Detection of Point Scatterers by Regularized Inversion of a Linear Ultrasound System Model

Tobias Gehrke, Heinrich M. Overhoff

Medical Engineering Laboratory, University of Applied Sciences Gelsenkirchen
tobias.gehrke@fh-gelsenkirchen.de

Abstract. Scatterer detection in medical ultrasound imaging can be formulated as an inverse problem concerning a linear system model based on point scatterers. The applicability of different methods of regularized inversion is examined and a novel regularization scheme specifically adapted to ultrasound imaging is introduced. The results show that standard inversion methods fail in ultrasound imaging while the proposed new method is a feasible way to improve axial resolution without amplifying erroneous signal components. The consequences of A-line detection by regularized inversion for B-mode imaging are examined by numerical simulations.

1 Introduction

In medical ultrasound imaging sound waves are emitted from a transducer into body tissue and partially scattered back at inhomogeneously distributed scatterers with individual scattering coefficients. Since scattering takes place at different depths within the tissue, the scattered waves of a single pulse-echo sequence contain a "fingerprint" of the tissue in axial direction, the so called A-line. A complete image is constructed by consecutive side-by-side arrangement of A-lines from repeated pulse-echo sequences. The reconstruction of a single A-line from the scattered data within an ultrasound image can be understood as the spatially sensitive estimation of scattering coefficients from the scattered waves. Conventional ultrasound imaging algorithms employ unspecific methods such as envelope detection and matched filtering for this task. The receive signal is however known to be the sum of blurred, scaled, and time delayed replica of the excitation signal, where the blurring is caused by the transducer's dynamic behaviour, diffraction and absorption. Inclusion of an analytic model for the dynamic system behaviour into the estimation process has the potential to partly reverse the blurring and thereby to improve the resolution of ultrasound images.

A common analytic model for the dynamic behaviour of an ultrasound imaging system is investigated with respect to the properties of image reconstruction by system inversion. Several standard approaches for system inversion are investigated with respect to to their robustness against erroneous receive data and their applicability in B-mode imaging. Furthermore, a novel inversion scheme, that is specifically adapted to ultrasound imaging by combining envelope detection with regularized inversion, is developed and tested.

2 Materials and Methods

2.1 Model for an Ultrasound Imaging System

Ultrasound imaging uses the process chain: emit signal - electromechanical emit transformation - wave propagation and scattering - electromechanical receive transformation - receive signal. A discrete linear model for the calculation of ultrasound B-mode images relates the transducer's electric receive signal $\boldsymbol{y}$ to the scattering behaviour of the tissue [1]:

$$\boldsymbol{y} = \boldsymbol{\Psi} \boldsymbol{s} \tag{1}$$

Considering a single A-line, the column vector $\boldsymbol{s}$ contains the scattering coefficients of scatterers at consecutive distances, with a zero element if no scatterer exists. The lower triangular Toeplitz matrix $\boldsymbol{\Psi}$ contains time shifted pulse-echo-wavelets. A pulse echo wavelet is the receive signal due to a single point scatterer of unit strengths. For brevity it is assumed here that the pulse-echo-wavelet is invariant to the scan direction.

2.2 Model Inversion

By inversion of the model for the ultrasound imaging system (1), the scattering vector

$$\tilde{\boldsymbol{s}} = \boldsymbol{\Psi}^{\#} \boldsymbol{y} \tag{2}$$

can be estimated. Matrix $\boldsymbol{\Psi}^{\#}$ reverses the blurring effect of $\boldsymbol{\Psi}$ and thereby provides an estimate with good axial resolution. In many cases $\boldsymbol{\Psi}^{\#}$ is of the form [2]

$$\boldsymbol{\Psi}^{\#} = \left(\boldsymbol{\Psi}^{\mathrm{T}} \boldsymbol{\Psi} + \lambda^2 \boldsymbol{H}\right)^{-1} \boldsymbol{\Psi}^{\mathrm{T}} \tag{3}$$

with matrix $\boldsymbol{H}$ specific to the inversion method.

Noise in $\boldsymbol{y}$ can cause large undesired oscillations of high frequency in the estimated solution $\tilde{\boldsymbol{s}}$. Thus $\boldsymbol{\Psi}^{\#}$ must ensure that $\tilde{\boldsymbol{s}}$ is insensitive to noise.

Unregularized Inversion The scattering vector can be estimated by solving (1) with the pseudoinverse of the system matrix, i.e. by choosing $\boldsymbol{\Psi}^{\#} = \boldsymbol{\Psi}^{\dagger}$ in (2) or $\boldsymbol{H} = 0$ in (3).

