

# Distributed Description Logics Revisited

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**Abstract.** Distributed Description Logics (DDLs) is a KR formalism that enables reasoning with multiple ontologies interconnected by directional semantic mapping (bridge rules). DDLs capture the idea of importing and reusing concepts between ontologies and thus combine well with intuitions behind Semantic Web.

We modify the original semantics of DDLs in order to cope with a modeling discrepancy that has been pointed out in the literature. We do so by introducing a new kind of bridge rules, which we call conjunctive. Using conjunctive bridge-rules instead of the normal ones solves the problem. All the basic properties that have been established for DDLs hold also for the adjusted framework. We also provide a transformational semantics for conjunctive bridge rules, and thus, at least theoretically, a decision procedure for the new semantics.

## 1 Introduction

Distributed description logic (DDL) is a KR formalism introduced by Borgida, Serafini and Taminin in [1,2,3], intended especially to enable reasoning between multiple ontologies connected by directional semantic mapping (bridge rules), built upon the formal, logical and well established framework of Description Logics (DLs). DDLs capture the idea of importing and reusing concepts between several ontologies. This idea combines well with the basic assumption of Semantic Web that no central ontology but rather many ontologies with redundant knowledge will exist [4].

It has been noted in [5] that DDLs and the derived framework of C-OWL [6] suffer from several drawbacks. Among these is the unintuitive behaviour in a modeling scenario outlined therein. We analyse this problem and cope with it by introducing a new kind of bridge rules with modified semantics. We then evaluate the new semantics with respect to the desiderata that have been postulated for DDLs. We also provide a transformational semantics for conjunctive bridge rules, and thus, at least theoretically, a decision procedure for the new semantics.

## 2 Distributed Description Logics

As introduced in [1,2,3], a DDL knowledge base consists of a distributed TBox  $\mathfrak{T}$  – a set of local TBoxes  $\{\mathcal{T}_i\}_{i \in I}$ , and a set of bridge rules  $\mathfrak{B} = \bigcup_{i,j \in I, i \neq j} \mathfrak{B}_{ij}$

between these local TBoxes, for some non-empty index-set  $I$ . Each of the local TBoxes  $\mathcal{T}_i$  is a collection of axioms called general concept inclusions (GCIs) in its own local DL  $\mathcal{L}_i$  of the form:  $i : C \sqsubseteq D$ . It is assumed that each  $\mathcal{L}_i$  is a sub-language of  $\mathcal{SHIQ}$  [7]. Each  $\mathfrak{B}_{ij}$  is a set of directed bridge rules from  $\mathcal{T}_i$  to  $\mathcal{T}_j$ . Intuitively, these are meant to “import” information from  $\mathcal{T}_i$  to  $\mathcal{T}_j$ , and therefore  $\mathfrak{B}_{ij}$  and  $\mathfrak{B}_{ji}$  are possibly, and expectedly, distinct. Bridge rules of  $\mathfrak{B}_{ij}$  are of two forms, *into*-bridge rules and *onto*-bridge rules (in the respective order):

$$i : A \overset{\sqsubseteq}{\equiv} j : G \quad , \quad i : B \overset{\sqsupseteq}{\equiv} j : H \quad .$$

Given a TBox  $\mathcal{T}$ , a hole is an interpretation  $\mathcal{I}^\epsilon = \langle \emptyset, \cdot^\epsilon \rangle$  with empty domain. Holes are used for fighting propagation of inconsistency. We use the most recent definition for holes, introduced in [3]. A distributed interpretation  $\mathfrak{I} = \langle \{\mathcal{I}_i\}_{i \in I}, \{r_{ij}\}_{i \in I, i \neq j} \rangle$  of a distributed TBox  $\mathfrak{T}$  consists of a set of local interpretations  $\{\mathcal{I}_i\}_{i \in I}$  such that for each  $i \in I$  either  $\mathcal{I}_i = (\Delta^{\mathcal{I}_i}, \cdot^{\mathcal{I}_i})$  is an interpretation of local TBox  $\mathcal{T}_i$  or  $\mathcal{I}_i = \mathcal{I}^\epsilon$  is a hole, and a set of domain relations  $r_{ij}$  between these domains – each  $r_{ij}$  is a subset of  $\Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_j}$ . We denote by  $r_{ij}(d)$  the set  $\{d' \mid \langle d, d' \rangle \in r_{ij}\}$  and by  $r_{ij}(D)$  the set  $\bigcup_{d \in D} r_{ij}(d)$ .

