales for the 4 many-valued attributes.

To get a flavour of the rich possibilities of conceptual scaling the reader is referred to Wolff [39], Ganter, Wille [16,18].

3 Conceptual System Theory

At first we give a short description of Conceptual System Theory developed by the author [42,45]. The basic notion in Conceptual System Theory is that of a Conceptual

Time System which is used as a quite general definition of a system. In contrast to system descriptions using laws Conceptual Time Systems describe the observations taken at 'real systems'. While system descriptions using laws (for example differential equations) do not refer in their theoretical framework to the 'dirty data' and the granularity of the data Conceptual Time Systems represent the data as well as a granularity tool, namely the conceptual scales. The implications and clauses in the resulting derived contexts yield the laws which are valid in these systems with respect to the chosen granularity.

3.1 Conceptual Time Systems

Definition: 'conceptual time system'
Let **G** be an arbitrary set and **T** := ((**G**, **M**, W, I_T), ($\mathbf{S}_m$ | m ∈ **M**)) and
C := ((**G**, **E**, V, I), ($\mathbf{S}_e$ | e ∈ **E**)) scaled many-valued contexts (on the same object set **G**). Then the pair (**T**, **C**) is called a *conceptual time system on **G***.
T is called the *time part* and **C** the *event part of (**T**, **C**).*

The elements of the set **G** are called *time granules* or *time objects*, interpreted as pieces of time like 'morning' which are described by *time measurements*, formally represented by the many-valued attributes of the many-valued context (**G**, **M**, W, I_T). The scales $\mathbf{S}_m$ of the time measurements m ∈ **M** describe the granularity of this conceptual 'language' about the values of the time measurements. The many-valued context (**G**, **E**, V, I) of the event part **C** is interpreted as the measurement table of the events of that system. The scales $\mathbf{S}_e$ of the events e ∈ **E** describe the chosen granularity for the values of the event measurements.

3.2 States in Conceptual Time Systems

In a conceptual time system (**T**, **C**) we would like to say 'The system is at time granule g in state s(g)'. This is introduced in the following definition where the object concepts of the derived context $\mathbf{K}_C$ are defined as the *states*. As to the time part we call the object concepts of the derived context $\mathbf{K}_T$ the *time states* or *time granule concepts*.

Definition: 'state space of a conceptual time system'
Let (**T**, **C**) be a conceptual time system and $\mathbf{K}_T$ and $\mathbf{K}_C$ the derived contexts of **T** and **C**. For each time granule g we define *the state s(g) of (**T**, **C**) at time granule g* by s(g) := γ_C(g) := the object concept of g in $\mathbf{K}_C$ and the *time state or time granule concept t(g) of (**T**, **C**) at time granule g* by t(g) := γ_T(g) := the object concept of g in $\mathbf{K}_T$.
The set **S**(**T**, **C**):= {s(g) | g ∈ **G** } is called *the state space of (**T**, **C**)*. We say that *the system (**T**, **C**) is at time granule g in the state s ∈ **S**(**T**, **C**) iff s = s(g).*

This definition yields the 'partition meaning' of states, namely, that the set **G** of time granules is partitioned by the extents of the states, or, equivalently, that a system is at each time granule in exactly one state. Clearly, for any two time granules g and h, s(g) = s(h) iff for each event e the values e(g) and e(h) have the same scale attributes and therefore the same object concept in the scale S_e.
For the following discussion we need the notion of a subsystem of a conceptual time system and an embedding of the concepts of a subsystem into the concept lattice of the given system.

3.3 States in Subsystems

Definition: 'subsystem of a scaled many-valued context'
Let **C** = ((**G**, **E**, V, I), ($\mathbf{S}_e$ | e ∈ **E**)) be a scaled many-valued context; for an arbitrary subset **R** ⊆ **E** let $I_{\mathbf{R}}$:= {(g,e,v) ∈ I | e ∈ **R**}. Then
C(**R**) := ((**G**, **R**, V, $I_{\mathbf{R}}$), ($\mathbf{S}_e$ | e ∈ **R**)) is called *the **R**-part of **C*** or *a subsystem of **C***.

Analogously we define the 'R-part of a formal context (G, M, I)' (where R ⊆ M); ([18], p. 98).

