Modeling of Didactic Relationships in the OntoMath^{Edu} Educational Mathematical Ontology

Olga A. Nevzorova[a], Marina V. Falileeva[a], Alexander V. Kirillovich[b], Liliana R. Shakirova[a], Evgeny K. Lipachev[a], Anastasiya E. Dyupina[a]

[a]*Kazan Federal University, Kazan, Russia*
[b]*Joint Supercomputer Center of the Russian Academy of Sciences, Kazan, Russia*

Abstract

We discuss the system of didactic relationships of the OntoMath^{Edu} educational mathematical ontology.

OntoMath^{Edu} is a central component of the digital educational platform under development at Kazan Federal University, which is intended for solving such tasks as automatic knowledge tests generation and automatic recommendation of educational materials according to an individual study plan. The ontology is organized into three layers: a foundational ontology layer, a domain ontology layer, and a linguistic layer. The domain ontology layer contains math language-independent concepts from secondary school mathematics curriculums. The concepts are provided with labels in Russian, English, Spanish and Tatar.

The system of didactic relationships reflects how the concepts are studied in the actual educational process according to a corresponding national curriculum. The system defines two basic didactic relations: the prerequisite relation and the relation between a concept and its educational level. An educational level is associated with the stage of student training and its profiling. The prerequisite relation determines the sequence of the studied concepts. Arrangement of concepts by educational levels, in turn, determines educational projections, i.e. projections of the ontology to national education systems. Currently, there are two projections of the ontology: on Russian and the UK education systems respectively. Educational projections can be used for curriculum planning, translation of a curriculum into other languages, and personalization of learning.

The didactic relationships were introduced in the 2nd release of OntoMath^{Edu}, which was presented at WEA 2021. While the WEA 2021 paper describes them from an engineering point of view, this paper does it from a methodological perspective.

Keywords

Prerequisite, educational projection, educational level, ontology, mathematical education, plane geometry, OntoMath^{Edu}

1. Introduction

Digital transformation of education is a multi-stage process that affects all levels and components of the educational system. This transformation relies primarily on promising digital technologies that offer new opportunities for solving educational problems, such as development of digital teaching materials, teaching tools and services, and knowledge assessment. New perspectives for education are associated with the use of artificial intelligence methods and, in particular, with the use of semantic technologies [1–7].

Artificial intelligence methods are quite actively used in creating intelligent learning systems for teaching mathematics. For example, the Active Math system [8] is a web-based learning environment that

DTTL-2021: International Workshop on Digital Technologies for Teaching and Learning, March 22-28, 2021, Kazan, Russia
EMAIL: onevzoro@gmail.com (O.A. Nevzorova); mmwwff@yandex.ru (M.V. Falileeva); alik.kirillovich@gmail.com (A.V. Kirillovich); liliana008@mail.ru (L.R. Shakirova); elipachev@gmail.com (E.K. Lipachev); anastasiya.dupina@yandex.ru (A.E. Dyupina)
ORCID: 0000-0001-8116-9446 (O.A. Nevzorova); 0000-0003-2228-7551 (M.V. Falileeva); 0000-0001-9680-449X (A.V. Kirillovich); 0000-0001-5758-4076 (L.R. Shakirova); 0000-0001-7789-2332 (E.K. Lipachev); 0000-0002-0917-3763 (A.E. Dyupina)

dynamically generates interactive teaching materials in mathematics which would be adapted to the goals, preferences, capabilities, and knowledge of the student. ALEKS [9] is an artificial intelligent learning and assessment system for math, chemistry, statistics, and accounting. The system uses an ontology (directed graph) whose nodes contain a certain set of topics for study.

After successfully studying the topic of the node, the student moves on to the next node in the graph. The learning goal is specified as the final (or intermediate) node. Cognitive Tutor Curriculum Complex [10] from Carnegie Learning Inc. is focused on the use of blended learning. It combines traditional teaching materials and MATHia software. The teaching materials are adapted to the interests of learners, and the proposed tasks have practical orientation. The solutions in Cognitive Tutor are an example of integration of digital technology, quality teaching materials, and achievements of cognitive sciences. AI in this system is used to continuously monitor learners and navigate individualized learning materials.

Currently, we are working on development of such intelligent learning system, an in-house digital educational mathematical platform of Kazan Federal University, based on the OntoMath^{Edu} [11, 12] educational mathematical ontology.

One of the tasks of this platform is to design an individualized student learning plan in studying mathematics and to generate recommendation of educational materials according to this plan.

When developing educational programs, a mandatory step is to determine the dependencies of educational modules and the sequence of their study. Semantic dependencies between educational modules are specified using the prerequisites of the academic discipline under consideration. A prerequisite is a connection between two components of an educational program, which means one component must be studied before the other.

Generating individual educational trajectories in digital educational environments is based on the data of the "digital portrait" of the student or his or her cognitive profile. Based on an individualized assessment, for example, on tests, a list of little or no knowledge in math is identified. After which, a sequence of educational material to be learned is formed based on the connections between concepts in the ontology of the subject area.

