A Boolean Lattice Based Fuzzy Description Logic in Web Computing

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Abstract. Contrasting to Description Logics (DLs), there are some inherent shortcomings in classical fuzzy Description Logics (FDLs). For example, they no longer satisfy the complementary laws. In this paper, to analyze these shortcomings derived from Zadeh semantics, an improved fuzzy set definition using Boolean lattices is approached, whereas, the analogous definition of Zadeh Fuzzy Set can only use a distributive lattice. Therefore, an improved FDL in Web Computing Environment, using Boolean lattices rather than the interval [0,1] to represent fuzziness, is constructed. Finally, some probable extensions of the chosen lattice are discussed.

Keywords: Fuzzy Description Logic, Zadeh Fuzzy Set, Web Computing.

1 Introduction

Web Computing are the ways we managing, organizing, sharing and exchanging web resources (e.g. data and services) on the Web by means of web technologies, e.g. Semantic Web and Web Services. Among these technologies, Description Logics (DLs) [1] provide a logical reconstruction of object-centric and frame-based knowledge representation languages as essential means to describe web resources and to infer on them; Furthermore, Fuzzy Description Logics (FDLs) [2] extend DLs with capabilities of representing and reasoning on fuzziness of Web resources.

In this paper, rather than going on advocating their advantages, we will talk about some shortcomings of classical FDLs and try to find a solution. We proceed as follows. In the following section, taking f-$\mathcal{ALC}$ [2] as example, we focus on classical FDLs' shortcomings (e.g. unsatisfactory of the complementary laws) derived from their Zadeh semantics. Then, in Section 3, an improved FDL in Web Computing, using a proper Boolean lattice rather than the interval [0,1] to represent fuzziness, is constructed. In Section 4, some probable extensions of the chosen lattice are discussed. At last, Section 5 concludes and presents some topics for further research.

2 Analysis on Shortcomings of Classical FDLs

2.1 The Problems

As mentioned in [2], complementary laws never hold in classical FDLs. To make it clear, let's take f-$\mathcal{ALC}$, a simple FDL whose detail can be found in [2], as an illustration. In f-$\mathcal{ALC}$, a fuzzy interpretation $\mathcal{I}$ satisfies some Zadeh style equations like: for all $d \in \Delta^{\mathcal{I}}$, $(\bot)^{\mathcal{I}}(d) = 0$, $(\top)^{\mathcal{I}}(d) = 1$, $(C \sqcap D)^{\mathcal{I}}(d) = min(C^{\mathcal{I}}(d), D^{\mathcal{I}}(d))$, $(C \sqcup D)^{\mathcal{I}}(d) = max(C^{\mathcal{I}}(d), D^{\mathcal{I}}(d))$, and $(\neg C)^{\mathcal{I}}(d) = 1 - C^{\mathcal{I}}(d)$. Obviously, $(C \sqcap \neg C)^{\mathcal{I}}(d) = 0$ and $(C \sqcup \neg C)^{\mathcal{I}}(d) = 1$ no longer hold. That is, $C \sqcap \neg C \neq \bot$ and $C \sqcup \neg C \neq \top$. What's more, serious to say, this is not the only issue, there are more implicit problems in classical FDLs, as depict in following examples.

Example 1. Consider the fuzzy KB $\mathcal{K}_1 = (\mathcal{T}_1, \mathcal{A}_1)$, where

$$\mathcal{T}_1 = \{Fowl \sqsubseteq Animal, Reptile \sqsubseteq Animal, Fowl \sqcap Reptile \sqsubseteq \bot\} \quad ,$$
$$\mathcal{A}_1 = \{archaeopteryx{:}Fowl = 0.6, archaeopteryx{:}Reptile = 0.3\} \quad .$$

Suppose α_1=$archaeopteryx{:}Fowl \sqcap Reptile$ and α_2=$archaeopteryx{:}Fowl \sqcup Reptile$. Easy to see, the conclusion $\alpha_1 = min\{0.6, 0.3\} = 0.3$ conflicts with the fuzzy axiom $Fowl \sqcap Reptile \sqsubseteq \bot$. The other conclusion $\alpha_2 = max\{0.6, 0.3\} = 0.6$ conflicts with the more reasonable fuzzy value $0.6 + 0.3 = 0.9$. Therefore, conjunction and disjunction of fuzzy concepts have the probability of causing conflicts in fuzzy KBs which are originally consistent.

