Limits on Estimating the Width of Thin Vessels in 3D Medical Images

Stefan Wörz and Karl Rohr

University of Heidelberg, IPMB, and DKFZ Heidelberg,
Dept. Bioinformatics and Functional Genomics, Biomedical Computer Vision Group
Im Neuenheimer Feld 580, 69120 Heidelberg, Germany
Email: {s.woerz,k.rohr}@dkfz.de

Abstract. This work studies limits on estimating the width of thin vessels in 3D medical images. Based on nonlinear estimation theory we analyze the minimal stochastic error of the width estimate caused by image noise. Given a 3D analytic model of the image intensities of a vessel, we derive a closed-form expression for the Cramér-Rao bound of the vessel width. We use the derived lower bound as a benchmark and compare it with previously proposed accuracy limits of three different approaches for vessel width estimation. Moreover, by experimental investigations we demonstrate that the derived lower bound can be achieved by fitting a 3D parametric intensity model directly to the image data.

1 Introduction

Heart and vascular diseases are one of the main causes of death for women and men in modern society. An abnormal narrowing of arteries (stenosis) is one of the main reasons of these diseases as the essential blood flow is hindered. In clinical practice, images of the human vascular system are acquired using different imaging modalities, for example, 3D Magnetic Resonance Angiography (MRA) or 3D Computed Tomography Angiography (CTA). Segmentation and quantification of vessels from 3D medical images is crucial for diagnosis, treatment, and surgical planning. An essential task is the accurate estimation of the width (diameter) of vessels, for example, to identify and quantify stenoses, in particular for thin vessels such as coronary arteries.

Concerning *thin* vessels, limits on the accuracy of estimating the vessel width have been addressed by different groups using different approaches (e.g., [1, 2, 3], see below for details). However, the results of these approaches have not yet been compared with each other. For a quantitative comparison of the proposed limits it would be preferable to have a benchmark. Analytic benchmarks for performance evaluation have been introduced for a different task in medical image analysis, namely, the localization of 3D landmarks [4]. However, an analytic benchmark for vessel width estimation has not yet been derived.

Based on nonlinear estimation theory, we have analyzed the minimal stochastic error of the width estimate caused by image noise. Given a 3D analytic model of the image intensities of a vessel, we have derived a closed-form expression for the Cramér-Rao bound (CRB) of the vessel width, which defines the minimal uncertainty. Note that the derivation of the CRB of the vessel width significantly differs from the derivation of the CRBs of the landmark models in [4]. The reason is that the here used cylindrical parametric intensity model and the required first order partial derivative are more complex (e.g., requiring a Bessel function). We employ the derived CRB as a benchmark and compare it with previously proposed accuracy limits of three different approaches for vessel width estimation [1, 2, 3]. Moreover, by experimental investigations we demonstrate that the derived lower bound can be achieved by fitting a 3D parametric intensity model directly to the image data.

2 Cramér-Rao Lower Bound of Cylinder Model

To derive a benchmark for performance evaluation of 3D vessel segmentation approaches, we use a 3D analytic model that represents the image intensities of a vessel. We propose to use a 3D Gaussian smoothed cylinder, which is well suited to model vessels of different widths (e.g., [2, 3]). The cylinder model comprises parameters for the width of the vessel (radius R), the image blur σ, and the image contrast a between the intensity levels of the vessel and the surrounding tissue. A 2D cross-section of this Gaussian smoothed 3D cylinder is defined as

$$g_{Disk}\left(x, y, R, \sigma\right) = \mathrm{Disk}\left(x, y, R\right) \; * \; G_{\sigma}^{2D}(x, y) \tag{1}$$

where $*$ denotes convolution, $\mathrm{Disk}\left(x, y, R\right)$ is a two-valued function with value 1 if $r \leq R$ and 0 otherwise (for $r = \sqrt{x^2 + y^2}$), as well as the 2D Gaussian function $G_{\sigma}^{2D}(x, y) = G_{\sigma}(x)\, G_{\sigma}(y)$, where $G_{\sigma}(x) = \left(\sqrt{2\pi}\sigma\right)^{-1} e^{-\frac{x^2}{2\sigma^2}}$. Extending the 2D disk in z-direction as well as including the image contrast a yields the cylinder model $g_{M,Cylinder}(x, y, z, R, \sigma, a) = a\, g_{Disk}(x, y, R, \sigma)$. Note that here we omit, without loss of generality, the 3D rigid transform used in [3].