In this paper, we discuss the system of didactic relationships of OntoMath^{Edu}. This system is intended for solve the aforementioned tasks, reflecting how the concepts of the ontology are studied in the actual educational process according to a corresponding national curriculum. The system defines two basic didactic relations: the prerequisite relation and the relation between a concept and its educational level. An educational level is associated with the stage of student training and its profiling. The prerequisite relation determines the sequence of the studied concepts according to the teaching standards of the corresponding education system [13–16]. Arrangement of concepts by educational levels, in turn, determines educational projections, i.e. projections of the ontology to national education systems.

The didactic relationships were introduced in the 2nd release of OntoMath^{Edu}, which was presented at WEA 2021 [17]. While [17] describes them from engineering and ontological points of view, this paper does it from methodological and pedagogical perspectives. Additionally, the paper describes our experiments on the automatic construction of prerequisite relationships.

2. The structure of OntoMath^{Edu} educational projections

To use the OntoMath^{Edu} ontology in educational applications, we introduced the concept of "educational projection" (hereinafter – projection). The projection of the OntoMath^{Edu} ontology onto the curricula of school mathematics is defined as a special set of the OntoMath^{Edu} ontology concepts and ontological relations between them. The set includes those concepts of the ontology that are formed from the concepts studied according to the curricula of school mathematics in the national system of mathematical education.

OntoMath^{Edu} is a component of OntoMath digital ecosystem [18], an ecosystem of ontologies, text analytics tools, and applications for mathematical knowledge management, including semantic search for mathematical formulas [19] and a recommender system for mathematical papers [20].

The ontology is organized in three layers: a foundational ontology layer, a domain ontology layer and a linguistic layer.

The domain ontology layer contains language-independent math concepts from the secondary school mathematics curriculum. The concepts are organized in two hierarchies: a hierarchy of objects and a hierarchy of reified relationships. The current version of OntoMath^{Edu} contains approximately 900 concepts from the secondary school Euclidean plane geometry curriculums. These concepts are studied in Russian schools in accordance with the Federal State Standard of Basic General Education, as well as in schools of the UK in accordance with the National Curriculum. The concepts are provided with labels in Russian, English, Spanish and Tatar.

When creating an educational projection of the OntoMath^{Edu} ontology that would reflect the information content of the most popular Russian textbooks in plane geometry for general secondary school of basic and specialized levels of training, we analyzed such textbooks as those written by L.S. Atanasyan, I.F. Sharygina et al. [21], G.K. Gordin [22], A.G. Myakishev [23], and others.

When creating an educational projection of the OntoMathEdu ontology that would reflect the information content of the most popular English textbooks, we analyzed, in particular, the National Curriculum of England including curricula for teaching mathematics, Standards of mathematical practice of the UK and other documents [24–29].