**Definition 1.** *For every  $i$  and  $j$ , a distributed interpretation  $\mathfrak{I}$  satisfies the elements of a distributed TBox  $\mathfrak{T}$  (denoted by  $\mathfrak{I} \models_\epsilon \cdot$ ) according to the following clauses:*

1.  $\mathfrak{I} \models_\epsilon i : C \sqsubseteq D$  if  $\mathcal{I}_i \models C \sqsubseteq D$ .
2.  $\mathfrak{I} \models_\epsilon \mathcal{T}_i$  if  $\mathfrak{I} \models_\epsilon i : C \sqsubseteq D$  for each  $C \sqsubseteq D \in \mathcal{T}_i$ .
3.  $\mathfrak{I} \models_\epsilon i : C \overset{\sqsubseteq}{\equiv} j : G$  if  $r_{ij}(C^{\mathcal{I}_i}) \subseteq G^{\mathcal{I}_j}$ .
4.  $\mathfrak{I} \models_\epsilon i : C \overset{\sqsupseteq}{\equiv} j : G$  if  $r_{ij}(C^{\mathcal{I}_i}) \supseteq G^{\mathcal{I}_j}$ .
5.  $\mathfrak{I} \models_\epsilon \mathfrak{B}$  if  $\mathfrak{I}$  satisfies all bridge rules in  $\mathfrak{B}$ .
6.  $\mathfrak{I} \models_\epsilon \mathfrak{T}$  if  $\mathfrak{I} \models_\epsilon \mathfrak{B}$  and  $\mathfrak{I} \models_\epsilon \mathcal{T}_i$  for each  $i$ .

If  $\mathfrak{I} \models_\epsilon \mathfrak{T}$  then we say that  $\mathfrak{I}$  is a (distributed) model of  $\mathfrak{T}$ . Finally, given  $C$  and  $D$  of some local TBox  $\mathcal{T}_i$  of  $\mathfrak{T}$ ,  $C$  is subsumed by  $D$  in  $\mathfrak{T}$  (denoted by  $\mathfrak{T} \models_\epsilon i : C \sqsubseteq D$ ) whenever, for every distributed interpretation  $\mathfrak{I}$ ,  $\mathfrak{I} \models_\epsilon \mathfrak{T}$  implies  $\mathfrak{I} \models_\epsilon i : C \sqsubseteq D$ .

### 3 The Problem

In [5] it is pointed out that certain properties of subsumption relations are not modeled properly by DDL. This problem is demonstrated by the following example that we borrow from [5].

*Example 1 ([5]).* Consider the ontology  $\mathcal{O}$ :

$$\begin{array}{ll} \text{NonFlying} \equiv \neg \text{Flying} \quad , & \text{Penguin} \sqsubseteq \text{Bird} \quad , \\ \text{Bird} \sqsubseteq \text{Flying} \quad , & \text{Penguin} \sqsubseteq \text{NonFlying} \quad . \end{array}$$

And the distributed counterpart of  $\mathcal{O}$ , divided into two ontologies  $\mathcal{O}_1$  (on the left) and  $\mathcal{O}_2$  (on the right):

$$\begin{aligned} \text{NonFlying}_1 &\equiv \neg\text{Flying}_1 , & 1 : \text{Bird}_1 &\stackrel{\sqsupseteq}{\Rightarrow} 2 : \text{Penguin}_2 , \\ \text{Bird}_1 &\sqsubseteq \text{Flying}_1 . & 1 : \text{NonFlying}_1 &\stackrel{\sqsupseteq}{\Rightarrow} 2 : \text{Penguin}_2 . \end{aligned}$$

