Process Representation in Temporal Concept Analysis

Karl Erich Wolff

Department of Mathematics, University of Applied Sciences
Schoefferstr. 3, D-64295 Darmstadt, Germany
wolff@mathematik.tu-darmstadt.de

Abstract. Process representation in Temporal Concept Analysis is described and related to other process descriptions. Temporal Concept Analysis [15] is based on Formal Concept Analysis (Ganter, Wille [2,3]). The basic tools are Conceptual Time Systems [10,11,14] introducing the new construction of state and phase spaces as concept lattices. Applications in industry and science using the new tool of conceptual movies are demonstrated.

1 Continuous, Discrete, and Conceptual Structures

The investigation of processes was very successful at first in physics where the observation of systems led to laws formulated in highly developed theories using as mathematical tools the continuum of real numbers, multidimensional vector spaces and partial differential equations. 'Natura non facit saltus' was accepted until 1900 when Planck introduced Quantum Theory. The successful development of discrete mathematics and its applications, for example in the computer sciences and in linguistics, led to a dissection into continuous and discrete methods.

General knowledge representation has to cover both, the continuous and the discrete case. A powerful tool for that purpose is Conceptual Knowledge Processing based on Formal Concept Analysis (Wille[8], Ganter, Wille [2,3]) which represents data tables by conceptual hierarchies, called concept lattices; they are (up to isomorphism) exactly the complete lattices. Therefore the complete lattice of all real numbers (including $+\infty$ and $-\infty$) with its usual ordering is covered as well as all finite lattices, and among them finite chains. That led the author [10] to a new conceptual description of time generalizing continuous and discrete time as well as (order-theoretic) multidimensional time – for example for the formal representation of a flight through several time zones.

General knowledge representation has to develop granularity tools. In Conceptual Knowledge Processing the main granularity tools are the conceptual scales [1] which are used to represent the objects of a many-valued context ('data table with arbitrary values') in the direct product of the concept lattices of the scales. The close connections among scaled many-valued contexts, information channels, linguistic variables in Fuzzy Theory, and knowledge bases in Rough Set Theory are investigated by the author [12,9,13].

2 Processes: Concepts, States, and Phases

For a common description of continuous and discrete processes the author [10,11] has introduced the notion of a *conceptual time system* as a pair (T,C) of two scaled many-valued contexts [1,3] on the same object set G of *time objects* where T is called the *time part* and C the *event part* of (T,C). The attributes in T are interpreted as time measurements, those in C as event measurements. For each attribute the concept lattice of its conceptual scale is a hierarchy for the values of that attribute and therefore yields a suitable frame for embedding objects at the same place, namely its object concept iff they have the same value (at the given attribute). The introduction of that granularity tool into the formal definition of a conceptual time system allows for the definition of *states* as the object concepts of the derived context [1,3] of the event part. The set of states is called the *state space*, the concept lattice of the event part is called the *general state space*. A *phase* is a pair (t(g),s(g)) where t(g) is the object concept of the time object g in the time part and s(g) is the state of g. A phase is a special element of the *general phase space* which is defined as the direct product of the concept lattice of the time part and the concept lattice of the event part. The elements of the general phase space are pairs ((A,B), (C,D)) of concepts, called *general phases*, where (A,B) describes a *general time granule* (for example 'week') by its extent A consisting of all time granules (for example '7 days') satisfying all conditions of the intent B; similarly the formal concept (C,D) is called a *general state* and we say that the *system (T,C) is during the general time granule (A,B) always in the general state (C,D)* iff $A \subseteq C$. That yields statements like: 'During the first week the system was in an optimal state'.

Conceptual time systems have been applied by the author in industry, psychology, and sociology.

3 Comparison to Other Process Definitions

In contrast to many other definitions of processes or systems like those given in Mathematical System Theory [4,5], in Automata Theory, in the theory of Petri Nets or in Knowledge Representation by Sowa [7] the definition of conceptual time systems

- covers the continuous case as well as the discrete case,
- has a simple and useful definition of states and phases which is related to time representation and to the granularity of the system description,
- relates the process description by data tables to the process description by laws through the implications of the derived context [14].

That leads to a unification of the above mentioned process theories, to a better understanding of the relation between data and laws, and to an easier and more flexible way to represent processes in practice. Applications in industry and science are demonstrated in the talk.

4 References

1. Ganter, B., R. Wille; Conceptual Scaling. In: F.Roberts (ed.) Applications of combinatorics and graph theory to the biological and social sciences,139-167. Springer Verlag, New York, 1989.

2. Ganter, B., R. Wille: Formale Begriffsanalyse: Mathematische Grundlagen. Springer-Verlag, Berlin-Heidelberg 1996.
3. Ganter, B., R. Wille: Formal Concept Analysis: mathematical foundations. (translated from the German by Cornelia Franzke) Springer-Verlag, Berlin-Heidelberg 1999.
4. Lin, Y.; General Systems Theory: A Mathematical Approach. Kluwer Academic / Plenum Publishers, New York, 1999.
5. Mesarovic, M.D., Takahara, Y.; General Systems Theory: Mathematical Foundations.Academic Press, London, 1975.
6. Sowa, J.F.; Knowledge Representation: Logical, Philosophical, and Computational Foundations. Brooks/Cole, Pacific Grove, CA.
7. Sowa, J.F.; Processes and Causality. http://www.bestweb.net/~sowa/ontology/causal.htm
8. Wille, R.: Restructuring lattice theory: an approach based on hierarchies of concepts. In: Ordered Sets (ed. I.Rival). Reidel, Dordrecht-Boston 1982, 445-470.
9. Wolff, K.E.: Conceptual Interpretation of Fuzzy Theory. In: Zimmermann, H.J.: EUFIT´98, 6th European Congress on Intelligent Techniques and Soft Computing, Aachen 1998, Vol. I, 555-562.
10. Wolff, K.E.: Concepts, States, and Systems. In: Dubois, D.M. (ed.): Computing Anticipatory Systems. CASYS´99 - Third International Conference, Liège, Belgium, August 9-14, 1999 American Institute of Physics Conference Proceedings 517 (selected papers), 2000, p. 83-97.
11. Wolff, K.E.: Towards a Conceptual System Theory. In: B. Sanchez, N. Nada, A. Rashid, T. Arndt, M. Sanchez (eds.) Proceedings of the World Multiconference on Systemics, Cybernetics and Informatics, SCI 2000, Vol. II: Information Systems Development, International Institute of Informatics and Systemics, 2000, p.124-132.
12. Wolff, K.E.: Information Channels and Conceptual Scaling. In: Stumme, Gerd (Hrsg.) Working with Conceptual Structures. Contributions to ICCS 2000. Shaker Verlag, Aachen, 2000, 277-283.
13. Wolff, K.E.: A Conceptual View of Knowledge Bases in Rough Set Theory. Proceedings of the Second International Conference on Rough Sets and Current Trends in Computing. Technical Report CS-2000-07, Department of Computer Science, University of Regina, 188-192.
14. Wolff, K.E.: Conceptual Descriptions of Systems: Data and Laws. Fourth International Conference on Computing Anticipatory Systems, CASYS'2000 . Liège, Belgium, August 7-12, 2000. To appear in the Proceedings of CASYS'2000.
15. Wolff, K.E.: Temporal Concept Analysis. 9th International Conference on Conceptual Structures (ICCS 2001). To appear in the Proceedings of the ICCS-01 Workshop on Concept Lattice for KDD.<