

$\mathcal{DL}clog$: A Hybrid System Integrating Rules and Description Logics with Circumscription ^{*}

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Abstract. In this paper, we propose $\mathcal{DL}clog$, a new hybrid formalism combining Description Logics and Logic Programming for Semantic Web serving as an extension to $\mathcal{DL} + log$ [19]. Negative dl-atoms are allowed to occur in the bodies of the rules, and we extend NM-Semantics of $\mathcal{DL} + log$ to evaluate dl-atoms with *circumscriptive models* of DL ontology in the sense of parallel circumscription rather than classical models. In this way, negative dl-atoms are treated in nonmonotonic way under Extended Closed World Assumption, and the formalism still remains faithful to NM-Semantics, DL and LP. Finally, we present decidability and complexity result for a restricted form of $\mathcal{DL}clog$.

1 Introduction

The problem of adding rules to Description Logics is currently a hot research topic, due to the Semantic Web applications of integrating rule-based systems with ontologies. Practically, most research work[3, 16–19] in this area focuses on the integration of Description Logic with datalog rules or its non-monotonic extensions, and $\mathcal{DL} + log$ [19] is a powerful result of a series of hybrid approaches: DL-log[16], r -hybrid KB[18], r^+ -hybrid KB[17], which define integrated models on the basis of single models of classical theory.

However, there is a severe limitation in $\mathcal{DL} + log$ that DL predicates cannot occur behind "not" in rules. This syntactical restriction makes it impossible to use rules to draw conclusions by the results currently underivable from DL ontology, which cannot satisfies practical needs such as closed world reasoning and modeling exceptions and defaults for DL predicates[12]. To overcome this limitation requires to introduce negative dl-atoms in the body of the rules and interpret them with a nonclassical semantics, which is a nontrivial generalization of NM-Semantics of $\mathcal{DL} + log$. As DL adopts Open World Assumption(OWA), we must introduce a kind of closed world reasoning to DL ontology to interpret the unknown results as negation. Such closed world reasoning must be transparently integrated into the framework of NM-Semantics in order that the generalized semantics remains faithful to NM-Semantics, DL and LP. Finally, the definition of the semantics should be concise, model-theoretical and if possible, decidable.

^{*} This work is supported by NSFC (60275024) and 973 Programme (2003CB317000) of People's Republic of China.

In this paper, a new hybrid formalism $\mathcal{DL}log$ is presented to achieve this goal. We allow DL predicates occurring behind "not" in rules, and extend NM-Semantics of $\mathcal{DL} + log$ to be Nonmonotonic Circumscriptive Semantics(NMC-Semantics). In NMC-Semantics, dl-atoms in rules are evaluated under the *circumscriptive models* of the DL ontology in the sense of McCarthy's parallel circumscription[10] rather than classical models as in NM-Semantics. Parallel circumscription is a general form of circumscription formalizing Closed World Assumption(CWA)[15], and is equivalent to Extended CWA[6] which avoids several anomalies in CWA. By parallel circumscription, "not" in negative dl-atoms are interpreted in a nonmonotonic manner closely similar to the treatment of "not" in Logic Programming(LP). As circumscriptive models serve as an intermediate models only used to evaluate dl-atoms, NMC-semantics remains faithful to DL, LP and NM-Semantics of $\mathcal{DL} + log$, in that users can switch $\mathcal{DL}log$ KB to any of these formalisms by syntactical restriction. When $\mathcal{DL}log$ is restricted to the form that ontologies are written in $\mathcal{ALCI}O$ or $\mathcal{ALC}QO$ and roles are not allowed to occur under "not" in rules, NMC-Satisfiability is $NEXP^{NP}$ -complete.

The rest of the paper is organized as follows. Section 2 presents the motivation of our formalism. Section 3 presents the syntax and semantics of $\mathcal{DL}log$. Section 4 presents the decidability and complexity results. Related work is presented in Section 5 and Section 6 ends with conclusion and future work. We assume that the readers be familiar with McCarthy's parallel circumscription[10, 14].

2 Motivation

In this section, we focus on clarifying the motivation of our solution. We analyze the semantic characterizations of "not" in LP, show how negative dl-atoms can be interpreted in similar way, and finally, present the solution to capture this semantics. Note that we adopt Standard Names Assumption[18], so we can use the same symbol to denote a constant and its interpretation.