Example 2. Consider another fuzzy KB $\mathcal{K}_2 = (\mathcal{T}_2, \mathcal{A}_2)$, where $\mathcal{T}_2 = \{C \sqcap D \sqsubseteq \bot\}$ and every model $\mathcal{I}$ of $\mathcal{K}_2$ should satisfy (*i*) for all $d \in \Delta^{\mathcal{I}}$, $C^{\mathcal{I}}(d) \leq D^{\mathcal{I}}(d)$; (*ii*) there always exists a $d \in \Delta^{\mathcal{I}}$ so that $C^{\mathcal{I}}(d) > 0$. We can infer that $C \sqsubseteq D$ and $C \neq \bot$, $D \neq \bot$, which contradict the axiom $C \sqcap D \sqsubseteq \bot$. In conclusion, the inclusion of fuzzy concepts sometime also leads to conflicts in originally consistent fuzzy KBs.

Well then, where do these problems lie in? We can find the answer in the Zadeh semantics of classical FDLs.

2.2 The Reason of Classical FDLs' Shortcomings

Coincidentally, Zadeh fuzzy set, under which the semantics of classical FDLs are defined, encounters same criticisms and should be treated as the root of abovementioned problems. To find the reason, we sketch a more sophisticate fuzzy set using Boolean lattice rather than the interval [0,1] to represent fuzziness.

Definition 1 (Boolean Lattice Based Fuzzy Set). Suppose X is the universe, and suppose $(\mathcal{L}, \preceq)$ is a Boolean lattice with operators as $\otimes$, $\oplus$, $\ominus$, and identity elements as o, $\mathrm{1}$, respectively. Then a *Boolean lattice based fuzzy set* (e.g. C and D) is a mapping from X to $\mathcal{L}$, satisfying: (*i*) for each $x \in X$, $\varnothing(x) = \mathrm{o}$,

$X(x)=1$ and $(\sim C)(x)=\ominus C(x)$, $(C\cap D)(x)=C(x)\otimes D(x)$, $(C\cup D)(x)=C(x)\oplus D(x)$;
(ii) $C\subseteq D$ iff for all $x\in X$, $C(x)\preceq D(x)$.

Here, for a fuzzy set C, $C(x)$ ($x\in X$) is a *Boolean lattice based membership degree* of *x* to C. What's more, we use a function $\sigma:\mathcal{L}\to[0,1]$ mapping Boolean lattice based membership degree (e.g. $C(x)$) to Zadeh membership degree (e.g. $\mu_C(x)=\sigma(C(x))$). Obviously, the Boolean lattice based fuzzy set satisfies all the set rules like a crisp set ever do, and the reason lies in its ability of modeling disjunction and inclusion relationships between fuzzy sets. Analogous to *Definition 1*, we can redefine Zadeh fuzzy set using a certain distributive lattice.

Definition 2 (Zadeh Fuzzy Set). Suppose X is the universe, and suppose $(\mathcal{L}',\preceq)$ is a lattice, all of whose elements are of format $[0,a]$ ($a\in[0,1]$) and satisfies: $[0,a_1]\preceq[0,a_2]$ iff $a_1\le a_2$. Then a *Zadeh fuzzy set* C can be represented as a mapping $C:X\to\mathcal{L}'$, satisfying: for any $x\in X$, $C(x)=[0,\mu_C(x)]$.

Isomorphic to interval [0,1], the lattice $\mathcal{L}'$ is no longer a Boolean lattice, because every element inside doesn't have its inverse element. What's more, since $\mathcal{L}'$ is also a linear order, we cannot model disjunction relationships between fuzzy sets. Therefore, in classical FDLs, Zadeh semantics blurs the overlapping relationships (e.g. disjunction, inclusion) between fuzzy concepts, and thus loses the information of these relationships. Essentially, these are the reason of abovementioned problems in classical FDLs.