2.1. Educational projections of the OntoMath^{Edu} ontology

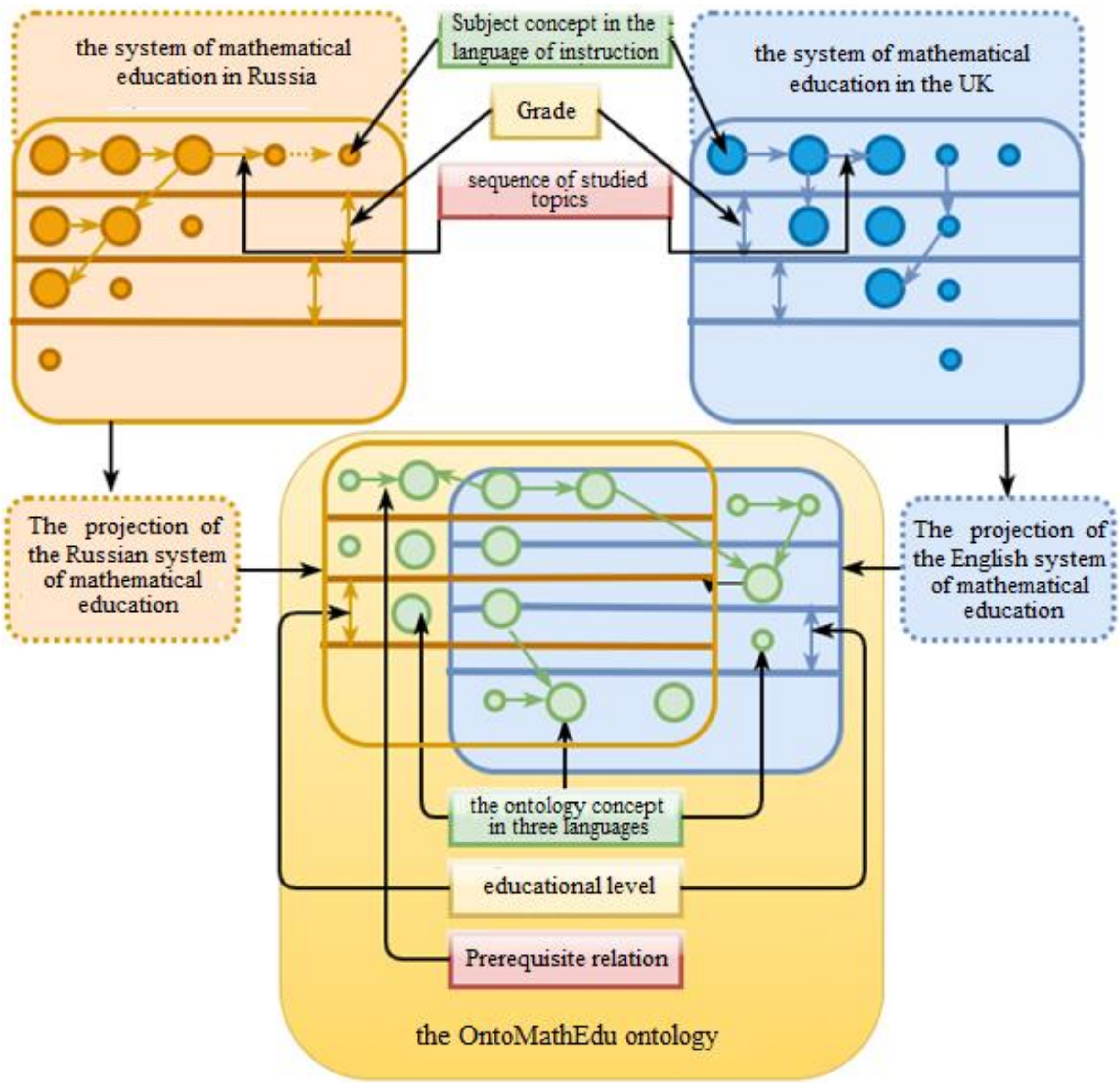

Figure 1: Model of interrelations of projections in various systems of mathematical education in the OntoMath^{Edu} ontology

At the moment, the multilingual mathematical OntoMath^{Edu} ontology has various projections; there are two large projections with math terminology in Russian and English and terminological relationships between two these systems.

Figure 1 shows that these projections do not exist separately from each other, but rather are connected by a system of concepts presented in both systems of teaching mathematics.

Thus, to add a new projection (for example, for curricula of Germany), it is required:

- to translate all the concepts of the OntoMath^{Edu} ontology into a new language,
- to mark up the information content in e-courses and textbooks by the concepts of the OntoMath^{Edu} ontology,
- to find the concepts in textbooks that are absent in the ontology,
- to add the new concepts to the ontology,
- at the final stage, to mark up the concepts by the labels of the new projection.

This approach allows not only for modeling sets of math terms in any education system through representation of projections, but also for replenishing this set by new methodological and didactic connections between the concepts, and therefore for creating new opportunities for learning.

As we noted earlier, our ontology has two projections and supports three term sets in English, Russian, and Tatar.

The projection of the Russian educational system supports two languages (Russian and Tatar) and the projection of the UK educational system supports English. Russia is a multilingual country where teaching is carried out in different languages of the Russian Federation. In Tatarstan (a region of Russia), teaching is carried out in Russian and in Tatar, but the system of teaching mathematics in the Russian Federation is the same for everyone; therefore we have one ontology projection for the Russian educational system. Thus, the structure of the OntoMath^{Edu} ontology in the form of a set of projections can be represented as in Figure 2.

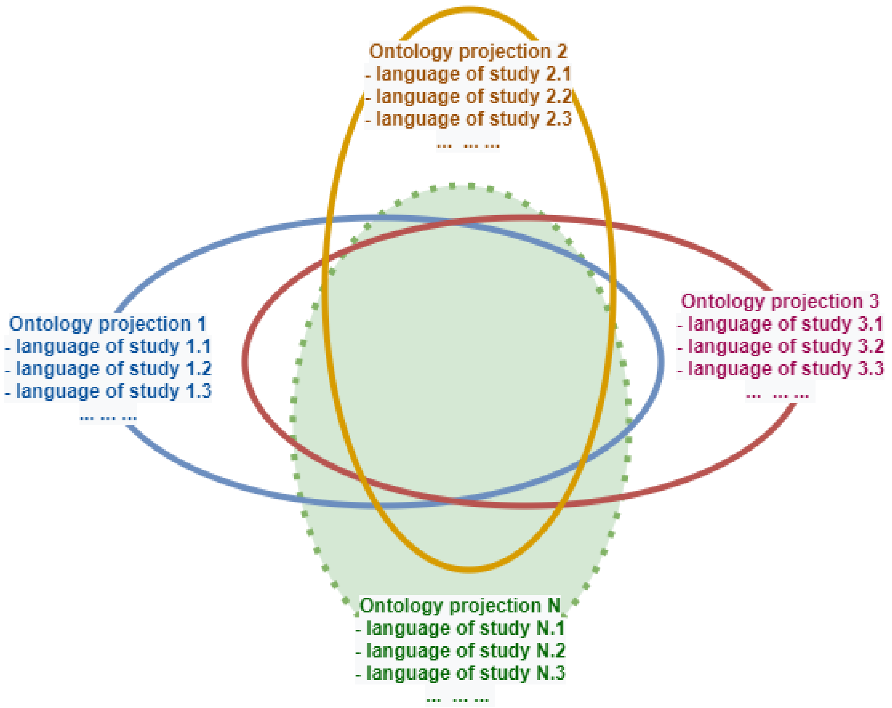

Figure 2: General structure of the multilingual educational mathematical ontology

