A Possibilistic Extension of Description Logics

Guilin Qi[1], Jeff Z. Pan[2], and Qiu Ji[1]

[1] Institute AIFB, University of Karlsruhe, Germany
{gqi,qiji}@aifb.uni-karlsruhe.de
[2] Department of Computing Science, The University of Aberdeen
Aberdeen AB24 3FX
jpan@csd.abdn.ac.uk

Abstract. Possibilistic logic provides a convenient tool for dealing with inconsistency and handling uncertainty. In this paper, we propose possibilistic description logics (DLs) as an extension of description logics. We give semantics and syntax of possibilistic description logics. Two kinds of inference services are considered in our logics and algorithms are provided for them. These algorithms are implemented using KAON2 reasoner.

1 Introduction

Dealing with uncertainty in the Semantic Web has been recognized as an important problem in the recent decades. Two important classes of languages for representing uncertainty are probabilistic logic and possibilistic logic. Arguably, another important class of language for representing uncertainty is fuzzy set theory or fuzzy logic. Many approaches have been proposed to extend description logics with probabilistic reasoning, such as approaches reported in [14, 12, 10]. These approaches can be classified according to ontology languages, the supported forms of probabilistic knowledge and the underlying probabilistic reasoning formalism. The work on fuzzy extension of ontology languages has also received a lot of attention (e.g., [16]). By contrast, there is relatively few work on combining possibilistic logic and description logic.

Possibilistic logic [5] or possibility theory offers a convenient tool for handling uncertain or prioritized formulas and coping with inconsistency. It is very powerful to represent partial or incomplete knowledge [4]. There are two different kinds of possibility theory: one is qualitative and the other is quantitative. Qualitative possibility theory is closely related to default theories and belief revision [7, 3] while quantitative possibility can be related to probability theory and can be viewed as a special case of belief function [8].

The application of possibilistic logic to deal with uncertainty in the Semantic Web is first studied in [13] and is then discussed in [6]. When we obtain an ontology using ontology learning techniques, the axioms of the ontology are often attached with confidence degrees and the learned ontology may be inconsistent [11]. In this case, possibilistic logic provides a flexible framework to interpret the confidence values and to reason with the inconsistent ontology under uncertainty.

However, there exist problems which need further discussion. First, there is no formal definition of the semantics of possibilistic description logics. The semantic extension of possibilistic description logic is not trivial because we need negation of axioms to define the *necessity measure* from a *possibility distribution*. Second, there is no implementation of possibilistic inference in description logics.

In this paper, we discuss possibilistic extension of description logics. Both syntax and semantics of possibilistic description logics are provided in Section 3. The inference services in possibilistic description logics are also given. After that, we provide algorithms for implementing reasoning problems in Section 4. Finally, we conclude this paper in Section 5.

We assume that the reader is familiar with description logics and refer to the description logic handbook [1] for more details.

2 Possibilistic Logic

Possibilistic logic [5] is a weighted logic where each classical logic formula is associated with a number in $(0,1]$. Semantically, the most basic and important notion is *possibility distribution* π: $\Omega \rightarrow [0,1]$, where Ω is the set of all classical interpretations. $\pi(\omega)$ represents the degree of compatibility of interpretation ω with available beliefs. From *possibility distribution* π, two measures can be determined, one is the possibility degree of formula ϕ, defined as $\Pi(\phi) = max\{\pi(\omega) : \omega \models \phi\}$, the other is the necessity or certainty degree of formula ϕ, defined as $N(\phi) = 1 - \Pi(\neg\phi)$.

At syntactical level, a *possibilistic formula* is a pair (ϕ, α) consisting of a classical logic formula ϕ and a degree α expressing certainty or priority [1]. A possibilistic knowledge base is the set of possibilistic formulas of the form $B = \{(\phi_i, \alpha_i) : i = 1, ..., n\}$. The classical base associated with B is denoted as B^*, namely $B^* = \{\phi_i | (\phi_i, \alpha_i) \in B\}$. A possibilistic knowledge base is consistent iff its classical base is consistent.