Definition: 'part embedding φ_R'
Let **K** := (G, M, I) be a formal context, and R ⊆ M, and **K**(R) the R-part of **K**. Then the mapping φ_R which maps each concept (A,B) of **K**(R) onto the concept φ_R((A,B)) := (A,$A^{\uparrow}$) of **K** with the same extent, but possibly a larger intent $A^{\uparrow}$ in **K** is called the *part embedding from **B**(**K**(R)) into **B**(**K**).*

The part embedding φ_R is a meet-preserving order-embedding ([18], p.98). It demonstrates the difficulties that arise, if one does not distinguish between the states of a system and the states of a subsystem. It was shown by the author in [42] that φ_R is not state preserving which led to the notion of 'general time granules' and 'general states' for arbitrary concepts of $\mathbf{K}_T$ and $\mathbf{K}_C$.

3.4 Phase Spaces

In classical Mathematical System Theory [20] a phase of a system is a pair (t, s) of a point t of time and a state s. This classical division is represented in the notion of a conceptual time system (**T**, **C**), which allows us to introduce the notion of *the phase space* of an arbitrary conceptual time system [42].

Definition: 'phase space of a conceptual time system'
Let (**T**, **C**) be a conceptual time system on **G** and $\mathbf{K}_T$ and $\mathbf{K}_C$ the derived contexts of **T** and **C**.
The apposition $\mathbf{K}_T|\mathbf{K}_C$ of the derived contexts is called the *phase context of (**T**, **C**).*
The concept lattice **B**($\mathbf{K}_T|\mathbf{K}_C$) is called *the phase space of (**T**, **C**).*

For any 'general time granule' (A,B) $\in$ **B**($\mathbf{K}_T$) and any 'general state' (C,D) $\in$ **B**($\mathbf{K}_C$) we say, that *the system (**T**, **C**) is during the general time granule (A,B) always in the general state (C,D)* iff A $\subseteq$ C (which is equivalent to φ_T(A,B) $\leq$ φ_C(C,D) in **B**($\mathbf{K}_T|\mathbf{K}_C$), where φ_T and φ_C are the part embeddings of **B**($\mathbf{K}_T$) and **B**($\mathbf{K}_C$) into **B**($\mathbf{K}_T|\mathbf{K}_C$)). And we say, that *the system (**T**, **C**) is during the general time granule (A,B) sometimes* [*never*] *in the general state (C,D)* iff A $\cap$ C $\neq \emptyset$
[resp. A $\cap$ C = $\emptyset$].

In Table 1 the formal context **K** is the apposition of $\mathbf{K}_T$ (with 4 time attributes) and $\mathbf{K}_C$ (with 3 event attributes). For time granule 1 the pair (t(1), s(1)) is the 'first' phase of this system, where t(1) = ({1}, {Monday, morning}) and s(1) = ({1,3}, {east, luminous}). By the previous definition (**T**, **C**) is during time granule 1 always in the state s(1) since φ_T(t(1)) = ({1}, {Monday, morning, east, luminous}) ≤ ({1,3}, {morning, east, luminous}) = φ_C(s(1)) in **B**(**K**) = **B**($\mathbf{K}_T|\mathbf{K}_C$).

3.5 Object Construction and Object – Time Duality

If we interpret the observations in the previous example as being caused by a single star, then this star could be nicely represented by the attribute concept of 'luminous'. The extent of this concept describes the time when this star was 'discovered' by the observation of its 'permanent' attribute 'luminous'. This star looks like a 'planet' since 'it moves' in the state space.
If we interpret the same observations as being caused by two stars, namely the 'morning star' which 'always' (Monday and Tuesday) occurs in the morning in the east, and the 'evening star' which 'always' occurs in the evening in the west, then these two stars look like fixed stars. The 'morning star' can be nicely represented as the formal concept ({1,3}, {morning, east, luminous}). Clearly, we could explain these observations also by four stars which might be represented as the object concepts of the four time granules.
This example shows that Conceptual Time Systems might be useful to understand from a philosophical point of view the role of objects, its construction from temporal phenomena and therefore its intimate relationship to our notion of time. For further discussions I call this relationship the 'object – time duality'. Is there any meaningful connection to the 'particle – wave duality' or the complementarity of energy and time?

4 Formal Representations of Time Objects and 'Usual' Objects

Clearly, since time granules are used as formal objects in conceptual time systems the introduction of 'usual' objects like 'morning star' leads to the problem of the formal representation of them. Let us assume that we have accepted two objects 'morning star' and 'evening star'; then we might introduce these two stars as subsystems of a system of some stars. The following definition describes such conceptual time systems with (some selected) subsystems or parts.

4.1 Conceptual Time Systems with Parts

In applications a conceptual time system with parts might describe not only its specified subsystems but also the relations among these subsystems, for example the flow of information among several participants of a computer network.