Simply applying NM-Semantics of $\mathcal{DL} + log$ to evaluate negative dl-atoms by classical models is implausible, as in following example. We use " \models_{NM} " to denote the satisfiability of NM-Semantics of $\mathcal{DL} + log$.

Example 1. For a KB $\mathcal{K} = (\mathcal{O}, \mathcal{P})$

\mathcal{O}	\mathcal{P}
$\neg seaside \equiv notSC$ $portCity(Barcelona)$	$seasideCity(x) \leftarrow portCity(x), O(x), not notSC(x)$ $O(Barcelona)$

We have two classical models of \mathcal{O} : \mathcal{I}_1 and \mathcal{I}_2 , where $notSC(Barcelona) \in \mathcal{I}_1$, $notSC(Barcelona) \notin \mathcal{I}_2$. Evaluating with \mathcal{I}_1 , the rule in \mathcal{P} is blocked due to negative dl-atoms, thus for the stable model \mathcal{J}_1 , $seasideCity(Barcelona) \notin \mathcal{J}_1$, while with \mathcal{I}_2 , the rule functions, with $seasideCity(Barcelona) \in \mathcal{J}_2$. As a result, $\mathcal{K} \not\models_{NM} seasideCity(Barcelona)$. However, we hope to obtain $seasideCity(Barcelona)$: since $notSC(Barcelona)$ is unknown to \mathcal{O} , the first rule should function.

The problem with the above example is that in the convention of nonmonotonic reasoning and LP, ground atom "not $p(x)$ " is interpreted as "if $\neg p(x)$ can be *consistently assumed*". Whether an assumption can be *consistently assumed* depends on two facts, which obviously cannot be achieved by NM-Semantics: (1) it is currently underivable and, (2) it is justified in the ultimate extension of the theory[9]. We hope to interpret $notSC(Barcelona)$ in this form. In Example 1, $notSC(Barcelona)$ is underivable from \mathcal{O} . When it comes to justification, we find that $\neg notSC(Barcelona)$ cannot be justified by the stable models of the rules because $notSC$ is not involved in the stable models. Alternatively, it should be justified by the fact that the extension of \mathcal{O} remains consistent after being completed with asserted assumptions, i.e. $\mathcal{I}_2 \cup \{\neg notSC(Barcelona)\}$ is consistent. Therefore, we claim that $\neg notSC(Barcelona)$ is *consistently assumed*. So the task we left with is to specify a proper way to complete the extension of the classical part. Naturally, CWA[15] is a candidate, in which a classical model is completed with the negation of underivable facts, but potential anomalies occur.

Example 2. For a $\mathcal{DL}clog$ KB $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ as follows.

\mathcal{O}	\mathcal{P}
$M \equiv P \sqcup Q$	$r(x) \leftarrow O(x), not P(x)$
$M(a)$	$r(x) \leftarrow O(x), not Q(x)$
	$O(a)$

With the standard CWA, the completed extension is $\mathcal{J} = \{\neg P(a), \neg Q(a), M(a)\}$ which is a contradiction: \mathcal{J} is no longer a model of \mathcal{O} . On the contrary, we would rather prefer to use two classical models \mathcal{J}_1 and \mathcal{J}_2 to evaluate dl-atoms, with $\{\neg P(a), Q(a)\} \subset \mathcal{J}_1$ and $\{P(a), \neg Q(a)\} \subset \mathcal{J}_2$.

A similar proposal is to evaluate negative dl-atoms by whether it is entailed from DL ontology, but integrating "interaction via entailment" complicates the framework of "interaction via single model" of NM-Semantics, and makes the resulted semantics tedious and ill-defined. Finally, our decision is to use Extended CWA[7]. Its semantics is model-theoretically formalized by parallel circumscription[10], and a circumscriptive model is also a classical model. Note that in Example 2, \mathcal{J}_1 and \mathcal{J}_2 are the models of a circumscribed theory $CIRC[\mathcal{O}; P, Q]$.