To determine a lower bound for estimating the vessel radius, we utilize the Fisher information matrix $\mathbf{F}$. We consider a cylindrical region-of-interest (ROI) of half-width (radius) w within the xy-plane and half-width w_z in z-direction (along the cylinder) around a position on the centerline of the cylinder. A cylindrical ROI as compared to a cubic (or cuboidal) ROI is a more natural choice for tubular structures and also reduces the complexity in the calculation of the involved integrals. Since here we are only interested in estimating the vessel radius R (assuming that the values of the remaining parameters are known), the information matrix $\mathbf{F}$ consists of one element. The Cramér-Rao lower bound (CRB) of the uncertainty is then given by (e.g., [4])

$$\sigma_{\hat{R}}^2 \geq \sigma_{CRB,\hat{R}}^2 = \mathbf{F}^{-1}. \tag{2}$$

The bound determines the minimal possible uncertainty of the estimated parameter R for a given level of image noise. For calculating the CRB in (2), the first order partial derivative of the cylinder model w.r.t. the radius R is required. Fortunately, whereas a closed-form solution of a Gaussian smoothed cylinder is not known, a closed-form solution of the required partial derivative can be derived:

$$\frac{\partial g_{M,Cylinder}(r,z,R,\sigma,a)}{\partial R} = a\,\frac{\sqrt{2\pi}R}{\sigma}\,G_\sigma\left(\sqrt{r^2+R^2}\right)\,I_0\left(\frac{rR}{\sigma^2}\right), \tag{3}$$

with I_0 being the modified Bessel function of the first kind (order 0). Assuming that the half-width w of the ROI (within the xy-plane) is much larger than the radius R and the standard deviation σ, i.e. $w \to \infty$, the closed-form expression of the CRB in (2) using (3) can be stated as

$$\sigma^2_{CRB,\hat{R}} = \frac{\sigma^2\sigma_n^2\,e^{\frac{R^2}{2\sigma^2}}}{2\pi a^2 R^2\,w_z\,I_0\left(\frac{R^2}{2\sigma^2}\right)}\mathrm{vox}^3 \tag{4}$$

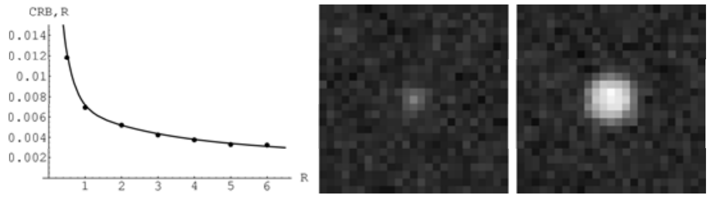
where σ_n denotes the standard deviation of the Gaussian image noise and *vox* denotes the spatial unit in 3D (i.e. one voxel is a cube with a size of one *vox* in each dimension). It can be seen that the precision increases (i.e. the bound decreases) with decreasing image noise σ_n as well as increasing image contrast a and size w_z of the 3D ROI along the cylinder, and depends in a more complicated way on the radius R and the image blur σ (compared to σ_n, a, and w_z). The limits of the CRB for $R \to 0$ and $R \to \infty$ are ∞ and 0, respectively. For example, Fig. 1 (left, black curve) visualizes the CRB given in terms of the standard deviation (square root of the variance) as a function of the radius R. In general, it turns out that the bound is monotonically decreasing as a function of the vessel radius.

To give an impression of the achievable accuracy, we state numerical examples of the CRB for thin vessels. For example, using $a = 100\,\mathrm{gr}$, $\sigma = 1\,\mathrm{vox}$, $\sigma_n = 5\,\mathrm{gr}$, and $w_z = 12\,\mathrm{vox}$, for vessel radii R of 0.5, 1, 2, and 3 vox the minimal uncertainties $\sigma_{CRB,\hat{R}}$ compute to 0.012, 0.007, 0.005, and 0.004 vox, respectively (*gr* denotes the unit of the intensities in grey levels). Thus, for thin vessels the precision is well in the subvoxel range. However, for very thin vessels (e.g., a width of 1 vox) in combination with extremely poor imaging conditions (i.e. a very poor signal-to-noise ratio), the uncertainty of the vessel radius can excess the radius of the vessel itself. For example, for an extremely low image contrast of $a = 5\,\mathrm{gr}$, a vessel radius of $R = 0.5\,\mathrm{vox}$, and a small size of the ROI along the cylinder of $w_z = 5\,\mathrm{vox}$, the CRB computes to 0.76 vox, which is about 50% larger in comparison to the radius. Note that the derived CRB in (4) does not impose a *fixed* limit for a minimal value of the vessel radius. A limit on the minimal radius can be derived based on the desired maximal uncertainty of the vessel radius (e.g., 5% or 0.1 vox). For example, for a maximal uncertainty of 5%, the limit of the minimal radius computes to $R = 0.34\,\mathrm{vox}$ (using the same settings as above), i.e. the minimal width of $2R = 0.68\,\mathrm{vox}$ is below image resolution.