When constructing a set of concepts from the field of plane geometry for educational purposes, it is necessary to analyze the following:

- whether the concepts being introduced belong to only one educational system;
- to highlight common concepts in the two educational systems;
- to introduce, if required, synonyms for the concept with the names adopted at different educational levels in the education system.

We have carefully thought through all these cases. Figure 3 shows some set of general concepts and sets of concepts belonging to only one of the two educational systems.

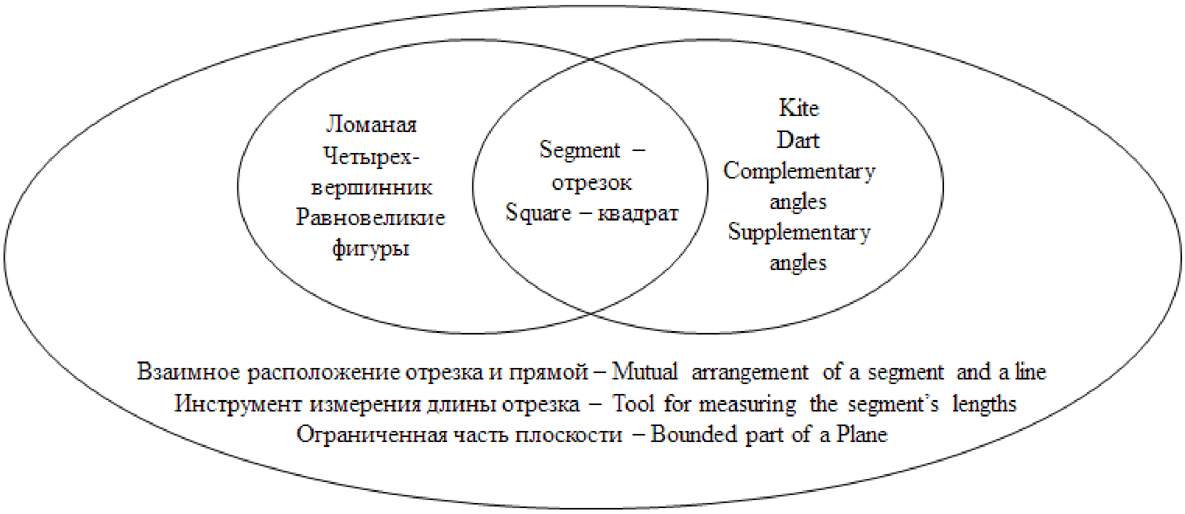

Figure 3: Examples of ontology concepts in the educational systems of Russia and of Great Britain

2.2. Didactic relations of the OntoMath^{Edu} ontology

At this stage, there are two types of educational relations relevant for the project: didactic and methodological.

Didactic principles determine the level of student knowledge and the sequence of topics to be included in the math program.

In the OntoMath^{Edu} ontology, didactic principles are formalized by relationships between educational levels as well as by prerequisite relations [17].

Methodological relationships determine methodical links between the studied concepts. In the OntoMath^{Edu} ontology, methodological relations are formalized by the relations of "ontological dependence", "has argument", "is defined by", "is determined by", "aboutness", etc. With the help of these relations, we determine basic classes and dependent objects which are defined through other objects (classes).

Educational levels reflect the needed volume of student knowledge in the math course in accordance with the educational system. The system of mathematical education in each country includes several educational levels.

As was mentioned earlier, in the current version of the OntoMath^{Edu} ontology, projections of two educational national systems for teaching math have been developed. We will take as an example the experience of courses in plane geometry. In the Russian Federation, the educational levels of teaching math are well represented in accordance with the division into classes (grade levels) from which the systematic teaching of plane geometry begins (see Table 1).

Table 1

The "Educational level" relation in the projection of the educational system of the Russian Federation

Grade levels in Russia	the average age of a student (years)	Educational level in the ontology
grade 7 of basic math course	13–14	7 class
grade 8 of basic math course	14–15	8 class
grade 8 of math majors	14–15	8 class (majors)
grade 9 of basic math course	15–16	9 class
grade 9 of math majors	15–16	9 class (majors)
additional math program	13–18	additional program

Additional program in mathematics includes math Olympiad training, participation in math contests and research activities in mathematics.

In the UK, in accordance with the UK teaching standards, four levels of training in mathematics are defined (see Table 2).