Definition: 'conceptual time system with parts'
Let (**T**, **C**) be a conceptual time system, **T** = ((**G**, **M**, W, I_T), ($\mathbf{S}_m$ | m ∈ **M**)),
C = ((**G**, **E**, V, I), ($\mathbf{S}_e$ | e ∈ **E**)) and P some index set. If for each p ∈ P $\mathbf{T}_p$ is the $\mathbf{M}_p$ –part of **T** for some $\mathbf{M}_p \subseteq \mathbf{M}$ and $\mathbf{C}_p$ is the $\mathbf{E}_p$ –part of **C** for some $\mathbf{E}_p \subseteq \mathbf{E}$ then (**T**, **C,** (($\mathbf{T}_p$, $\mathbf{C}_p$) | p ∈ P)) is called a *conceptual time system with parts from P*.

observ.	day	day time	morning star		evening star	
			space	brightness	space	brightness
1	Monday	morning	east	luminous	/	/
2	Monday	evening	/	/	west	luminous
3	Tuesday	morning	east	luminous	/	/
4	Tuesday	evening	/	/	west	luminous

Table 4: A conceptual time system with parts 'morning star' and 'evening star'

In Table 4 the time part of the 'morning star' is described by the columns 1,2,3 and the event part by the columns 1,4,5.
The corresponding scales are chosen as nominal scales for 'day' and 'day time' and as the following scales (scaling also the missing value "/") for the columns 4,5,6,7 :

	east
east	×
/	

	luminous
luminous	×
/	

	west
west	×
/	

Table 5: The scales for columns 4, 5 (7), and 6.

The set P of parts is chosen as {morning star, evening star}, their time parts are chosen as **T**, their event parts are described by columns 4, 5 respectively 6, 7 (together with the corresponding scales).

4.2 Subposition of Parts: Actual Objects

In many applications objects are introduced as something which is relatively stable, but sometimes objects are changing quite rapidly with time. Then we usually combine such an object with a 'piece of time' as for example 'my father in his youth' or the 'morning star at Monday morning'.

If we like to use the parts as new objects, then it is very natural to combine a part p and a time granule g to a pair (p,g). This is done in the following example.

observation	day	day time	space	brightness
(ms,1)	Monday	morning	east	luminous
(ms,2)	Monday	evening	/	/
(ms,3)	Tuesday	morning	east	luminous
(ms,4)	Tuesday	evening	/	/
(es,1)	Monday	morning	/	/
(es,2)	Monday	evening	west	luminous
(es,3)	Tuesday	morning	/	/
(es,4)	Tuesday	evening	west	luminous

Table 6: The subposition of parts: The pairs (part, time granule) as 'actual objects'

In Table 6 the information of Table 4 (where the parts are graphically represented using apposition of their tables) is now represented using subposition of their tables. (For a definition of apposition and subposition the reader is referred to [18], p. 40.)
In general we take some of the elements of P×G as time objects of a new conceptual time system with a suitable time part (representing some common aspect of the time parts $\mathbf{T}_p$, $p \in P$) and a suitable event part (representing some common aspect of the event parts $\mathbf{E}_p$, $p \in P$) usually with suitable refinements in the scales. Then we obtain a 'conceptual time system with actual objects' in the sense of the following definition.

Definition: 'conceptual time system with actual objects'
Let P and G be sets and $\Pi \subseteq P \times G$, then a conceptual time system (**T**,**C**) on Π is called a *conceptual time system with actual objects from P×G.* P is called the set of *objects* or *particles*, G the set of *time points* and Π the set of *actual objects* (which is the set of time granules of (**T**,**C**)).
Clearly, each conceptual time system with actual objects can be transformed (in many ways) into a conceptual time system with parts. While in conceptual time systems with parts for each time granule g the intent of g (in the derived context) contains all the information given in all parts, 'this information' is distributed over the intents of all actual objects (p,g). The details of these constructions are not explained here.

5. Applications

Conceptual time systems with actual objects can be used to represent the development of many objects in a common phase space or – omitting the time part – in a common state space. Classical examples are a swarm of birds or a physical system of particles. The author has applied this method in cooperation with a computer firm in the production of wafers based on process data of a reactor; it was applied in a chemical plant for the visualization of a chemical process in a distillation column [45]. Now we describe two applications in greater detail, one in an air-conditioning plant and the other in a therapeutic treatment of an anorectic young woman.