Singular Value Truncation A matrix whose pseudoinverse provides a noise insensitive estimation $\boldsymbol{s}_{\nu}^{\mathrm{tsvd}}$ can be obtained by truncating the ν smallest singular values of $\boldsymbol{\Psi}$. The resulting matrix $\boldsymbol{\Psi}_{\nu}$ is the closest rank-ν approximation to $\boldsymbol{\Psi}$ [2, 3] and thus the system characteristics is largely maintained. The corresponding regularized inverse in (2) becomes $\boldsymbol{\Psi}^{\#} = \boldsymbol{\Psi}_{\nu}^{\dagger}$.

Small Solution Norm Condition Approximately solving the system (1) subject to a small solution norm provides a noise robust estimate [2, 4]:

$$\tilde{\boldsymbol{s}}_{\lambda}^{\text{tik}} = \underset{\boldsymbol{s}\in\mathbb{R}^n}{\operatorname{argmin}} \left\{ \|\boldsymbol{\Psi s} - \boldsymbol{y}\|_2^2 + \lambda^2 \|s\|_2^2 \right\} \tag{4}$$

The regularization parameter λ adjusts the trade-off between a small residual norm and a small solution norm. This method, known as Tikhonov regularization, can be given in terms of a regularized inverse (3) with $\boldsymbol{H}$ being the unity matrix.

Small First Derivative Constraint A smooth and thereby noise insensitive estimate is obtained by minimizing

$$\tilde{\boldsymbol{s}}_{\lambda}^{\text{sfd}} = \underset{\boldsymbol{s}\in\mathbb{R}^n}{\operatorname{argmin}} \left\{ \|\boldsymbol{\Psi s} - \boldsymbol{y}\|_2^2 + \lambda^2 \|\boldsymbol{D s}\|_2^2 \right\} \tag{5}$$

with matrix $\boldsymbol{D}$ being a discrete approximation of the first derivative operator and $\boldsymbol{H} = \boldsymbol{D}^{\mathrm{T}}\boldsymbol{D}$ (3) [2, 3].

Envelope as Solution Norm Weighting Envelope detection is commonly performed for A-line computation. It is computationally simple and relatively insensitive to noise but provides only moderate resolution. To increase the resolution of the A-lines, envelope detection and regularized inverse filtering can be combined, by regarding the envelope of the receive signal $\tilde{\boldsymbol{s}}^{\text{env}} = \text{env}(\boldsymbol{y})$ as a rough pre-estimate, which is refined by a system inversion process. Thereto Tikhonov regularization is generalized such that the estimated solution is weighted before its norm is calculated

$$\tilde{\boldsymbol{x}}_{\beta,\lambda}^{\text{tew}} = \underset{\boldsymbol{x}\in\mathbb{R}^n}{\operatorname{argmin}} \left\{ \|\boldsymbol{\Psi s} - \boldsymbol{y}\|_2^2 + \lambda^2 \|\boldsymbol{L s}\|_2^2 \right\} \tag{6}$$

The weighting matrix

$$\boldsymbol{L} = \text{diag}(\boldsymbol{l}) = \text{diag}\left(1 - \frac{(1-\beta)\tilde{\boldsymbol{s}}^{\text{env}}}{\max(\tilde{\boldsymbol{s}}^{\text{env}})}\right) \tag{7}$$

causes samples $\tilde{\boldsymbol{s}}^{\text{env}} = 0$ to contribute completely to the norm $\|\boldsymbol{L s}\|_2^2$, while samples belonging to large pre-estimated scattering strengths $\tilde{\boldsymbol{s}}^{\text{env}} \approx \max(\tilde{\boldsymbol{s}}^{\text{env}})$ are suppressed by the weighting. Thereby it is ensured that oscillations at positions without scatterers are suppressed without affecting the system inversion in the vicinity of scatterers. The influence of the pre-estimate is controlled by $\beta \in [0, 1]$. Expression as a regularized inverse in terms of (3) yields $\boldsymbol{H} = \boldsymbol{L}^{\mathrm{T}}\boldsymbol{L}$.

2.3 Simulation Settings

The inversion methods are tested with a synthetic example consisting of two point scatterers at $z = 1.2\ mm$ and $z = 2.3\ mm$ of different scattering strengths.

The pulse-echo-wavelet has been determined experimentally. Gaussian noise is added to the simulated receive signal. The optimal regularization parameters are determined empirically.