To make improvements, some papers introduces other semantics of fuzzy connectives (e.g. Lukasiewicz semantics) [4], others using lattice representing fuzziness [5,6]. According *Definition 1*, to represent fuzziness, it's more applicable to use Boolean lattice than the general ones. In the following section, a proper Boolean lattice in Web Computing environment is founded and is used to build a Boolean lattice based FDL named lf-$\mathcal{ALC}$.

3 lf-$\mathcal{ALC}$: a Boolean Lattice Based FDL in Web Computing

3.1 Choosing of Boolean Lattice

Consider fuzzy KB $\mathcal{K}_1$ in *Example 1*, how do we deem that an archaeopteryx is a fowl in a degree about 60%, and is a reptile in 30%. Commonly, this is done in a voting approach. Suppose we sampled a certain number of (e.g. 100) famous paleontologists. Because of their different backgrounds, different major and different research experiences, even different believes, it is reasonable that their viewpoints on whether archaeopteryx belongs to fowl or reptile will vary each other. And, the last thing to do is to collect and synthesize their different viewpoints into a more reasonable fuzzy result (e.g. the fuzzy assertions in $\mathcal{K}_1$).

Currently, for the sake of the universality of web entities (i.e. almost every people has their agents on the Web delegate their words and actions[1]) and the connectedness among them, such a voting approach can be enforced in Web Computing environment. A decision-making entity (called decision-maker) can put forwards its questionable assertion (e.g. whether archaeopteryx belongs to fowl) to some chosen entities (called observers, accordingly) and receives their responses (i.e. their viewpoint). Suppose $O = \{o_1, o_2, ...o_n\}$ ($n \geq 1$) is the set of chosen observers, then we have a Boolean lattice $(\mathcal{L}_O, \subseteq)$ as O's power set. Moreover, suppose, each element of $\mathcal{L}_O$ is the set of those observers who are for the questionable assertion, then $\mathcal{L}_O$ can be a proper Boolean lattice for fuzziness representation in Web Computing environment.

3.2 Syntax, Semantics and Inference Problems of lf-$\mathcal{ALC}$

Here, by contrast to f-$\mathcal{ALC}$, lf-$\mathcal{ALC}$ is built on Boolean lattice $\mathcal{L}_O$. To make it general, we rename $(\mathcal{L}_O, \subseteq)$ as $(\mathcal{L}, \preceq)$, and rename its set operators as $\otimes$, $\oplus$, $\ominus$, it's identity elements $\varnothing$, O as o, $\mathrm{1}$, respectively. In this way, we can use another better Boolean lattice to substitute $\mathcal{L}_O$ whenever necessary.

Consider three alphabets of symbols, *primitive concepts* (denoted A), *primitive roles* (denoted R), and *individuals* (denoted a and b)[2]. A *concept* (denoted C or D) of lf-$\mathcal{ALC}$ is build out according to the following syntax rules:

$$C, D \rightarrow \bot | \top | A | C \sqcap D | C \sqcup D | \neg C | \forall R.C | \exists R.C$$

A *Boolean Lattice based fuzzy interpretation* of lf-$\mathcal{ALC}$ is now a pair $\mathcal{I} = (\Delta^{\mathcal{I}}, \bullet^{\mathcal{I}})$, where $\Delta^{\mathcal{I}}$ is as for the crisp case, the *domain*, whereas $\bullet^{\mathcal{I}}$ is an *Boolean lattice based interpretation function* mapping (*i*) individuals as for the crisp case, i.e., $a^{\mathcal{I}} \neq b^{\mathcal{I}}$ if $a \neq b$; (*ii*) a fuzzy concept C into a Boolean Lattice based membership function $C^{\mathcal{I}} : \Delta^{\mathcal{I}} \rightarrow \mathcal{L}$; (iii) a fuzzy role R into a Boolean Lattice based membership function $R^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \rightarrow \mathcal{L}$.