5.1 A Phase Space of an Air-Conditioning Plant

In this application an air-conditioning plant in the pharmaceutical industry was investigated to control the behavior of the plant. Therefore several temperature time series were collected. The usual visual inspection of these discrete data represented in 'curves' finds some phases which occur nearly every day, for example the phase that during the afternoon the temperature outside and inside is relatively high.
To represent such phases systematically we constructed several conceptual time systems with different scales based on the same many-valued context describing the original temperature measurements taken at each hour of the first six months (=182 days) of the year 2000 using 4368 = 182 x 24 hours as time objects. Each hour is described in the time part by three attributes 'month', 'day', and 'hour' where the values of 'hour' range from 0 to 23. Clearly, the temperature measurements form the event part.
To demonstrate the method we take from these data only the first three days and only two temperatures, namely the outside temperature and the temperature in one of the production rooms. For the 72 = 3 x 24 hours we use labels, for example '100' for the midnight hour of the first day, and '323' for the last hour of the third day. The line diagram in Figure 2 shows the phase space of a conceptual time system using an ordinal scale with five concepts for the attribute 'hour' and two ordinal scales each with three concepts for the temperatures inside and outside.
The process starts at midnight of the first day (in the phase of time granule 100) where the temperatures outside and inside are low. The system changes to another phase at time granule 108, but remains in the same state. After time granule 110 the system changes to a state with a medium inside temperature. The change to the phase of 112 shows that the transitions between two phases clearly need not follow the lines of the line diagram. Omitting the 'hour'-attributes we get the projection of this phase space onto the state space (described by the six points in the upper-left part) which can be embedded into a 3 x 3 –grid spanned by the two 3-chains for the temperatures.
A typical phase space implication: hour:>=16 ⇒ Temp-outside:<=6.0.

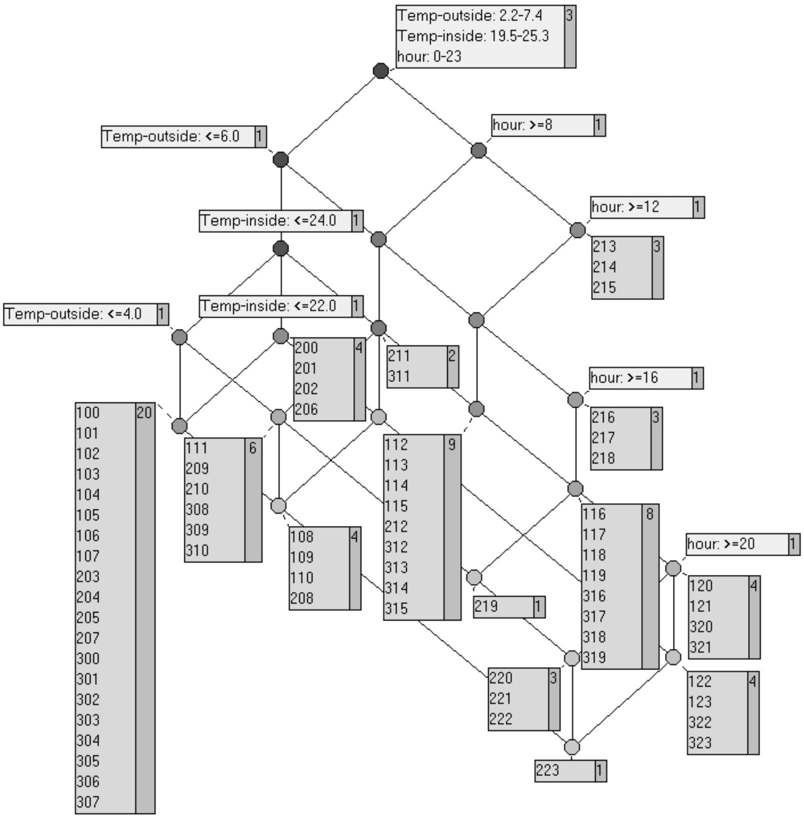

Figure 2: A phase space of an air-conditioning plant

5.2 A State Space of a Therapeutic Process of an Anorectic Young Woman

In the following example a conceptual time system with actual objects is described. We represent a therapeutic process of an anorectic young woman over a period of about three years [34,35]. During the course of treatment at four points of time the patient was asked to describe herself and her family using a repertory grid. This is a many-valued context where (in this example) the objects are the members of the family denoted by SELF (the patient), FATHER, MOTHER, together with IDEAL (self-ideal of the patient). The many-valued attributes are self-chosen pairs of constructs like 'rational - emotional' and the values are marks from 1 to 6, for example '1' for 'very rational' and '6' for 'very emotional'. Application of Formal Concept Analysis led to four concept lattices – one for each time point. Since the chosen attributes changed from time point to time point there was no common formal framework for the representation of the whole process. Therefore the 'main information' in these four repertory grids was described by the therapeut using a suitable common set of attributes for all points of time. The following line diagram demonstrates the state space of the corresponding conceptual time system with 16 actual objects (for example SELF1 denotes SELF at time point 1).