Table 2
The “Educational level” relation in the projection of the educational system of the UK

Grade levels in the UK	the average age of a student (years)	Educational level in the ontology
learning math (1–2 years)	5–7	Key stage 1
learning math (3–6 years)	8–11	Key stage 2
learning math (7–9 years)	12–14	Key stage 3
learning math (10–11 years)	15–16	Key stage 4

Let us consider some principal differences in teaching plane geometry in schools in Russia and the UK. In the UK, plane geometry is part of a united math course in which all math topics are presented in a joint textbook. There are topics using algebraic concepts in plane geometry, and plane geometry is studied in parallel with solid geometry. In Russia, plane geometry is an independent math course from the 7th grade.

Any concept can belong to only one educational level of a particular education system. At the same time, there are concepts that doesn’t belong to any educational level at all, or belong to an educational level of only one education system.

Such concepts appear in the following cases:

1) If it is necessary to formalize the represented domain in more detail (for example, the concept “Bounded part of the plane” is needed only as a union of concepts of different geometric figures that have such characteristic as area. Thus, this concept does not have the “Educational level” property.

2) In situation when we translate a concept from the language of one educational system into the language of the other. However, some translated concepts from the former are not used in the latter. For example, the “Supplementary angles” concept is translated into Russian as “an angle that complements up to an angle of 180 degrees”, but this concept is not independent in the math educational system of Russia. Therefore this concept has the “Educational level” property only in the educational system of the UK with the value equal to “Key Stage 2” (Figure 4).

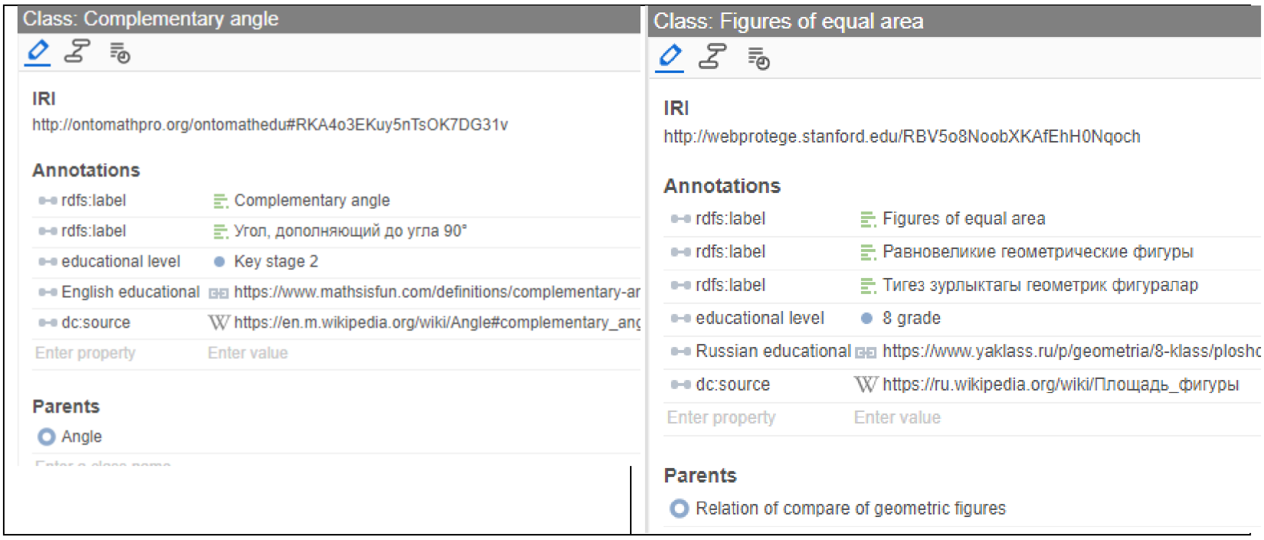

Figure 4: Concepts in the OntoMath^{Edu} ontology that are presented only in one education system

The “Educational level” properties of concepts allow linking the educational levels of training of students in the projections of the ontology.

For example, the union of concepts with the values of “7 grade”, “8 grade” and “8 class (majors)” of the “Educational level” property allows selecting concepts corresponding to the level of training of students who completed their studies in grade 8 of math majors.

Thus, the “Educational level” property allows additional grouping concepts within educational projections.

2.3. Automated construction of prerequisite relationships

In educational ontology, this relation is of particular importance, since the continuity of the studied concepts is the leading principle of the theory of learning.

To construct instances of the “prerequisite” relation in the ontology, we used textbooks recommended for studying plane geometry in Russian schools.

The essence of the proposed method is as follows. It is necessary to highlight the sequence of the studied concepts in the textbook using the OntoMath^{Edu} ontology and then to form pairs of adjacent or closely located concepts in this sequence. Then we checked the constructed pairs and selected those of them that are in the "prerequisite" relation.

Such experiment was carried out with the textbook of plane geometry by Atanasyan et al. [21] using the “OntoIntegrator” ontological-linguistic system [30].

The general scheme of the computational experiment is shown in Figure 5.