3 $\mathcal{DL}clog$: Syntax and Semantics

Based on the motivation above, we present the syntax and semantics of $\mathcal{DL}clog$.
Syntax

We partition the alphabet of predicates Σ into three mutually disjoint sets $\Sigma = \Sigma_C \cup \Sigma_R \cup \Sigma_D$, where Σ_C is an alphabet of concepts names, Σ_R is an alphabet of role names and Σ_D is an alphabet of Datalog predicates. The syntax of $\mathcal{DL}clog$ KB is defined as follows.

Definition 1. A hybrid knowledge base \mathcal{K} is a pair $(\mathcal{O}, \mathcal{P})$, where \mathcal{O} is the a description logic ontology $(\mathcal{T}, \mathcal{A})$ as its TBOX and ABOX, and \mathcal{P} is a finite set

of Datalog^{∇,∨} rules of following form:

$$\begin{aligned} \mathcal{R} : & H_1(X_1) \vee \dots \vee H_k(X_n) \leftarrow RB_1(Y_1), \dots, RB_m(Y_m), \\ & \text{not } RB_{m+1}(Y_{m+1}), \dots, \text{not } RB_s(Y_s), \\ & CB_1(Z_1), \dots, CB_n(Z_n), \\ & \text{not } CB_{n+1}(Z_{n+1}), \dots, \text{not } CB_t(Z_t). \end{aligned}$$

where $H_i(X_i)$, $RB_i(Y_i)$, $CB_i(Z_i)$ are atoms, X_i , Y_i , and Z_i are vectors of variables. Let \mathcal{C} denote a set of countably infinite constant names. And

- each H_i is either a DL predicate or a Datalog predicate.
- each RB_i is a Datalog predicate, and $RB_i(Y_i)$ is called a rule-atom.
- each CB_j is a DL predicate, $CB_j(Z_j)$ is called a dl-atom.
- (DL-safeness) every variable of \mathcal{R} must appear in at least one of the $RB_i(Y_i)$ ($1 \leq i \leq m$).

In this version of our work, we only consider $\mathcal{DL}clog$ with DL-safeness. Introducing weak safeness[19] will be included in our future work. Besides, we use $M(\mathcal{P})$ to denote the set of DL predicates occurring under "not" in \mathcal{P} .

Let P and Z be two disjoint sets of predicates and A is the first order theory containing P and Z . Use $CIRC[A; P; Z]$ to denote the parallel circumscription of A , in which the extensions of the predicates in P are minimized and the interpretations of the predicates in Z are fixed[10, 14]. We use $CIRC[A; P]$ to denote the parallel circumscription $CIRC[A; P; Z]$ with $Z = \emptyset$, indicating that all other predicates' interpretations can vary to support the minimization.

Semantics

Like $\mathcal{DL}+log$, in the definition of the semantics of $\mathcal{DL}clog$ we adopt *Standard Names Assumption*[18, 19]: every interpretation is defined over the same fixed, countably infinite domain Δ , and the alphabet of constant \mathcal{C} is such that it is in the same one-to-one correspondence with Δ in each interpretation. Under SNA, with a bit abuse of the notation, we can use the same symbol to denote both a constant and its semantic interpretation, and reformulate the notion of satisfaction in FOL accordingly. Such reformulation doesn't change any standard DLs' consequences. See [18, 12] for the details.

Given a set of constants \mathcal{C} , the *ground instantiation of \mathcal{P} with respect to \mathcal{C}* , denoted by $gr(\mathcal{P}, \mathcal{C})$, is the program obtained from \mathcal{P} by replacing every rule \mathcal{R} in \mathcal{P} with the set of rules obtained by applying all possible substitutions of variables in \mathcal{R} with constants in \mathcal{C} .

Based on the motivation discussed in Section 2, we present the definition of semantics. Following is a generalization of the projection Π in $\mathcal{DL}+log$ to the case of negative dl-atoms.