As it is argued in [5], while the concept **Penguin** of  $\mathcal{O}$  is not satisfiable, the corresponding concept **Penguin**<sub>2</sub> of  $\mathcal{O}_2$  is. The problem is that, in a perfectly sane interpretation, each instance  $x \in \text{Penguin}_2^{\mathcal{I}_2}$ , is assigned to two distinct elements of  $\Delta^{\mathcal{I}_1}$ , say  $y_1$  and  $y_2$ , by  $r$ , one instance of **Bird**<sub>1</sub> and the other one of **NonFlying**<sub>1</sub>. Note that this is possible even if **Bird**<sub>1</sub> <sup>$\mathcal{I}_1$</sup>  and **NonFlying**<sub>1</sub> <sup>$\mathcal{I}_1$</sup>  are disjoint as required by ontology  $\mathcal{O}_1$ . We agree with [5] that it is intuitive to expect that bridge rules retain certain properties that GCIs have. So, we would expect **Penguin**<sub>2</sub> to be unsatisfiable, as we made it a “subconcept of two imported concepts” **Bird**<sub>1</sub> and **NonFlying**<sub>1</sub> which in their original ontology  $\mathcal{O}_1$  are disjoint.

Let us generalize the problem illustrated by Example 1 a bit further. We have two local TBoxes in  $\mathfrak{T}$ , say  $\mathcal{T}_i$  and  $\mathcal{T}_j$ , and we have two onto-bridge rules from  $i$  to  $j$ ,  $i : C \stackrel{\sqsupseteq}{\Rightarrow} j : G \in \mathfrak{B}$  and  $i : D \stackrel{\sqsupseteq}{\Rightarrow} j : H \in \mathfrak{B}$ . The problem is that the inclusion  $(G \sqcap H)^{\mathcal{I}_j} \subseteq r_{ij} \left( (C \sqcap D)^{\mathcal{I}_i} \right)$  does not necessarily hold in every model of  $\mathfrak{T}$ , as we would have expected. The source of our intuition here is indeed the fact that the respective inclusion  $(G \sqcap H)^{\mathcal{I}} \subseteq (C \sqcap D)^{\mathcal{I}}$  holds in every model  $\mathcal{I}$  in the case when  $C$ ,  $D$ ,  $G$  and  $H$  are all local concepts of some  $\mathcal{T}$  and instead of the bridge rules we have two GCIs  $G \sqsubseteq C \in \mathcal{T}$  and  $H \sqsubseteq D \in \mathcal{T}$ . We push our generalization even further and expect the respective to hold in case if  $n > 0$  onto-bridge rules are involved. Please note that this issue does not arise in case of into-bridge rules (see Theorem 3 below).

To justify this generalization, we offer Example 2 in which two distinct pairs of concepts are bridged by two onto-bridge rules.

*Example 2.* Consider two ontologies  $\mathcal{O}_1$  and  $\mathcal{O}_2$  with the following GCIs (and also possibly some other):

$$1 : \text{Tokaji}_1 \sqcap \text{Selection}_1 \sqsubseteq \text{DessertWine}_1 , \quad 2 : \text{SixPuttony}_2 \sqsubseteq \text{Tokaji}_2 \sqcap \text{Selection}_2 .$$

In order to import knowledge from  $\mathcal{O}_1$  to  $\mathcal{O}_2$  we add the following bridge rules:

$$\begin{aligned} 1 : \text{Tokaji}_1 &\stackrel{\sqsupseteq}{\Rightarrow} 2 : \text{Tokaji}_2 , & 1 : \text{DessertWine}_1 &\stackrel{\sqsubseteq}{\Rightarrow} 2 : \text{SweetWine}_2 , \\ 1 : \text{Selection}_1 &\stackrel{\sqsupseteq}{\Rightarrow} 2 : \text{Selection}_2 . \end{aligned}$$

