Beam Hardening Correction with an Iterative Scheme using an Exact Backward Projector and a Polychromatic Forward Projector

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Abstract. In computed tomography (CT), reconstructions from cone-beam (CB) data acquired with a polychromatic X-ray device show so called beam hardening artifacts. Beam hardening artifacts are highly undesirable for a medical diagnosis, because details in the reconstructed image are severely disturbed or completely lost. In this work, we demonstrate the significant reduction of beam hardening artifacts by using an iterative reconstruction scheme which consists of a backward and a forward projector. For the backward projector, an exact reconstruction approach was used. The forward projector was extended by a polychromatic model to mimic a realistic X-ray device. The presented experiments use simulated CB data to restrict the evaluation to beam hardening artifacts. The discussion is focused on CB data acquired along a helical trajectory.

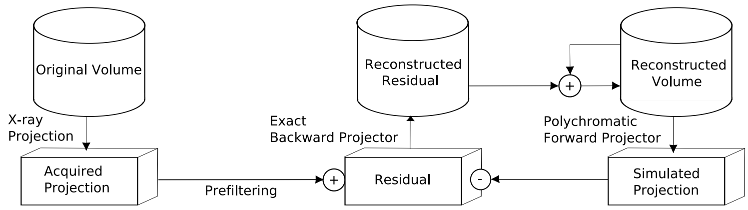
1 Introduction

Real CT systems are equipped with X-ray beams which emit photons of different energy and frequency. However, most reconstruction approaches do not properly consider the non-linear nature of the polychromatic X-rays. This leads to severe beam hardening artifacts with streaks, flares and inhomogenities in the vicinity of high contrast structures which makes a medical diagnosis difficult or even impossible.

In this work, we present an iterative scheme which significantly reduces beam hardening artifacts. The algorithm consists of two components: (i) The backward projector and (ii) the forward projector. For the backward projector, an exact reconstruction method is used. The forward projector is extended by a polychromatic model which mimics the nature of a realistic X-ray device. We restrict the evaluation to beam hardening artifacts by using simulated CB data, acquired along a helical trajectory.

The organisation of the paper is as follows. Section II gives an overview of the state of the art and discusses advances of our approach. In Section III, the iteration process together with the backward and forward projector are explained. Experiments and results are presented in Section IV. Section V gives a final discussion and describes future directions.

Fig. 1. The iterative reconstruction process (see also [7])



2 State of the art and new contribution

Approaches to correct beam hardening artifacts can be divided into three categories: (i) Dual-energy, (ii) preprocessing and (iii) postprocessing methods.

Dual-energy approaches are theoretically elegant, but they require CB data from two different X-ray beam spectra to calculate the complete energy dependency needed for beam hardening correction [1]. Thus, the amount of CB data and consequently the radiation dose is doubled compared to the other approaches. Preprocessing approaches mostly assume that soft tissues have a similar energy dependency as water. This presumption allows the mapping between monochromatic and polychromatic projection values [2]. Postprocessing correction methods are based on the assumption that every material attenuation coefficient of the volume can be described as a linear combination of two known substances like bone and water. They are mostly used in iterative algorithms which estimate the coefficients of the linear combination for each voxel under consideration [3, 4, 5, 6].

The presented approach can be classified into category (iii). The iteration shows similarities with the approach proposed in [7]. However, instead of using an approximate backward projector we combined the iteration process with an exact backward projector [8, 9]. This has the advantage that the so called cone-beam artifacts, arising from an incompletely filled Radon space, are already compensated by the backward projector and therefore do not influence the iteration process. Moreover, the forward projector is extended by a polychromatic model to significantly reduce beam hardening artifacts.

3 Methods

3.1 Iterative reconstruction

The general structure of iterative reconstruction is shown in Fig. 1. It consists of four steps: (i) The acquired X-ray projections are prefiltered. (ii) The residual is computed between the filtered X-ray projections and the simulated X-ray projections. (iii) The residual is backprojected by using the exact backward projector and added to the reconstructed volume of the previous iteration. (iv) The reconstructed volume is forward projected by using the polychromatic forward projector. The resulting simulated projections are used to compute the residual in the next iteration. Before the iteration begins, the reconstructed residual, the reconstructed volume and the simulated X-ray projections are initialized with zeros.

Prefiltering of Projection Data The prefiltering of the acquired X-ray projections is necessary to stabilize the iteration process [10]. The acquired X-ray projections contain the whole range of representable frequencies. However, the backward projector and the forward projector involve interpolation steps which act like a low pass filter to the simulated projection data. Consequently, the residual contains high frequencies which accumulate in the reconstructed volume. To avoid this, the acquired X-ray projections have to be low pass filtered prior to the iteration process.

Exact Backward Projector For the backward projector, an exact reconstruction approach according to [9] was used. Exact approaches are based on a completely filled Radon space and provide excellent image quality without cone-beam artifacts in a monochromatic setup. However, they are not immune to beam hardening artifacts when polychromatic X-ray projections are involved.