For each individual a (resp. each individual pair (a,b)), $C^{\mathcal{I}}(a^{\mathcal{I}})$ (resp. $R^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}})$) represents those observers who is for the crisp assertion $a : C$ (resp. $(a,b) : R$) w.r.t. interpretation $\mathcal{I}$. Furthermore, we use $\mu_C^{\mathcal{I}}(a^{\mathcal{I}})$ as the Zadeh membership degree of a belongs to C w.r.t. $\mathcal{I}$. Reasonably, we have

$$\mu_C^{\mathcal{I}}(a^{\mathcal{I}}) = \frac{|C^{\mathcal{I}}(a^{\mathcal{I}})|}{\log_2 |\mathcal{L}|}\ ^{3} \quad . \qquad \textbf{(1)}$$

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[1] Of cause, we are somewhat overstated, but we trust such a situation is expectable.
[2] Metavariables may have a subscript or superscript.
[3] To a crisp set X , $|X|$ means the cardinality of X .

as the proportion between those observers who are for $a:C$ and the overall observers. Similarly, $\mu_R^{\mathcal{I}}(a^{\mathcal{I}},b^{\mathcal{I}})$, the Zadeh membership degree w.r.t. $\mathcal{I}$, should be

$$\mu_R^{\mathcal{I}}(a^{\mathcal{I}},b^{\mathcal{I}}) = \frac{|R^{\mathcal{I}}(a^{\mathcal{I}},b^{\mathcal{I}})|}{\log_2|\mathcal{L}|} \quad . \qquad \textbf{(2)}$$

Additionally, the Boolean lattice based interpretation function $\bullet^{\mathcal{I}}$ has to satisfies the following equations: for all $d \in \Delta^{\mathcal{I}}$, $(\bot)^{\mathcal{I}}(d) = 0$, $(\top)^{\mathcal{I}}(d) = 1$, and

$$\begin{array}{rcl}
(C \sqcap D)^{\mathcal{I}}(d) & = & C^{\mathcal{I}}(d) \otimes D^{\mathcal{I}}(d) \\
(C \sqcup D)^{\mathcal{I}}(d) & = & C^{\mathcal{I}}(d) \oplus D^{\mathcal{I}}(d) \\
(\neg C)^{\mathcal{I}}(d) & = & \ominus C^{\mathcal{I}}(d) \\
(\forall R.C)^{\mathcal{I}}(d) & = & \underset{d' \in \Delta^{\mathcal{I}}}{\otimes}(\ominus R^{\mathcal{I}}(d,d') \oplus C^{\mathcal{I}}(d')) \\
(\exists R.C)^{\mathcal{I}}(d) & = & \underset{d' \in \Delta^{\mathcal{I}}}{\oplus}(R^{\mathcal{I}}(d,d') \otimes C^{\mathcal{I}}(d'))
\end{array}$$

Note that for an individual a, $(\forall R.C)^{\mathcal{I}}(a^{\mathcal{I}})$ represents those observers who believe that either there's no other individual b satisfies the assertion $(a,b):R$, or every individual b satisfying $(a,b):R$ also satisfies $b:C$. Accordingly, $(\exists R.C)^{\mathcal{I}}(a^{\mathcal{I}})$ represents those observers who believe that there exists an individual b satisfies both $(a,b):R$ and $b:C$. Obviously, lf-$\mathcal{ALC}$ satisfies all rules like the crisp case.