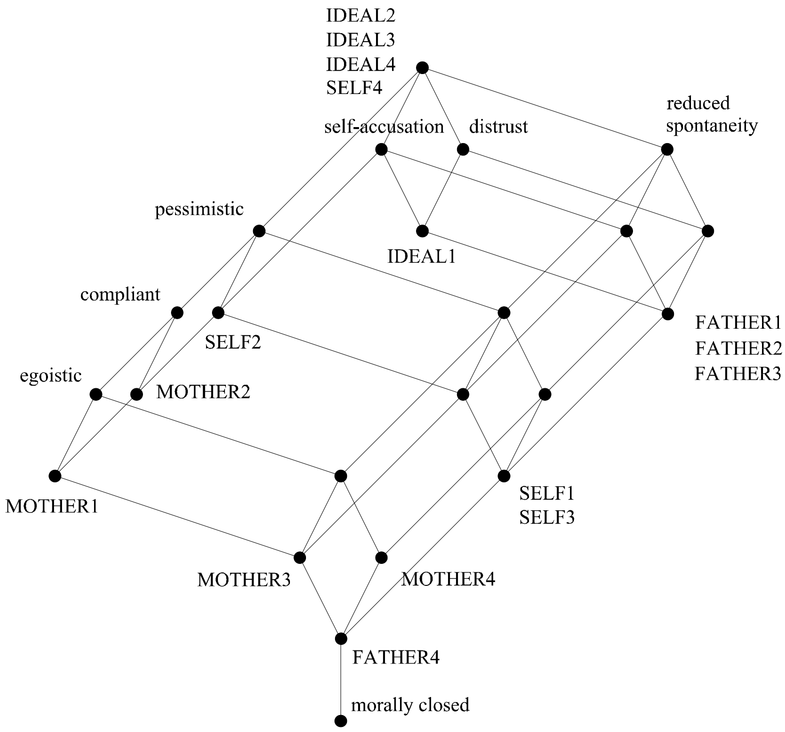

Figure 3: A state space of a therapeutic process of an anorectic young woman

From the line diagram in Figure 3 we see for example that SELF1 and SELF3 are in the same state described by the attributes 'pessimistic', 'self-accusation', 'distrust', and 'reduced spontaneity'. SELF2 has only the first two of these attributes while SELF4 has none of the used attributes just like her IDEAL at the time points 2, 3, and 4. The development of the FATHER is quite remarkable since he is stable during the first three time points in the state 'self-accusation', 'distrust', and 'reduced spontaneity' and in the forth time point he is judged as having all of the used attributes except the attribute 'morally closed' which was used for none of the 'actual persons'.
This investigation is based on a long cooperation with my friend Norbert Spangenberg [34,35] at the Sigmund-Freud-Institute Frankfurt.

6 Future Research in Temporal Concept Analysis

The actual state of Formal Concept Analysis and Conceptual System Theory seems to be a promising starting point for a new discipline in Conceptual Knowledge Processing namely Temporal Concept Analysis understood as the theory of temporal phenomena described with tools of Formal Concept Analysis. On the basis of a clear definition of states and phases as formal concepts a general description of temporal phenomena in all areas of science can be developed.
One of the next steps will be a conceptual transition theory such that transitions in automata theory [1,11] and in the theory of Petri Nets [7,29,30] can be understood in a temporal conceptual framework. Future research is necessary for the conceptual understanding of temporal logics, temporal dependencies and temporal relational structures.
For the application of Temporal Concept Analysis programs have to be developed which represent complex temporal systems in easily understandable visualizations. One of the most promising possibilities are conceptual movies introduced by the author [42,45]. A diploma thesis on conceptual movies was written by Freimuth [13].

Acknowledgments:
I like to thank Prof. Walter Sedelow for his encouragement and his help during the development of this research area and for his suggestion to introduce the name 'Temporal Concept Analysis'.
Thanks also to the firm NAVICON GmbH Frankfurt for the permission for using their excellent programs on Conceptual Knowledge Processing in their NAVICON DECISION SUITE.

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