Polychromatic Forward Projector In real CT systems, the X-ray beam is polychromatic and consists of photons of different energy. The beam is defined by its bremsstrahlung spectrum $S_{\text{in}}(E)$. The material attenuation $\mu(\boldsymbol{x}, E)$ depends on the location $\boldsymbol{x}$ and on the energy E of the photons. Low energy photons are more attenuated than high energy photons. The polychromatic projection value $p_\lambda(\boldsymbol{\theta})$ of each X-ray with direction $\boldsymbol{\theta}$, emitted from an X-ray source at position $\mathbf{a}(\lambda)$ can be computed with the non-linear attenuation law [11]:

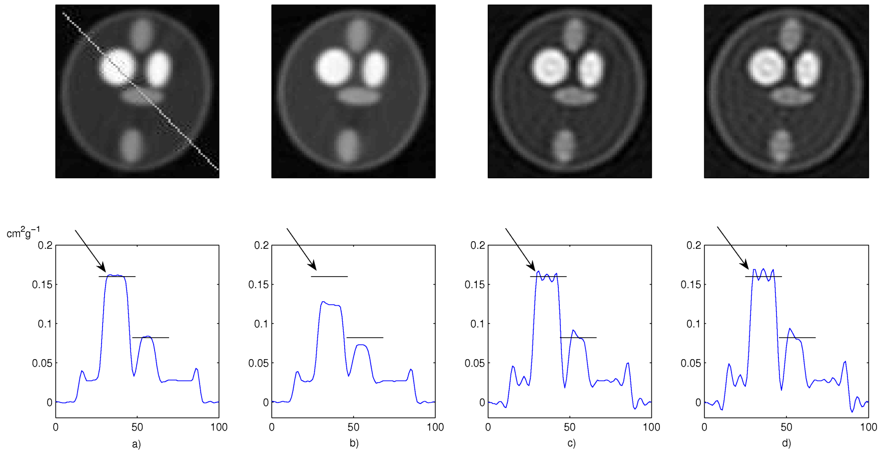
$$p_\lambda(\boldsymbol{\theta}) = -\ln \frac{n_{\text{out}}}{n_{\text{in}}} = -\ln \frac{\int_0^\infty S_{\text{in}}(E) \exp\left(-\int_0^\infty \mu(\boldsymbol{a}(\lambda) + t\boldsymbol{\theta}, E)\, dt\right)\, dE}{\int_0^\infty S_{\text{in}}(E)\, dE} \tag{1}$$

While the bremsstrahlung spectrum of the X-ray beam $S_{\text{in}}(E)$ is usually known, the material attenuation $\mu(\boldsymbol{x}, E)$ of the traversed material is lost after the backward projection because the backward projector interprets the polychromatic projection values in a monochromatic sense without considering the energy dependency. Thus, the reconstructed volume consists of attenuation values which correspond to unknown energies. To compute the unknown material attenuation $\mu(\boldsymbol{x}, E)$, the effective energy E_{eff} can be used. The effective energy describes the monochromatic energy at which a given material produces the same projection value as if a polychromatic X-ray beam was used [11]. We assume, that the effective energy E_{eff} equals the energy of the X-ray beam. The material attenuation $\mu(\boldsymbol{x}, E)$ is then computed as a linear combination of soft tissue and bone, for which the material attenuation spectra are known (see also [4]):

$$\mu(\boldsymbol{x}, E) = v \cdot \mu_{\text{bone}}(E) + (1 - v) \cdot \mu_{\text{soft}}(E) \tag{2}$$

For each voxel $\mathbf{x}$, the linear combination coefficient can be computed by solving Eq. (2) for v while setting $E = E_{\text{eff}}$.

Fig. 2. Iterative reconstruction of a phantom of different materials: (a) monochromatic case after 1 iteration (40.61 keV), (b) - (d) polychromatic case (80 kV) after 1 (b), 4 (c) and 10 iterations (c)



4 Results

The effectiveness of the proposed iterative approach is shown based on a water phantom with inlays of varying shape and density. A helical source trajectory was chosen to provide a complete set of CB data for the exact backward projector. The projections were created with an analytical forward projector (DRASIM, Siemens AG, Medical Solutions, Forchheim, Germany). Two cases are evaluated: (i) In the monochromatic case, the corresponding effective energy $E_{\text{eff}} = 40.61\,\text{keV}$ was estimated by using a water phantom. (ii) In the polychromatic case, the tube acceleration voltage was set to 80 kV. For both experiments, volumes of 128^3 voxels were reconstructed.

Fig. 2 shows the reconstruction results. In the top of the figure, the volume slices are shown while in the bottom, the corresponding profiles are depicted. The profiles are determined along the line which is plotted in the top left volume slice. As a reference, Fig. 2a shows the reconstruction result after the first iteration for the monochromatic case. The first iteration corresponds to an application of the exact backward projector without involving the forward projector. Because the charge of the X-ray quanta is known, the attenuation coefficients can be used to determine the phantom material. The hardest material shows the desired attenuation coefficient $\mu_{\text{silicon}}(E_{\text{eff}}) = 1.59\,\text{cm}^2\text{g}^{-1}$. These attenuation coefficients should also be achieved for the polychromatic case. Fig. 2b shows the result after the first iteration for the polychromatic case. Here, the characteristic beam hardening artifacts like inhomogenities and too low attenuation coefficients in comparison to the monochromatic case can be clearly seen. Fig. 2c and 2d show the result after the 4th and 10th iteration respectively for the polychromatic case. After the 4th iteration, the preferred attenuation coefficients are already reached and the beam hardening artifacts are completely compensated while the iteration converges to the result of Fig. 2a. The overshoots can be blamed on the prefiltering of the acquired X-ray projections and on the Gibbs phenomenon.

5 Discussion

We have shown that beam hardening artifacts are significantly reduced by our approach. The exact backward projector and a polychromatic forward projector nicely complement each other. While the backward projector is free of cone-beam artifacts, the forward projector is used to get rid of beam hardening artifacts. We believe that it is possible to extend the forward projector to model other physical effects like scatter and noise and to reduce salting image artifacts in a similar manner as demonstrated here.

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