A *terminological axiom* is either a *fuzzy concept specialization* of the form $A \sqsubseteq C$, or a *fuzzy concept definition* of the form $A \equiv C$. An interpretation $\mathcal{I}$ *satisfies* $A \sqsubseteq C$ iff for all $d \in \Delta^{\mathcal{I}}$, $A^{\mathcal{I}}(d) \preceq C^{\mathcal{I}}(d)$; Similarly for $A \equiv C$. A *fuzzy assertion* is an expression of the form $\langle \alpha \succeq c_1 \rangle$ or $\langle \alpha' \preceq c_2 \rangle$, where α is an crisp assertion, c_1 and c_2 are values in $\mathcal{L}$, but α' is an crisp assertion of the form $a:C$ only. An interpretation $\mathcal{I}$ *satisfies* $\langle a:C \succeq c \rangle$ (resp. $\langle (a,b):R \succeq c \rangle$) iff $C^{\mathcal{I}}(a^{\mathcal{I}}) \succeq c$ (resp. $R^{\mathcal{I}}(a^{\mathcal{I}},b^{\mathcal{I}}) \succeq c$). Similarly for $\preceq$.

A *fuzzy Knowledge Base* (KB) is pair $\mathcal{K} = (\mathcal{T},\mathcal{A})$, where $\mathcal{T}$ and $\mathcal{A}$ are finite sets of fuzzy terminological axioms and fuzzy assertions, respectively. An interpretation $\mathcal{I}$ *satisfies* (is a *model* of) $\mathcal{T}$ (resp. $\mathcal{A}$) iff $\mathcal{I}$ satisfies each element in $\mathcal{T}$ (resp. $\mathcal{A}$), while $\mathcal{I}$ *satisfies* (is a *model* of) $\mathcal{K}$ iff $\mathcal{I}$ satisfies both $\mathcal{T}$ and $\mathcal{A}$. A fuzzy KB $\mathcal{K}$ *fuzzy entails* a fuzzy assertion $\langle \alpha \succeq c \rangle$ (denoted $\mathcal{K} \vDash \langle \alpha \succeq c \rangle$) iff every model of $\mathcal{K}$ also satisfies $\langle \alpha \succeq c \rangle$ (similarly for $\preceq$). What's more, to the inference problems, the extended Tableau algorithm with complexity of PSPACE-complete in [5] is also valid for lf-$\mathcal{ALC}$, since lf-$\mathcal{ALC}$ can be treated as a special case of the lattice based FDLs in this paper.

Example 3. Suppose $O = \{o_1, o_2, o_3\}$, consider the fuzzy KB $\mathcal{K}_3 = (\varnothing, \mathcal{A}_3)$, where

$$\mathcal{A}_3 = \{archaeopteryx\text{:}Fowl\text{=}\{o_1,o_2\}, archaeopteryx\text{:}Reptile\text{=}\{o_3\}\} \quad .$$

Then $\mu^{\mathcal{I}}_{Fowl}(archaeopteryx^{\mathcal{I}}) = 2/3 \approx 0.6$, $\mu^{\mathcal{I}}_{Reptile}(archaeopteryx^{\mathcal{I}}) = 1/3 \approx 0.3$. And, suppose $\alpha = archaeopteryx{:}\neg Reptile$, we have $\mathcal{K}_3 \models <\alpha = \{o_1, o_2\}>$.

4 Extensions of the Boolean Lattice in lf-$\mathcal{ALC}$

In Web Computing, to the decision-maker's aspect, it's reasonable to prefer one observer than another. Therefore, we can attach each observer a weight representing the decision-maker's preference. For example, suppose we have n ($n \geq 1$) observers $O = \{o_1, o_2, \ldots o_n\}$ and a weight function $\gamma : O \rightarrow [0,1]$ assigning each observer a proper weight, then, to a crisp assertion $a : C$, the Zadeh membership degree w.r.t. interpretation $\mathcal{I}$ becomes

$$\mu^{\mathcal{I}}_C(a^{\mathcal{I}}) = \frac{\sum_{o \in C^{\mathcal{I}}(a^{\mathcal{I}})} \gamma(o)}{\sum_{o \in O} \gamma(o)} \quad . \qquad (3)$$

In addition, it's hard to mandate observers to only provide unambiguous answers. In most cases, the observers can only give fuzzy viewpoint, e.g. an archaeopteryx is only about 60% possibilities to be a fowl. As a result, it is necessary to transform each observer's fuzzy opinion (e.g. $\langle \alpha \geq 0.7 \rangle$) in to a Boolean lattice format, although the original Zadeh style opinion has already lost the information of set overlapping relationships (as mentioned in Section 2). Here we using a simple four-valued Boolean lattice $(\mathcal{L}_4, \leq_4)$ (with operators as $\otimes_4$, $\oplus_4$, $\ominus_4$), customized from Belnap's four-valued logic [7], to illustrate the transforming process.