We argue, that intuitively **SixPuttony**  $\sqsubseteq$  **SweetWine** should hold in  $\mathcal{O}_2$ . This is not the case however, since  $(\text{Tokaji}_2 \sqcap \text{Selection}_2)^{\mathcal{I}_2} \subseteq r_{12} \left( (\text{Tokaji}_1 \sqcap \text{Selection}_1)^{\mathcal{I}_1} \right)$  does not necessarily hold in every distributed model, as discussed above.

In the following, we introduce an alternative kind of onto-bridge rules with slightly modified semantics. We then show that this semantics follows the intuitions outlined above (Theorem 2 below).

## 4 Conjunctive Bridge Rules

We address the problem outlined above by introducing new form of onto-bridge rules. We call these new bridge rules *conjunctive* and the original form *normal*. We introduce the following syntax for them:

$$i : D \overset{\exists}{\rightsquigarrow} j : H .$$

In the following, we use  $i : D \overset{\exists}{\rightsquigarrow} j : H$  to denote onto-bridge rules that are possibly of both kinds, either conjunctive or normal.

**Definition 2.** *The semantics of conjunctive onto-bridge rules is established by adding the following clause to Definition 1:*

$$7. \mathfrak{I} \models_{\epsilon} i : C \overset{\exists}{\rightsquigarrow} j : G \text{ if for each } i : D \overset{\exists}{\rightsquigarrow} j : H \in \mathfrak{B}, r_{ij}(C^{\mathcal{I}_i} \cap D^{\mathcal{I}_i}) \supseteq G^{\mathcal{I}_j} \cap H^{\mathcal{I}_j}.$$

Our choice of adding new kind of onto-bridge rules instead of simply replacing the old semantics is to underline the fact that both kinds can co-exist and be used according to the modeling scenario and the intentions of the ontology editor. Also, it is not yet clear, how the usage of conjunctive bridge rules affects the computational complexity of the framework. It surely introduces a significant number of additional conditions to verify. Hence it might be desirable to be allowed to choose the exact form of a bridge rule according to the modeling scenario.

## 5 Properties of Conjunctive Bridge Rules

We first show that conjunctive bridge rules are somewhat strictly stronger, in a sense, than normal bridge rules. That is, all the semantic implications caused by normal bridge rules are also in effect if conjunctive bridge rules are used instead. With conjunctive bridge rules, we have some more implications in addition.

**Theorem 1.** *Given a distributed TBox  $\mathfrak{T}$  with a set of bridge rules  $\mathfrak{B}$  and some local TBoxes  $\mathcal{T}_i$  and  $\mathcal{T}_j$  such that  $i \neq j$  and  $i : C \overset{\exists}{\rightsquigarrow} j : G \in \mathfrak{B}$ , for each distributed interpretation  $\mathfrak{I}$  such that  $\mathfrak{I} \models_{\epsilon} \mathfrak{T}$  it holds that  $r_{ij}(C^{\mathcal{I}_i}) \supseteq G^{\mathcal{I}_j}$ .*

The next theorem provides a characterization of conjunctive bridge rules. It says, that if we bridge between several pairs of concepts with conjunctive onto-bridge rules, say  $i : C_1 \overset{\exists}{\rightsquigarrow} j : G_1, \dots, i : C_n \overset{\exists}{\rightsquigarrow} j : G_n$ , then the implications caused to the pairs of concepts pair-wise, do propagate to intersections  $C_1 \sqcap \dots \sqcap C_n$  and  $G_1 \sqcap \dots \sqcap G_n$  of these concepts. This does not hold for normal bridge rules however, as demonstrated by Examples 1 and 2. It follows that indeed the choice of conjunctive bridge rules does solve the problem outlined by the examples.