A-line detection by regularized inversion with envelope solution norm weighting is included into a linear-array ultrasound imaging system, that has been emulated in the simulation software Field II [5]. A virtual phantom consisting of seven axially aligned point scatterers in $5mm$ distance to each other is used to evaluate the feasibility of the reconstruction algorithm for B-mode image reconstruction. As a comparison the same simulation has also been performed with a conventional envelope detection algorithm.

3 Results

Unregularized inversion and regularization by first derivative constraint were not able to produce meaningful results. The estimated scattering strengths obtained by the other three methods are shown in Fig. 1. The achieved axial resolutions, signal to noise ratios (SNR), and peak to sidelobe ratios (PSR) are summarised in Tab. 1.

The simulated B-mode image obtained by inversion with envelope solution norm weighting is displayed in Fig. 2(a). As a comparison the ultrasound image of the same phantom obtained by conventional envelope detection is shown in Fig. 2(b). Note that the grey scale in both images is applied after log-compression. Regarding the central axial image axis reveals that the axial resolution is significantly improved by regularized inversion compared to envelope detection. This however comes at the cost of strong artefacts in vertical direction that appear as white "wings" in image Fig. 2(a).

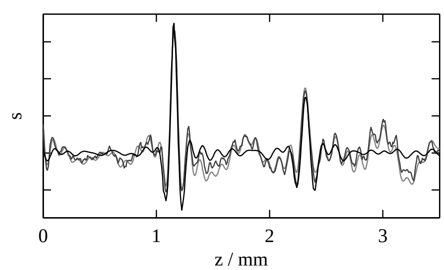
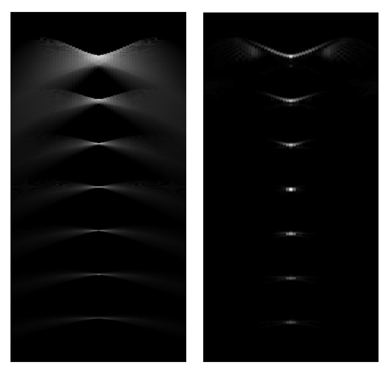


Fig. 1. A-lines estimated from noisy receive signals. Obtained by singular value truncation (light grey), Tikhonov Regularization (dark grey) and by regularized inversion with envelope weighting (black)



(a) (b)

Fig. 2. Simulated ultrasound images of seven point scatterers. Obtained by regularized inversion with envelope weighting (a) and by conventional envelope detection (b)

Table 1. Characteristic values of estimated A-lines

	Resolution [mm]	SNR [dB]	PSR [dB]
Unregularized	0.008	< 0	∞
Truncation	0.054	20.4	22.9
Tikhonov	0.054	21.0	24.8
Derivative	0.054	2.0	17.6
Weighting	0.054	34.4	26.4

4 Discussion

Unregularized inversion and regularization relying on a small first derivative of the solution are not applicable for ultrasound imaging; the first owing to the noise susceptibility of the system matrix's pseudoinverse, the second since the scattering vector consists of sharp peaks, which contradict the specification of a smooth solution. Expectedly singular value truncation and Tikhonov regularization gave similar results due to the close connection between filtering of singular components and solution norm reduction [3, 5]. Both do not provide SNR and PSR sufficient for ultrasound imaging. Regularized inverse filtering with envelope weighting is able to enhance the axial resolution compared to envelope or matched filter detection without degrading SNR and PSR to much and is thus the suitable candidate for an application in medical ultrasound imaging.

The applicability of the envelope weighting algorithm for B-mode imaging is currently limited due to strong artefacts. These artefacts are owed to the fact that the system model is one-dimensional along the individual A-lines, i.e. do not account for diffraction of the wave during its propagation through the tissue. It is expected that inclusion of diffraction into the system model reduces the vertical artefacts significantly.

References

1. Shen J, Ebbini ES. A new coded excitation ultrasound imaging system: Part I: Basic principles. IEEE Trans Ultrason Ferroelectr Freq Control. 1996;43(1):131–40.
2. Hansen PC. Rank-Deficient and Discrete Ill-Posed Problems: Numerical Aspects of Linear Inversion. Philadelphia: SIAM; 1998.
3. Vogel CR. Computational Methods for Inverse Problems. Philadelphia: SIAM; 2002.
4. Hansen PC. The truncated SVD as a method of regularization. BIT Numeric Math. 1987;27(4):534–53.
5. Jensen JA. Field: A program for simulating ultrasound systems. Proc 10th Nordic-Baltic Conf Biomed Imaging. 1996; p. 351–3.