Before talking about $\mathcal{L}_4$, we first introduce some components that compose the elements of it. Firstly, the elements of set $\{t, f\}$ represent the attitude of each observer: for or against. Secondly, the elements of set $\{\top, \bot\}$ [4] represent the confidence degree of the observer: $\top$ means that he is sure on his attitude because he has enough information (i.e. proof), and $\bot$ means he is unsure on his attitude because his information (i.e. proof) is relatively insufficient. Thus, we get elements in $\mathcal{L}_4$ as $t_\top$, $t_\bot$, $f_\bot$ and $f_\top$, with different means as surely trust, unsurely trust, unsurely distrust and surely distrust, respectively. What's more, $f_\top \leq_4 t_\bot \leq_4 t_\top$, $f_\top \leq_4 f_\bot \leq_4 t_\top$, $\ominus_4 t_\top =_4 f_\top$ and $\ominus_4 t_\bot =_4 f_\bot$.

Suppose the observers opinions on assertion α are in formats of $\langle \alpha \geq c_1 \rangle$ or $\langle \alpha \leq c_2 \rangle$ where $c_1, c_2 \in [0,1]$. Here, $\langle \alpha \geq c_1 \rangle$ (resp. $\langle \alpha \leq c_2 \rangle$) means that the observer thinks the Zadeh membership degree behind α is c_1 (resp. c_2) and he is for (resp. against) α . Then, the process transforming such opinions to elements in $\mathcal{L}_4$ is as following. Construct a function $\beta : O \rightarrow [0,1]$ representing the dividing point between assurance and diffidence. It is necessary to point out that, for each observer

[4] The symbols here do not represent the top concept and bottom concept in FDLs.

$o \in O$, $\beta(o)$ should be founded by negotiation between decision-maker and o before their interaction and remains unchanged throughout the whole interactive process. Therefore, for each $o \in O$, the mapping from his Zadeh style opinion to elements in $\mathcal{L}_4$ is shown in *Fig. 1*.

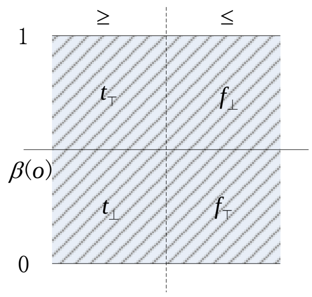

Fig. 1. Mapping from Zadeh style opinion of observer o to elements of $\mathcal{L}_4$

Example 4. Suppose for observer $o \in O$, $\beta(o) = 0.6$ and his opinion on assertions $\alpha_1 = a : C$, $\alpha_2 = a : D$ are $\langle \alpha_1 \geq 0.4 \rangle$, $\langle \alpha_2 \leq 0.8 \rangle$, respectively. According to *Fig. 1*, we have $\langle \alpha_1 =_4 t_\perp \rangle$, $\langle \alpha_2 =_4 f_\perp \rangle$.

To verify the soundness of the mapping in *Fig. 1*, we only need to verify the cases of conjunction, disjunction of $t_\perp$ and $f_\perp$, since other rule are obviously holding. Consider *Example 4* again, the opinions $\langle \alpha_1 \geq 0.4 \rangle$ and $\langle \alpha_2 \leq 0.8 \rangle$ mean that observer o thinks a belongs to C in 40% degree and belongs to D in 80%, but with different attitudes. Thus, the values of $a : C \sqcap D$ and $a : C \sqcup D$ should be $min(0.4, 0.8) = 0.4$ and $max(0.4, 0.8) = 0.8$, respectively. Moreover, it's reasonable that o prefer to distrust the result of $a : C \sqcap D$ and to trust $a : C \sqcup D$'s result. That is to say, $a : C \sqcap D \leq 0.4$ and $a : C \sqcup D \geq 0.8$, which are in agreement with Boolean equalities $t_\perp \otimes_4 f_\perp =_4 f_\top$, $t_\perp \oplus_4 f_\perp =_4 t_\top$.