**Theorem 2.** *Given a distributed TBox  $\mathfrak{T}$  with a set of bridge rules  $\mathfrak{B}$  and some local TBoxes  $\mathcal{T}_i$  and  $\mathcal{T}_j$  such that  $i \neq j$ , if for some  $n > 0$  the bridge rules  $i : C_1 \overset{\exists}{\rightarrow} j : G_1, \dots, i : C_n \overset{\exists}{\rightarrow} j : G_n$  are all part of  $\mathfrak{B}$  then for every distributed interpretation  $\mathfrak{I}$  such that  $\mathfrak{I} \models_{\epsilon} \mathfrak{T}$  it holds that*

$$r_{ij} \left( (C_1 \sqcap \dots \sqcap C_n)^{\mathcal{I}_i} \right) \supseteq (G_1 \sqcap \dots \sqcap G_n)^{\mathcal{I}_j} .$$

The next theorem shows that the corresponding characterization indeed holds for normal into-bridge rules, hence there is no need to introduce conjunctive into-bridge rules.<sup>1</sup>

**Theorem 3.** *Given a distributed TBox  $\mathfrak{T}$  with a set of bridge rules  $\mathfrak{B}$  and some local TBoxes  $\mathcal{T}_i$  and  $\mathcal{T}_j$  such that  $i \neq j$ , if for some  $n > 0$  the bridge rules  $i : C_1 \overset{\sqsubseteq}{\rightarrow} j : G_1, \dots, i : C_n \overset{\sqsubseteq}{\rightarrow} j : G_n$  are all part of  $\mathfrak{B}$  then for every distributed interpretation  $\mathfrak{I}$  such that  $\mathfrak{I} \models_{\epsilon} \mathfrak{T}$  it holds that*

$$r_{ij} \left( (C_1 \sqcap \dots \sqcap C_n)^{\mathcal{I}_i} \right) \subseteq (G_1 \sqcap \dots \sqcap G_n)^{\mathcal{I}_j} .$$

## 6 Transformational Semantics

It follows that the problem of deciding subsumption with respect to a distributed knowledge base that allows conjunctive bridge rules is reducible to the case with normal bridge rules only (Theorem 4 below). As a tableaux decision procedure is known for the latter case (see [2,3]), this result provides us with reasoning support for DDLs with conjunctive bridge-rules. However, the transformation leads to quadratic blowup in the number of bridge rules in the worst case, and so the computational properties of the overall procedure may not be satisfiable. This suggests further investigation of reasoning in presence of conjunctive bridge rules.

**Theorem 4.** *Given a distributed TBox  $\mathfrak{T}$  with a set of bridge rules  $\mathfrak{B}$  that contains conjunctive bridge rules, let  $\mathfrak{T}'$  and  $\mathfrak{B}'$  be obtained in two steps:*

1. *adding  $i : C \sqcap D \overset{\exists}{\rightarrow} j : G \sqcap H$  to  $\mathfrak{B}$  for each pair of  $i : C \overset{\exists}{\rightarrow} j : G \in \mathfrak{B}$  and  $i : D \overset{\exists}{\rightarrow} j : H \in \mathfrak{B}$ ,*
2. *removing all conjunctive bridge rules from  $\mathfrak{B}$ .*

*Then for every  $i \in I$  and for every two concepts, say  $C$  and  $D$ , of  $\mathcal{T}_i$  it holds that  $\mathfrak{T} \models_{\epsilon} i : C \sqsubseteq D$  if and only if  $\mathfrak{T}' \models_{\epsilon} i : C \sqsubseteq D$ .*

Given the reduction, it is now clear that the expressive power of the framework is not enhanced by addition of conjunctive bridge rules. We argue, however, that conjunctive bridge rules still are an interesting update since using them instead of normal onto-bridge rules guarantees intuitive behaviour of the semantics, as demonstrated in Examples 1 and 2 and formally established by Theorem 2.

<sup>1</sup> We are indebted to one of the anonymous referees for pointing this out.