3 Vessel Width Estimation by Model Fitting

We have carried out an experimental investigation to analyze how the theoretical bound relates to practice. To this end we have generated 3D images based on

Fig. 1. Theoretical and experimental precision for estimating the radius R as a function of the radius R (left) as well as 2D axial sections of 3D synthetic cylinders of radii $R = 1$ (middle) and $R = 3$ (right), using $a = 100\,\text{gr}$, $\sigma = 1\,\text{vox}$, $\sigma_n = 5\,\text{gr}$, and $w_z = 12\,\text{vox}$.



the 3D cylinder model with additive Gaussian noise for different radii $R = 0.5, 1, 2, 3, 4, 5, 6\,\text{vox}$ and using the same parameter settings as above (i.e. $a = 100\,\text{gr}$, $\sigma = 1\,\text{vox}$, $\sigma_n = 5\,\text{gr}$). For example, Fig. 1 shows 2D axial sections of 3D synthetic cylinders of radii $R = 1\,\text{vox}$ (middle) and $R = 3\,\text{vox}$ (right). To estimate the radius of the vessel we apply a model fitting approach [3] using a cylindrical ROI with a size of $w = w_z = 12\,\text{vox}$. Since a closed-form solution of the Gaussian smoothed cylinder $g_{M,Cylinder}$ is not known, we here numerically integrate the cylinder model to utilize the same model for the theoretical analysis and the experiments (instead of using an analytic approximation as in [3]). In total, for each value of the radius we carried out 1000 experiments (randomly varying the initial parameters and the added Gaussian noise) and determined the precision $\sigma_{\hat{R}}$ of the radius as the standard deviation of the estimated radii. The results are represented by the dots in Fig. 1 (left). In addition, the black curve indicates the theoretical precision according to the derived CRB $\sigma_{CRB,\hat{R}}$. It turns out that the agreement between the theoretical and the experimental values is very good, in particular, for thin vessels (e.g., $R = 0.5\,\text{vox}$), i.e. the derived lower bound can indeed be achieved experimentally. The agreement is even more remarkable since the analytic derivation does not consider discretization effects due to sampling and quantization, while in the experiments discrete images have been used.

4 Comparison of Different Limits

We have also compared the accuracy limits of three different vessel segmentation approaches with the CRB from above. Hoogeveen *et al.* [1] studied accuracy limits in determining the vessel width from time-of-flight (TOF) and phase-contrast (PC) 3D MRA images. Experiments were based on 3D synthetic TOF and PC MRA images as well as on real images, which were generated by using phantom tubes with known diameters. For measuring the vessel width, the criteria full-width-half-maximum and full-width-at-10% were applied for TOF and PC images, respectively. The authors state that for both TOF and PC MRA images a minimal radius of about 1.5 vox is required for accurate estimation of the vessel width (allowing a maximal error of the estimated vessel width of 5%).

Sato *et al.* [2] developed a differential vessel segmentation approach based on a multi-scale line filter. A Hessian-based line filter is applied to different scales of a 3D image and vessels are extracted based on these filter responses.

To determine the width of a vessel, the filter responses are compared to filter responses of an ideal vessel model. It turns out that a maximum response of the multi-scale filter, which is required to estimate the vessel width, is inherently not obtainable for thin vessels with a radius below 1.39 vox.

In [3], we developed a model fitting approach for vessel segmentation based on an analytic 3D parametric intensity model. We use a 3D Gaussian smoothed cylinder to model the image intensities of a vessel and the surrounding tissue within a ROI. Since a closed-form solution of a Gaussian smoothed cylinder is not known, we have developed an accurate approximation based on the Gaussian function and the Gaussian error function. To segment a vessel we utilize an incremental process based on least-squares model fitting as well as linear Kalman filtering. We have applied the cylinder model to segment vessels in 3D MRA and 3D CTA images of the human. However, we obtain ambiguous estimates of the vessel width for a radius below about 1.72 vox. The reason is that for this value we automatically switch the used approximation in our approach.

Note that, in contrast to Hoogeveen *et al.* [1], in the approach of Sato *et al.* [2] as well as in our model fitting approach [3], an accuracy limit is given by the approach itself (note also that in both approaches the limits are stated assuming a standard deviation of the Gaussian image smoothing of 1 vox). In comparison, as discussed above, the derived CRB in (4) does not impose a fixed limit for a minimal value of the vessel radius such as in [2, 3]. Moreover, since the theoretically derived CRB has been experimentally achieved for thin vessels, in particular, for a radius of 0.5 vox, we conclude that the limit of $R \approx 1.5$ vox proposed by all three approaches [1, 2, 3] is not a fundamental limit.

5 Conclusion

We have analyzed limits for estimating the vessel width of thin vessels. Given a 3D analytic model of the image intensities of a vessel, we have derived a closed-form expression for the Cramér-Rao bound of the vessel radius. In addition, we have compared the derived Cramér-Rao bound with previously proposed limits of three different approaches. Moreover, by experimental investigations, we have demonstrated that the derived lower bound can indeed be achieved by model fitting of a 3D parametric intensity model.

References

1. Hoogeveen RM, Bakker CJG, Viergever MA. Limits to the Accuracy of Vessel Diameter Measurement in MR Angiography. J Magn Reson Imaging 1998;8:1228–1235.
2. Sato Y, Yamamoto S, Tamura S. Accurate Quantification of Small-Diameter Tubular Structures in Isotropic CT Volume Data Based on Multiscale Line Filter Responses. In: Procs MICCAI; 2004. p. 508–515.
3. Wörz S, Rohr K. A New 3D Parametric Intensity Model for Accurate Segmentation and Quantification of Human Vessels. In: Procs MICCAI; 2004. p. 491–499.
4. Rohr K. Fundamental Limits in 3D Landmark Localization. In: Procs IPMI; 2005. p. 286–298.