Practical Proof Methods for Combined Modal and Temporal Logics

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For a number of years, temporal and modal logics have been applied outside pure logic in areas such as formal methods, theoretical computer science and artificial intelligence. In our research we are particularly interested in the use of modal logics in the characterisation of complex components within software systems as *intelligent* or *rational* agents. This approach allows the system designer to analyse applications at a much higher level of abstraction. In order to reason about such agents, a number of theories of rational agency have been developed, for example the BDI (Rao and Georgeff 1991) and KARO (van Linder, van der Hoek, and Meyer 1996) frameworks. The leading agent theories and formal methods in this area all share similar logical properties, more precisely, they all exhibit (i) an *informational* component, being able to represent an agent's beliefs (by the modal logic $\mathsf{KD45}$) or knowledge (by the modal logic $\mathsf{S5}$), (ii) a *dynamic* component, allowing the representation of dynamic activity (by temporal or dynamic logic), and, (iii) a *motivational* component, often representing the agents desires, intentions or goals (by the modal logic KD).

While many of the basic properties of such combinations of modal and temporal or dynamic logics are well understood (Baader and Ohlbach 1995; Fagin et al. 1996; Gabbay 1996; Wolter 1998), very little work has been carried out on practical proof methods for such logics.

Our aim in recent work has been to develop proof methods that are general enough to capture a wide range of combinations of temporal and modal logics, but still provide viable means for effective theorem proving. Currently, we are investigating an approach with the following properties:

- The approach covers the combination of discrete, linear, temporal logic with extensions of multi-modal K_m by any combination of the axiom schemata $\mathsf{4}$, $\mathsf{5}$, B, D, and T. This extends the results presented in (Dixon, Fisher and Wooldridge 1998; Wooldridge, Dixon, and Fisher 1998).
- Instead of combining two calculi operating according to the same underlying principles, like for example two tableaux-based calculi, we combine two different approaches to theorem-proving in modal and temporal logics, namely the translation approach for modal logics (using first-order resolution) and the SNF approach for temporal logics (using modal resolution).
- The particular translation we use has only recently been proposed by de Nivelle (1999) and can be seen as a special case of the T-encoding introduced by Ohlbach (1998). It allows for conceptually simple decision procedures for

extensions of K4 by ordered resolution without any reliance on loop checking or similar techniques.

In more detail, this approach consists of (i) a normal form transformation of formulae of the combined logics into sets of so-called *SNF_K clauses* (similar to those presented in Dixon et. al. 1998), (ii) a translation of modal subformula in SNF_K clauses into a first-order language, and (iii) a calculus $\mathsf{C}_{\mathsf{MTL}}$ for the combined logic which can be divided into standard resolution inference rules for first-order logic and a modified version of the temporal resolution inference rules of Fisher (1991).

The calculus $\mathsf{C}_{\mathsf{MTL}}$ provides a decision procedure for combinations of the basic multi-modal logic K_m and its extensions by arbitrary combinations of the axiom schemata 4, 5, B, D, and T with linear, temporal logic.

For a more detailed description of our approach see Hustadt, Dixon, Schmidt, and Fisher (2000). Related work on proof methods for the KARO framework of agency can be found in Hustadt, Dixon, Schmidt, Fisher, Meyer, and van der Hoek (2000).

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