## 7 Evaluation and Comparison

In [1,2,3] various intuitions on how the semantics of a distributed DL environment, such as DDL, should behave are presented. The original semantics of DDLs has been evaluated with respect to these desiderata throughout [1,2,3]. We proceed with evaluating the new framework with respect to these desiderata.

First of all, monotonicity is a desired property, that is, the requirement that bridge rules do not delete local subsumptions as postulated in [1,2].

**Theorem 5 (Monotonicity).** *In every distributed TBox  $\mathfrak{T}$  that also allows conjunctive bridge-rules it holds that  $\mathcal{T}_i \models A \sqsubseteq B \implies \mathfrak{T} \models_\epsilon i : A \sqsubseteq B$ .*

Another desired property is that there is no backflow of information against the direction of bridge rules. This property (we use the version of [3]) also holds in the presence of conjunctive bridge-rules.

**Theorem 6 (Directionality).** *Given a distributed TBox  $\mathfrak{T}$  that allows conjunctive bridge rules in its set of bridge rules  $\mathfrak{B}$ , if there is no directed path of bridge rules from  $\mathcal{T}_i$  to  $\mathcal{T}_j$  in  $\mathfrak{T}$ , then  $\mathfrak{T} \models_\epsilon j : C \sqsubseteq D$  if and only if  $\mathfrak{T}' \models_\epsilon j : C \sqsubseteq D$ , where  $\mathfrak{T}'$  is obtained by removing  $\mathcal{T}_i$  from  $\mathfrak{T}$  as well as removing all bridge-rules involving  $\mathcal{T}_i$  from  $\mathfrak{B}$ .*

Another interesting desideratum that has been postulated in [2] is that one should be only able to add new knowledge by combination of into- and onto-bridge rules.

**Desideratum 1 (Strong directionality)** *If either for all  $k \neq i$ ,  $\mathfrak{B}_{ki}$  contains no into-bridge rules or for all  $k \neq i$ ,  $\mathfrak{B}_{ki}$  contains no onto-bridge rules, then  $\mathfrak{T} \models_\epsilon i : A \sqsubseteq B$  implies  $\mathcal{T}_i \models A \sqsubseteq B$ .*

Unfortunately, this does not hold for DDLs, with or without conjunctive bridge rules. As a counterexample consider the distributed TBox of Example 1 and replace all bridge-rules therein by conjunctive ones. This setting counters the desideratum. Using the reduction of Theorem 4 one obtains an equivalent knowledge base with no conjunctive bridge rules that still counters the desideratum.

Yet another interesting desideratum for DDLs is that local inconsistency that occurs in some of the local TBoxes does not spread and pollute the whole system. In [3] a precise characterization of how inconsistent local TBoxes affect a DDL knowledge base is given. We confirm this property also in presence of conjunctive bridge rules.

**Theorem 7 (Local inconsistency).** *Given a distributed TBox  $\mathfrak{T}$  that also allows conjunctive bridge rules,  $\mathfrak{T} \models_\epsilon i : C \sqsubseteq D$  if and only if for any  $J \subseteq I$  not containing  $i$ ,  $\mathfrak{T}(\epsilon_J) \models_d i : C \sqsubseteq D$ , where  $\models_d$  is a kind of entailment that does not allow holes, and  $\mathfrak{T}(\epsilon_J)$  is obtained from  $\mathfrak{T}$  by removing each  $\mathcal{T}_j$ ,  $j \in J$ , and adding  $\{D \sqsubseteq \perp \mid j : C \overset{\exists}{\rightsquigarrow} i : D \in \mathfrak{B} \wedge j \in J\}$  to each  $\mathcal{T}_i$ ,  $i \in I \setminus J$ .*

Two desiderata of [1,2,3] show how subsumption is propagated along bridge rules. Since only one onto-bridge rule is involved here, it follows immediately that these desiderata also hold when a conjunctive onto-bridge rule is used.