Definition 2. Given an interpretation \mathcal{I} over an alphabet of predicates $\Sigma' \subset \Sigma$ and a ground program $gr(\mathcal{P}, \mathcal{C})$ over the predicates in Σ , the projection of $gr(\mathcal{P}, \mathcal{C})$ with respect to \mathcal{I} , denoted by $\Pi(gr(\mathcal{P}, \mathcal{C}), \mathcal{I})$ is the program obtained from $gr(\mathcal{P}, \mathcal{C})$ as follows. Let $r(t)$ denote the literal (either unary or binary) in $gr(\mathcal{P}, \mathcal{C})$, for each rule $\mathcal{R} \in gr(\mathcal{P}, \mathcal{C})$,

- delete \mathcal{R} if there exists an atom $r(t)$ in the head of \mathcal{R} such that $r \in \Sigma'$ and $t \in r^{\mathcal{I}}$;
- delete each atom $r(t)$ in the head of \mathcal{R} such that $r \in \Sigma'$ and $t \notin r^{\mathcal{I}}$;
- delete \mathcal{R} if there exists a positive literal $r(t)$ in the body of \mathcal{R} such that $r \in \Sigma'$ and $t \notin r^{\mathcal{I}}$;
- delete each positive literal $r(t)$ in the body of \mathcal{R} such that $r \in \Sigma'$ and $t \in r^{\mathcal{I}}$;
- delete \mathcal{R} if there exists a negative atom $\text{not } r(t)$ in the body of \mathcal{R} such that $r \in \Sigma'$ and $t \in r^{\mathcal{I}}$;
- delete each negative atom $\text{not } r(t)$ in the body of \mathcal{R} such that $r \in \Sigma'$ and $t \notin r^{\mathcal{I}}$;

Based on this definition, dl-atoms can be eliminated by projecting with an interpretation on $\Sigma_R \cup \Sigma_C$. Then one can obtain its stable model via *Gelfond-Lifschitz reduction*[7]. In following we define the Nonmonotonic Circumscriptive Semantics(NMC-Semantics) for $\mathcal{DL}clog$.

Definition 3. (*Nonmonotonic Circumscriptive Semantics, NMC-Semantics*) For a hybrid KB $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ and \mathcal{C} the set of individuals explicitly stated in \mathcal{O} , let $\mathcal{U}, \mathcal{V}, \mathcal{W}$ be sets of interpretations on language $\Sigma_C \cup \Sigma_R, \Sigma_C \cup \Sigma_R$ and Σ_D , respectively. A structure $\mathcal{M} = (\mathcal{U}, \mathcal{V}, \mathcal{W})$ is the Nonmonotonic Circumscriptive Model (NMC-Model) of \mathcal{K} , denoted as $\mathcal{M} \models_{NMC} \mathcal{K}$, if and only if

- for each $\mathcal{I} \in \mathcal{U}, \mathcal{I} \models \mathcal{O}$.
- for each $\mathcal{I} \in \mathcal{V}, \mathcal{I} \models CIRC[\mathcal{O}; M(\mathcal{P})]$.
- for each $\mathcal{J} \in \mathcal{W}, \mathcal{J}$ is a stable model of $\Pi(\text{gr}(\mathcal{P}, \mathcal{C}), \mathcal{I}_c)$ where $\mathcal{I}_c \in \mathcal{V}$.

We call $\mathcal{U}, \mathcal{V}, \mathcal{W}$ the classical part, circumscriptive part, and stable part of the NMC-model. \mathcal{K} is NMC-satisfiable if and only if it has an NMC-model without any part as \emptyset . c denotes a tuple of constants. A ground atom $p(c)$ is NMC-entailed by \mathcal{K} , denoted as $\mathcal{K} \models_{NMC} p(c)$, if and only if

- if p is a DL predicate, for each interpretation $\mathcal{I} \in \mathcal{U}, \mathcal{I} \models p(c)$.
- if p is a rule predicate, for each interpretation $\mathcal{J} \in \mathcal{W}, \mathcal{J} \models p(c)$.

We use an example to illustrate this treatment.

Example 3. We use the hybrid KB of Example 1, and analyze three cases.

1. Querying $\text{seasideCity}(\text{Barcelona})$. With NMC-Semantics, negative dl-atom "not notSC(x)" is satisfied in circumscriptive models containing $\neg \text{notSC}(\text{Barcelona})$, in which notSC is circumscribed. This is the only model used for evaluating dl-atoms. Thus we obtain $\mathcal{K} \models_{NMC} \text{seasideCity}(\text{Barcelona})$.
2. Query $\text{notSC}(\text{Barcelona})$. As in Example 1, \mathcal{O} have two models \mathcal{I}_1 and \mathcal{I}_2 . Thus we have $\mathcal{O} \not\models_{NMC} \text{notSC}(\text{Barcelona})$ rather than $\mathcal{O} \models_{NMC} \neg \text{notSC}(\text{Barcelona})$. We can see that in NMC-Semantics, circumscriptive models don't affect reasoning with \mathcal{O} , which remains to be monotonic and classical.
3. Circumscriptive models can introduce nonmonotonicity to reasoning with rules by negative dl-atoms, Once we add "notSC(Barcelona)" into \mathcal{O} , we obtain $\mathcal{K} \models_{NMC} \neg \text{seasideCity}(\text{Barcelona})$.

