Deciding Fluted Logic with Resolution

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Fluted logic is a solvable fragment of first-order logic which arose as a by-product of Quine's predicate functor logic [13, 14]. It is defined as follows. Let $X_i = \{x_1, \ldots, x_i\}$ denote an ordered set of variables. By definition, an *atomic fluted formula* over X_i is an n-ary atom $P(x_l, \ldots, x_i)$, with $l = i - n + 1$ and $n \leq i$. Now, define *fluted formulae* by: (i) any atomic fluted formula over X_i is a fluted formula over X_i, (ii) if φ is a fluted formula over X_{i+1}, then $\exists x_{i+1}\varphi$ and $\forall x_{i+1}\varphi$ are fluted formulae over X_i, and (iii) any Boolean combination of fluted formulae over X_i is a fluted formula over X_i.

The way decidability is obtained in fluted logic is an interesting contrast to other solvable first-order fragments which are more well-known. Fragments considered until the sixties usually involve some form of restriction on quantification. In prefix classes such as the Bernays-Schönfinkel class, and the initially extended Ackermann class, the initially extended Gödel class the quantifier prefixes are restricted, to $\exists^*\forall^*$, $\exists^*\forall\exists^*$ and $\exists^*\forall\forall\exists^*$. In Maslov's class K (more precisely, in the dual class $\overline{K}$) there is a restriction on universal quantification. In the guarded and loosely guarded fragments, which were introduced more recently, quantifiers are restricted to conditional quantifiers of the form $\exists\overline{y}G(\overline{x},\overline{y}) \wedge \varphi$ or $\forall\overline{y}G(\overline{x},\overline{y}) \rightarrow \varphi$, where $G(\overline{x},\overline{y})$ is a guard formula satisfying certain restrictions ($G(\overline{x},\overline{y})$ is an atom in the case of the guarded fragment). Other decidable classes such as the monadic class and FO^2 are defined over predicate symbols with bounded arity. By contrast, in the case of fluted logic decidability is obtained by imposing an ordering on variables and arguments. To our knowledge, of the mentioned logics, fluted logic has thus far not been studied in the context of resolution. For all other logics mentioned above resolution-based decision procedures have been proposed, in some cases even several different refinements exist [1, 2, 5, 9].

Fluted logic is also of interest for its relationship to non-classical logics. Fluted logic may be viewed as a generalisation of propositional modal logic, just as the guarded fragments can. The properties fluted logic is known to share with modal logics are decidability and the finite model property [10–12]. From a modal perspective an advantage of fluted logic over the guarded fragment is that relational atoms may be negated. This means that extended modal logics such as Boolean modal logic [3] and other enriched modal logics, as well as expressive description logics like $\mathcal{ALB}$ (without converse) [8], which cannot be embedded in the guarded fragment, can be embedded in fluted logic. Interestingly, translations of propositional modal formulae by both the relational translation and a variation of the functional translation (described and used in [4, 6]) are fluted formulae.

In [15] we characterise fluted logic by a new class of clauses, called the class of *fluted clauses*. We present a decision procedure for this class which is based on an ordering refinement of resolution and an additional *separation rule*. This is a new inference rule which does dynamic renaming. It replaces a clause $C \vee D$ by two

clauses which involve a newly introduced literal. Formally:

$$\frac{N \cup \{C \vee D\}}{N \cup \{\neg q(x_1, \ldots, x_n) \vee C,\ q(x_1, \ldots, x_n) \vee D\}}$$

provided (N denotes a set of clauses) (i) the clause $C \vee D$ is separable into clauses C and D, that is, $\mathrm{var}(C) \not\subseteq \mathrm{var}(D)$ and $\mathrm{var}(D) \not\subseteq \mathrm{var}(C)$, (ii) $\mathrm{var}(C) \cap \mathrm{var}(D) = \{x_1, \ldots, x_n\}$ for $n \geq 0$, and (iii) q does not occur in N, C or D. The rule is sound, in general, and resolution extended by this rule remains complete, if it is not applied infinitely often. Separation is essential for our decision procedure, since it allows us to transform certain problematic fluted clauses into so-called *strongly fluted clauses*. A strongly fluted clause is a fluted clause that contains a literals which includes all the variables of the clause. When inference is restricted to such literals (i) the number of variables in any derivable clause is finitely bounded, in particular, the number of variables does not exceed the number of variables in the original clause set. To show termination, it is usually sufficient [9] to show in addition that (ii) there is a bound on the depth of terms occurring in derived clauses. Because separation introduces new predicate names during the derivation, in our case we also need to show that (iii) there is a bound on the number of applications of the separation rule. Exhibiting (ii) and (iii), along with verifying the deductive closure of the class of (strongly) fluted clauses are the most difficult parts of the termination proof. The difficulty can be attributed to the fact that the depth of terms can grow during the derivation, as is the case for some other solvable clausal classes, for example, those associated with Maslov's dual class $\overline{K}$ [1, 7].

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