**Theorem 8 (Simple subsumption propagation).** *If  $i : C \overset{\sqsupseteq}{\rightsquigarrow} j : G \in \mathfrak{B}$  and  $i : D \overset{\sqsupseteq}{\rightsquigarrow} j : H \in \mathfrak{B}$  then  $\mathfrak{T} \models_{\epsilon} i : C \sqsubseteq D \implies \mathfrak{T} \models_{\epsilon} j : G \sqsubseteq H$ .*

**Theorem 9 (Generalized subsumption propagation).** *If  $i : C \overset{\sqsupseteq}{\rightsquigarrow} j : G \in \mathfrak{B}$  and  $i : D_k \overset{\sqsupseteq}{\rightsquigarrow} j : H_k \in \mathfrak{B}$ , for  $1 \leq k \leq n$  then  $\mathfrak{T} \models_{\epsilon} i : C \sqsubseteq \bigsqcup_{k=1}^n D_k$  implies  $\mathfrak{T} \models_{\epsilon} j : G \sqsubseteq \bigsqcup_{k=1}^n H_k$ .*

So far we have evaluated the adjusted DDLs framework, with respect to the desiderata postulated for DDLs in [1,2,3]. We have showed that all the desiderata that are satisfied for the original framework also hold when conjunctive bridge rules are present. Moreover, we introduce a variant of the Generalized subsumption propagation desideratum, in which concept intersection is involved instead of concept union. We consider this a desired property and are pleased to report that it also holds for DDLs (with or without conjunctive bridge rules allowed).

**Theorem 10 (Subsumption propagation over concept intersection).** *If  $i : C \overset{\sqsupseteq}{\rightsquigarrow} j : G \in \mathfrak{B}$  and  $i : D_k \overset{\sqsupseteq}{\rightsquigarrow} j : H_k \in \mathfrak{B}$ , for  $1 \leq k \leq n$  then  $\mathfrak{T} \models_{\epsilon} i : C \sqsubseteq \prod_{k=1}^n D_k$  implies  $\mathfrak{T} \models_{\epsilon} j : G \sqsubseteq \prod_{k=1}^n H_k$ .*

## 8 Related Work

Besides of DDLs of Borgida, Serafini and Taminin [1,2,3], another major contribution to distributed and modular ontologies is the approach of Cuenca Grau et al. [8,5] where a combination of several ontologies using  $\mathcal{E}$ -connections [9] is proposed. In this framework, link relations – inter-ontology roles between local ontologies – are favored instead of bridge rules. While both are related [9,10], each maintains its own primary intuitions – in DDLs inter-ontology subsumption is modeled directly with bridge rules, while the preference of links in the latter framework has lead to such results as automated ontology decomposition [11].

An extension of DDLs called C-OWL has been introduced by Bouquet et al. in [6]. Several improvements were suggested, including a richer family of bridge rules, allowing bridging between roles, etc. Also, Ghidini and Serafini in [12,13] enrich DDLs with heterogenous mappings, that is mappings between concepts and roles.

## 9 Conclusion and Future Work

We have proposed an adjustment/extension of DDLs of [1,2,3] in order to address an issue noted in [5]. We have introduced so called conjunctive onto-bridge rules with modified semantics; there is no need for conjunctive into-bridge rules. Even

if the expressive power of the framework does not grow when conjunctive onto-bridge rules are added, using them instead of normal onto-bridge rules guarantees that the unintuitive behaviour of the semantics does not occur any more. All desired properties that hold for DDLs, as established in [1,2,3], also hold when conjunctive bridge rules are added. We have postulated one additional property which holds with and without conjunctive bridge rules as well. We have also provided a transformational semantics for conjunctive bridge rules and so, at least theoretically, a decision procedure, given the known results for DDLs [2,3].

Other interesting issues regarding distributed ontologies that we would like to address include evaluation of the adjusted DDL framework; and further investigation of reasoning algorithms and computational properties.

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