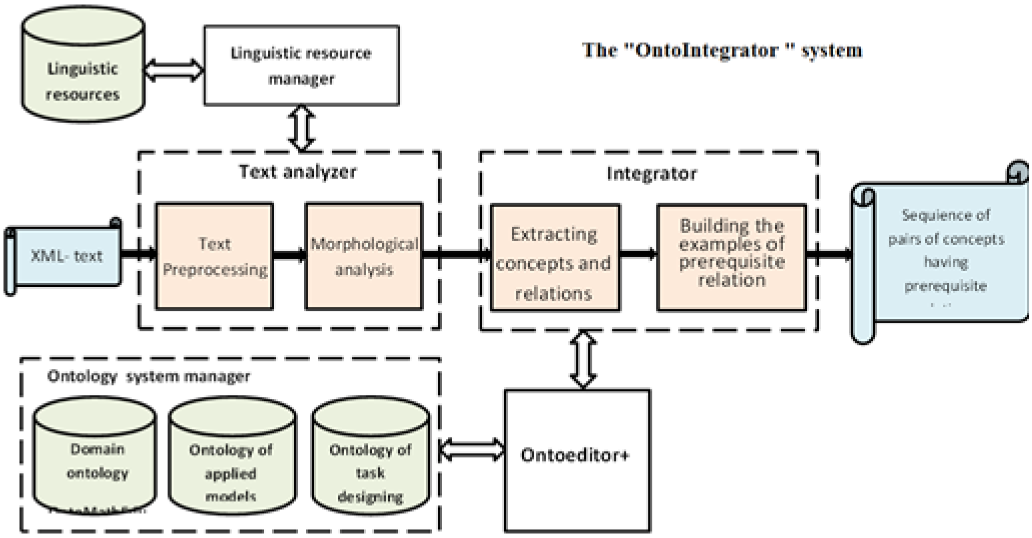

Figure 5: The general scheme of the computational experiment in the “OntoIntegrator” system

The concept sequence was extracted from the textbook using the modules of the “OntoIntegrator” system, such as:

- preprocessing of source data in xml format;
- data cleaning and extracting text content from specified xml-tags;
- morphological analysis of the text content obtained;
- extracting concepts from the text using the OntoMath^{Edu} ontology. Concepts were extracted from significant fragments, such as headings of sections, paragraphs, definitions, formulations of theorems, properties of theorems, etc.;
- sorting the constructed concept sequence and automatically forming concept pairs as instances of prerequisite relations.

In given experiment, the sequence of 125 concepts was obtained.

After an expert assessment of the constructed sequence, 53 pairs of concepts having prerequisite relations were selected. Only 8 pairs of these were previously presented in the ontology. According to the results of the experiment, 45 new prerequisite relations among concepts from the list given in Table 3 were added to the ontology.

Table 3
Prerequisite relations among concepts (in the last column, symbol 0 means absence of prerequisite relationship in the ontology)

Concept 1	Concept 2	Prerequisite relationship (0 – new, 1 – already existing)
line segment	line	1
line segment	point	1
ray (half line)	line	0
angle	ray (half line)	0
straight angle	angle	0
right angle	angle	0
comparison of line segments	line segment	0
congruent figures	geometric figure	0
congruent figures	translation	0
congruent line segment	line segment	0
midpoint of a line segment	congruent line segment	0
congruent angles	angle	0
supplementary angle	angle	0
angle bisector	congruent angles	0
length of a line segment	line segment	0
perpendicular lines	right angle	0
vertical angles	angle	0
perimeter of a triangle	triangle	0
congruent triangles	triangle	0
first criteria for congruence of triangles	congruent triangles	1
theorem	assertion	0
perpendicular	perpendicular lines	0
angle bisector of a triangle	angle bisector	1
angle bisector of a triangle	triangle	1
height of a triangle	perpendicular	1
height of a triangle	triangle	1
median of a triangle	midpoint of a side of a triangle	1
median of a triangle	triangle	0
equal sides of a triangle	a side of a triangle	0
Isosceles triangle	equal sides of a triangle	0
the isosceles triangle property	Isosceles triangle	0

second criteria for congruence of triangles	egual triangles	0
third criteria for congruence of triangles	egual triangles	0
parallel lines	line	0
criteria for parallel lines	parallel lines	0
alternate angles	parallel lines	0
alternate angles	angle	0
corresponding angles	angle	0
corresponding angles	angle	0
axiom	assertion	0
consecutive angles	angle	0
consecutive angles	parallel lines	0
triangle angle sum theorem	an angle of a triangle	0
exterior angle of a triangle	adjacent angle	0
exterior angle of a triangle	an angle of a triangle	0
acute triangle	triangle	0
acute triangle	acute angle	0
obtuse triangle	triangle	0
obtuse triangle	obtuse angle	0
right triangle	triangle	0
right triangle	right angle	0
triangle inequality	a side of a triangle	0

3. Conclusion

The approach to designing educational ontologies proposed in this paper is a new step in the development of models of individual digital space in the system of school math education. The development of these models makes it possible to put into practice intelligent recommender systems for teaching.