NMC-Semantics is the generalization of $\mathcal{DL}+log$. When there are no negative dl-atoms, we have $\mathcal{U} = \mathcal{V}$, and $\mathcal{DL}clog$ is reduced to $\mathcal{DL} + log$ in both syntax and semantics. Obviously, we have the following result.

Proposition 1. *For a $\mathcal{DL}clog$ KB $\mathcal{K} = (\mathcal{O}, \mathcal{P})$, when there are no negative dl-atoms in \mathcal{P} , \mathcal{K} is NMC-satisfiable iff \mathcal{K} is NM-satisfiable in the sense of $\mathcal{DL}+log$.*

NMC-Semantics is also faithful to DL and LP, in that DL and LP are the restricted forms of $\mathcal{DL}clog$.

Proposition 2. *For a $\mathcal{DL}clog$ KB $\mathcal{K} = (\mathcal{O}, \mathcal{P})$, (i) if $\mathcal{P} = \emptyset$, \mathcal{K} is NMC-satisfiable iff \mathcal{O} is classically satisfiable. (ii) if $\mathcal{O} = \emptyset$, \mathcal{K} is NMC-satisfiable iff \mathcal{P} has stable model(s).*

NMC-Semantics is nonmonotonic. Negative dl-atoms evaluated by circumscriptive models add nonmonotonicity features to rules. Consequently, reasoning with DL is monotonic and with OWA, while reasoning with rules is nonmonotonic and with CWA.

Proposition 3. *Given a $\mathcal{DL}clog$ KB $\mathcal{K} = (\mathcal{O}, \mathcal{P})$, \mathcal{O}' , \mathcal{P}' are sets of DL formulae and rules such that $\mathcal{O} \cup \mathcal{O}'$ and $\mathcal{P} \cup \mathcal{P}'$ are consistent. $\mathcal{K}' = (\mathcal{O} \cup \mathcal{O}', \mathcal{P} \cup \mathcal{P}')$, and $p(c)$ is a query such that $\mathcal{K} \models_{NMC} p(c)$. Then (i) $\mathcal{K}' \models_{NMC} p(c)$ holds if $p(c)$ is a DL query. (ii) $\mathcal{K}' \models_{NMC} p(c)$ may not hold if $p(c)$ is a rule query.*

4 Decidability and Complexity

In this section, we restrict $\mathcal{DL}clog$ to the case that the DL ontology is written in $\mathcal{ALC}\mathcal{IO}$ or $\mathcal{ALC}\mathcal{QO}$, and roles are not allowed under "not" in rules. We leave the decidability of general $\mathcal{DL}clog$ relative to NMC-Satisfiability as an open problem.

As the satisfiability of Datalog ^{\neg, \vee} program \mathcal{P} , which is NEXP^{NP} -Complete in [4], can be trivially reduced to the NMC-satisfiability of (\emptyset, \mathcal{P}) , we have following proposition.

Proposition 4. *NMC-satisfiability of $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ is NEXP^{NP} -hard.*

Furthermore, we remind of the exponential hierarchy $\text{NEXP} \subseteq \text{NP}^{\text{NEXP}} \subseteq \text{NEXP}^{\text{NP}} \subseteq 2\text{-EXP}$, and obtain following result.