Thus, Boolean lattice $\mathcal{L}_O$ can be extended to a new one $\mathcal{L}'_O = (\mathcal{L}_4)^n$ ($n =| O |$), whose partial order $\preceq$, operators $\otimes$, $\oplus$, $\ominus$, and identity elements o, $\mathrm{1}$ can be defined as: for any elements $p = (q_1, q_2, .., q_n)$ and $p' = (q'_1, q'_2, .., q'_n)$ in $\mathcal{L}'_O$, (*i*) $p \preceq p'$ iif for each i ($1 \leq i \leq n$), $q_i \leq_4 q'_i$; (*ii*) $\ominus p = (\ominus_4 q_1, \ominus_4 q_2, .., \ominus_4 q_n)$; (*iii*) $p \otimes p' = (q_1 \otimes_4 q'_1, q_2 \otimes_4 q'_2, .., q_n \otimes_4 q'_n)$; (*iv*) $p \oplus p' = (q_1 \oplus_4 q'_1, q_2 \oplus_4 q'_2, .., q_n \oplus_4 q'_n)$; (*v*) $p = \mathrm{o}$ iif for each i ($1 \leq i \leq n$), $q_i =_4 f_\top$; (*vi*) $p = \mathrm{1}$ iff for each i ($1 \leq i \leq n$), $q_i =_4 t_\top$.

If we use $\mathcal{L}'_O$ as the Boolean lattice of lf-$\mathcal{ALC}$, it's obvious that all the things still valid excepting Zadeh membership degree, which needs a new definition. Let's take the crisp assertion $a : C$ as an example. Suppose for $p \in \mathcal{L}'_O$ and $q \in \mathcal{L}_4$, $Ind(p, q)$ represent the indices set of $\mathcal{L}_4$ values in p which are equal to q. Then, the Zadeh membership degree w.r.t. interpretation $\mathcal{I}$ becomes (without weight function)

$$\mu_C^{\mathcal{I}}(a^{\mathcal{I}}) = \frac{\sum_{i \in Ind(C^{\mathcal{I}}(a^{\mathcal{I}}), t_{\top})} 1 + \sum_{i \in Ind(C^{\mathcal{I}}(a^{\mathcal{I}}), t_{\perp}) \cup Ind(C^{\mathcal{I}}(a^{\mathcal{I}}), f_{\perp})} \beta(o_i)}{|O|} . \quad \textbf{(4)}$$

or (with weight function)

$$\mu_C^{\mathcal{I}}(a^{\mathcal{I}}) = \frac{\sum_{i \in Ind(C^{\mathcal{I}}(a^{\mathcal{I}}), t_{\top})} \gamma(o_i) + \sum_{i \in Ind(C^{\mathcal{I}}(a^{\mathcal{I}}), t_{\perp}) \cup Ind(C^{\mathcal{I}}(a^{\mathcal{I}}), f_{\perp})} \beta(o_i)\gamma(o_i)}{\sum_{j=1}^{|O|} \gamma(o_j)} . \quad \textbf{(5)}$$

5 Conclusions

By analysis on the shortcoming of classical FDLs, we find a feasible improving method, i.e. using Boolean lattices rather than [0,1] to represent fuzziness. As a result, a proper Boolean lattice in Web Computing environment is constructed and is utilized to build an improved FDL, named lf-$\mathcal{ALC}$. And finally, some probable extensions of the chosen lattice are discussed. Still, there are many work deserves further research. For example, because of its limited expressiveness, it is necessary to extend both lf-$\mathcal{ALC}$ and its Boolean lattice; even, the applications of lf-$\mathcal{ALC}$ in Web Computing Environment are also necessary.

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