The article describes new solutions obtained in the development of the OntoMath^{Edu} educational ontology. In current version, the ontology contains structured knowledge from plane geometry course.

To represent national education systems and educational levels, a model of ontology projections was established and prerequisite relations for two projections concerned with the systems of math education in Russia and in the UK were introduced.

Further development of this project will be aimed at replenishing the ontology with new concepts and relations and also at developing new educational applications, including systems for multilingual education, as well as full-featured systems for teaching school math courses.

4. Acknowledgements

The main contribution of this paper is a methodological interpretation of the OntoMath^{Edu} didactic relationships as well as experiments on the automatic construction of prerequisites. This work was carried out by O. Nevzorova, M. Falileeva, L. Shakirova, E. Lipachev and A. Dyupina at KFU and funded by RFBR according to the project No 19-29-14084. The ontological model underlying this work was developed by A. Kirillovich at JSC RAS and funded by the state assignment to the Federal State Institution “Scientific Research Institute for System Analysis of the Russian Academy of Sciences” for scientific research.

5. References

[1] D. Taibi, G. Fulantelli, S. Dietze, and B. Fetahu, Educational Linked Data on the Web – Exploring and Analysing the Scope and Coverage, in: D. Mouromtsev, M.D’Aquin (Eds.) Open Data for Education: Linked, Shared, and Reusable Data for Teaching and Learning, volume 9500 of Lecture Notes in Computer Science. Springer, Cham, 2016, pp. 16–37. URL: https://doi.org/10.1007/978-3-319-30493-9_2.

[2] S. Nahhas, O. Bamasag, M. Khemakhem, and N. Bajnaid, Added Values of Linked Data in Education: A Survey and Roadmap, Computers 7 (2018). https://doi.org/10.3390/computers7030045.

[3] Ch. Lange, Ontologies and languages for representing mathematical knowledge on the Semantic Web, Semantic Web 4 (2013), pp. 119–158. URL: https://doi.org/10.3233/SW-2012-0059.

[4] M. Kohlhase, A Data Model and Encoding for a Semantic, Multilingual Terminology of Mathematics, in: S.M. Watt, et al. (Eds.) Proceedings of the International Conference on Intelligent Computer Mathematics (CICM 2014), volume 8543 of Lecture Notes in Computer Science. Springer, Cham, 2014, pp. 169–183. URL: https://doi.org/10.1007/978-3-319-08434-3_13.

[5] O. Nevzorova, N. Zhiltsov, A. Kirillovich, E. Lipachev, OntoMath^{PRO} Ontology: A Linked Data Hub for Mathematics, in: Klinov P., Mouromstev D. (Eds.) Proceedings of the 5th International Conference on Knowledge Engineering and Semantic Web (KESW 2014), volume 468 of Communications in Computer and Information Science, Springer, Cham, 2014, pp. 105–119. URL: https://doi.org/10.1007/978-3-319-11716-4_9.

[6] D. Tzoumpa, S. Mitropoulos, Semantic Web Technologies for Ontologies Description: Case study in Geometry Education, in: Proceedings of the 5th South-East Europe Design Automation, Computer Engineering, Computer Networks and Social Media Conference (SEEDA-CECNSM 2020), IEEE, 2020, pp.1–5. URL: https://doi.org/10.1109/SEEDA-CECNSM49515.2020.9221781.

[7] J.Q. Anderson, Individualisation of Higher Education: How Technological Evolution can Revolutionise Opportunities for teaching and learning, International social science journal 64 (2013), pp. 305–316. URL: https://doi.org/10.1111/issj.12046.

[8] P. Libbrecht, E. Melis, C. Ullrich, The ActiveMath learning environment: system description, in: S. Linton, R. Sebastiani (Eds.) Proceedings of 9th Symposium on the Integration of Symbolic Computation and Mechanized Reasoning (Calculemus'01), 2001, pp. 173–177.

[9] K.R Cheney; S.D. Craig, C. Anderson, A. Bargagliotti, A.C. Graesser, A. Sterbinsky, T. Okwumabua, X. Hu, Closing the knowledge gap in mathematics among sixth grade students using ALEKS, in M. Koehler and P. Mishra (Eds.) Proceedings of Society for Information Technology & Teacher Education International Conference (SITE 2011), AACE, 2011, pp. 1425–1427. URL: https://www.learntechlib.org/primary/p/36493/.

[10] J.R. Anderson, A.T. Corbett, K.R. Koedinger, R. Pelletier, Cognitive tutor: Lesson learned, The Journal of the Learning Sciences 4 (1995), pp. 167–207. doi:10.1207/s15327809jls0402_2.