Proposition 5. *For a restricted $\mathcal{DL}clog$ KB $\mathcal{K} = (\mathcal{O}, \mathcal{P})$, deciding satisfiability of NMC-Semantics of \mathcal{K} is NEXP^{NP} -complete.*

Proof. (Sketch) NMC-Satisfiability can be determined by calling three oracles in a non-deterministic framework (due to the consideration of space, the proof of correctness is omitted here.). For all the ground dl-atoms in $gr(\mathcal{P}, \mathcal{C})$, guess its partition (G_P, G_N) . (1) Check the satisfiability of following $\mathcal{ALC}\mathcal{IO}$ or $\mathcal{ALC}\mathcal{QO}$ KB: $\mathcal{O} \cup \{C(a) | C(a) \in G_P\} \cup \{\neg C(a) | C(a) \in G_N\} \cup \{\exists R.b(a) | R(a, b) \in G_P\} \cup \{\neg \exists R.b(a) | R(a, b) \in G_N\}$ in PSPACE or NEXP^{NP} [20]. (2) The dl-atoms in

$gr(\mathcal{P}, \mathcal{C})$ can be evaluated with (G_P, G_N) in polynomial time, and obtain a program \mathcal{P}_D without dl-atoms. Checking the existence of stable models for \mathcal{P}_D is in NEXP^{NP} [4]. (3) Let $G_{NP} = \{r(t) | r \in M(\mathcal{P}) \wedge r(t) \in G_P\}$, $G_{NN} = \{r(t) | r \in M(\mathcal{P}) \wedge r(t) \in G_N\}$ and $\neg G_{NN} = \{\neg r(t) | r(t) \in G_{NN}\}$, check whether there exists a interpretation \mathcal{I}_c such that $\mathcal{I}_c \models \text{CIRC}[\mathcal{O}; M(\mathcal{P})]$ and $G_{NP} \cup \neg G_{NN} \subseteq \mathcal{I}_c$ is NEXP^{NP} by being reduced to checking the satisfiability of $\text{ALC}\mathcal{I}\mathcal{O}$ or $\text{ALC}\mathcal{Q}\mathcal{O}$ with counting formulae[1]. As it is a non-deterministic process, we obtain the upper bound of NEXP^{NP} . Together with Proposition 4, we finish the proof for completeness.

5 Related Work

DLclog follows from a series of "r-hybrid" work [16–19] which are hybrid approaches defining integrated models on the basis of single models of classical theory. We inherit the framework of these methods. By introducing negative dl-atoms and use parallel circumscription to evaluate them, we obtain the semantics with nonmonotonic features treating negative dl-atoms, while it remains faithful to previous work and to both DL and LP.

In dl-program[3], DL predicates in rules are treated as queries to the ontology, in which the evaluation of DL-atoms in rules are actually by entailment of DL ontology rather than models, as in our method. Thus, the semantics framework is different from ours. Besides, *DLclog* cannot pass facts from rules to DL ontology.

CLP[8] is a method by introducing open domain to Answer Set Programming. It allows DL-predicates occurring under "not" in rules, but the models of rules must be organized in tree-like manner to obtain decidability. Finally, CLP can be used to simulate several expressive DLs. This is a homogenous method.

Recently, there are some "full-integration" methods proposed. [2] proposed to use First Order Autoepistemic Logic[5] as a host language to accommodate DL and rules. [12, 13] proposed to build a hybrid KB in the framework of the logic of MKNF. Both of the methods treat DL and LP in a uniform logic rather than integrating existing formalisms, and by introducing modal operators, dl-atoms are treated in precisely the same way as rule-atoms. Compared with these work, instead of extending language, our formalism is based on a hybrid, modular semantics integrating classical semantics, circumscription and stable model.

6 Conclusion and Future Work

In this paper, we present a hybrid formalism *DLclog* as both semantic and syntactic extension of Rosati's $\mathcal{DL} + \text{log}$ by allowing negative dl-atoms occurring in the body of the rules. To obtain the stable models of the rules, dl-atoms are evaluated by the circumscriptive models of the DL ontology in the sense of parallel circumscription. In this way, the negative dl-atoms are treated nonmonotonically and is closely similar to the treatment of "not" in LP. This formalism strengthens the nonmonotonic expressing and reasoning ability of $\mathcal{DL} + \text{log}$, and remains faithful to the NM-Semantics of $\mathcal{DL} + \text{log}$, DL and LP. Besides, when

roles do not occur under "not" in rules and ontologies are written in *ALC_{IO}* and *ALC_{QO}*, NMC-satisfiability is a NEXP^{NP} -complete problem.

As our future work, we will compare *DL_{log}* with full integration methods.

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