[11] A. Kirillovich, O. Nevzorova, M. Falileeva, E. Lipachev, L. Shakirova, OntoMath^{Edu}: Towards an Educational Mathematical Ontology, in: E. Brady, et al. (Eds.) Workshop Papers at 12th Conference on Intelligent Computer Mathematics (CICM-WS 2019), Prague, Czech Republic, 8–12 July 2019, CEUR Workshop Proceedings, volume 2634, CEUR-WS, 2020. URL: http://ceur-ws.org/Vol-2634/WiP1.pdf

[12] A. Kirillovich, O. Nevzorova, M. Falileeva, E. Lipachev, L. Shakirova, OntoMath^{Edu}: A Linguistically Grounded Educational Mathematical Ontology, in: C. Benzmüller, B. Miller (Eds.) Proceedings of the 13th International Conference on Intelligent Computer Mathematics (CICM 2020), Bertinoro, Italy, July 26–31, 2020, volume 12236 of Lecture Notes in Computer Science, Springer, 2020, pp. 157–172. URL: https://doi.org/10.1007/978-3-030-53518-6_10.

[13] Federal`ny`e gosudarstvenny`e obrazovatel`ny`e standarty`. Osnovnoe obshhee obrazovanie (5–9 kl.). URL: https://fgos.ru/#b3ac23ba5e3cfc8ef, https://fgos.ru/LMS/wm/wm_fgos.php?id=osnov.

[14] The national curriculum. URL: https://www.gov.uk/national-curriculum.

[15] National curriculum in England: mathematics programmes of study. URL: https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study/national-curriculum-in-england-mathematics-programmes-of-study.
[16] Standards for Mathematical Practice. URL: http://www.corestandards.org/Math/Practice/.
[17] A. Kirillovich, M. Falileeva, O. Nevzorova, E. Lipachev, A. Dyupina, L. Shakirova, Prerequisite Relationships of the OntoMath^{Edu} Educational Mathematical Ontology, in: J.C. Figueroa-García, et al. (Eds.) Proceedings of the 7th Workshop on Engineering Applications (WEA 2020), Communications in Computer and Information Science, Springer, 2021 (accepted).
[18] A. Elizarov, A. Kirillovich, E. Lipachev, O. Nevzorova, Digital Ecosystem OntoMath: Mathematical Knowledge Analytics and Management, in: L. Kalinichenko, S. Kuznetsov, Y. Manolopoulos (Eds.) Revised Selected Papers of the XVIII International Conference on Data Analytics and Management in Data Intensive Domains (DAMDID/RCDL 2016), volume 706 of Communications in Computer and Information Science, Springer, 2017, pp. 33–46. URL: https://doi.org/10.1007/978-3-319-57135-5_3.
[19] A. Elizarov, A. Kirillovich, E. Lipachev, O. Nevzorova, Semantic Formula Search in Digital Mathematical Libraries, in: Proceedings of the 2nd Russia and Pacific Conference on Computer Technology and Applications (RPC 2017), IEEE, 2017, pp. 39–43. URL: https://doi.org/10.1109/RPC.2017.8168063.
[20] A.M. Elizarov, A.V. Kirillovich, E.K. Lipachev, A.B. Zhizhchenko, N.G. Zhil'tsov, Mathematical Knowledge Ontologies and Recommender Systems for Collections of Documents in Physics and Mathematics, Doklady Mathematics 93 (2016) 1–3. URL: https://doi.org/10.1134/S1064562416020174.
[21] L.S. Atanasyan, V.F. Butuzov, S.B. Kadomcev, Geometriya 7-9 klassy`uchebn. dlya obshheobrazovat. Uchrezhdenij, Prosveshhenie, Moscow, 2010.
[22] E.K. Gordin, Geometriya. Planimetriya. 7–9 klassy`, MCzNMO, Moscow, 2006.
[23] A.G. Myakishev, E`lementy` geometrii treugol`nika, MCzNMO, Moscow, 2002.
[24] J. Cummins, J.A. Carter, G.J. Cuevas, R. Day, C. Malloy, Glencoe Geometry, Virginia Student Edition. McGraw-Hill/Glencoe, 2012.
[25] A.X. Gantert, Amsco's Geometry. AMSCO School Publications, Incorporated, 2008.
[26] R.L. Larson, L. Boswell, L. Stiff, Heath Geometry an Integrated Approach. Teacher’s Edition, 1998.
[27] KS2 Maths SATs Study Book: For the 2021 Tests (Collins KS2 SATs Practice). Collins KS2, 2015.
[28] KS3 Maths Complete Revision & Practice – Higher (with Online Edition) (Paperback). CGP Books, 2014.
[29] R. Joinson, Exercises in KS3 Mathematics Levels 5-6 (Spiral bound), Sumbooks, 2001.
[30] O. Nevzorova, V. Nevzorov, Ontology-Driven Processing of Unstructured Text. In: S. Kuznetsov, A. Panov (Eds.) Proceedings of the 17th Russian Conference on Artificial Intelligence (RCAI 2019), volume 1093 of Communications in Computer and Information Science, Springer, 2019, pp. 129–142. URL: https://doi.org/10.1007/978-3-030-30763-9_11.