Given a possibilistic knowledge base B and $\alpha \in (0,1]$, the α-cut (strict α-cut) of B is $B_{\geq\alpha} = \{\phi \in B^* | (\phi, \beta) \in B \textit{ and } \beta \geq \alpha\}$ ($B_{>\alpha} = \{\phi \in B^* | (\phi, \beta) \in B \textit{ and } \beta > \alpha\}$). The *inconsistency degree* of B, denoted $Inc(B)$, is defined as $Inc(B) = max\{\alpha_i : B_{\geq\alpha_i} \textit{ is inconsistent}\}$.

There are two possible definitions of inference in possibilistic logic.

Definition 1. *Let B be a possibilistic knowledge base.*

– *A formula ϕ is said to be a* plausible consequence *of B, denoted by $B \vdash_P \phi$, iff $B_{>Inc(B)} \vdash \phi$.*
– *A formula ϕ is said to be a* possibilistic consequence *of B to degree α, denoted by $B \vdash_\pi (\phi, \alpha)$, iff the following conditions hold: (1) $B_{\geq\alpha}$ is consistent, (2) $B_{\geq\alpha} \vdash \phi$, (3) $\forall \beta > \alpha$, $B_{\geq\beta} \nvdash \phi$.*

---

[1] In possibilistic logic, the weight of a possibilistic formula (ϕ, a) can be also considered as possibility degree of the formula. However, in most applications of possibilistic logic, we often consider the weight as certainty degree.

3 Possibilistic Description Logics

In this section, we define the semantics and syntax of possibilistic DLs and inference problems of it. We do not specify the underlying DL language, which can be any (decidable) description logic.

3.1 Syntax

The syntax of possibilistic DL is based on the syntax of classical DL. A *possibilistic axiom* is a pair (ϕ, α) consisting of an axiom ϕ and a weight $\alpha \in (0, 1]$. A *possibilistic RBox* (resp., *TBox*, *ABox*) is a finite set of possibilistic axioms (ϕ, α), where ϕ is an RBox (resp., TBox, ABox) axoim. A possibilistic DL knowledge base $\mathcal{B} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$ consists of a possibilistic RBox $\mathcal{R}$, a possibilistic TBox $\mathcal{T}$ and a possibilistic ABox $\mathcal{A}$. We use $\mathcal{R}^*$ to denote the classical DL axioms associated with $\mathcal{R}$, i.e., $\mathcal{R}^* = \{\phi_i : (\phi_i, \alpha_i) \in \mathcal{R}\}$ ($\mathcal{T}^*$ and $\mathcal{A}^*$ can be defined similarly). The classical base $\mathcal{B}^*$ of a possibilistic DL knowledge base is $\mathcal{B}^* = (\mathcal{R}^*, \mathcal{T}^*, \mathcal{A}^*)$. A possibilistic DL knowledge base $\mathcal{B}$ is said to be inconsistent if and only if its classical base $\mathcal{B}^*$ is inconsistent.

Given a possibilistic DL knowledge base $\mathcal{B} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$ and $\alpha \in (0, 1]$, the α-cut of $\mathcal{R}$ is $\mathcal{R}_{\geq\alpha} = \{\phi \in \mathcal{B}^* | (\phi, \beta) \in \mathcal{R} \textit{ and } \beta \geq \alpha\}$ (the α-cut of $\mathcal{T}$ and $\mathcal{A}$, denoted as $\mathcal{T}_{\geq\alpha}$ and $\mathcal{A}_{\geq\alpha}$, can be defined similarly). The strict α-cut of $\mathcal{R}$ (resp., $\mathcal{T}$, $\mathcal{A}$) can be defined similarly as the strict cut in possibilistic logic. The α-cut (resp., strict α-cut) of $\mathcal{B}$ is $\mathcal{B}_{\geq\alpha} = (\mathcal{R}_{\geq\alpha}, \mathcal{T}_{\geq\alpha}, \mathcal{A}_{\geq\alpha})$ (resp., $\mathcal{B}_{>\alpha} = (\mathcal{R}_{>\alpha}, \mathcal{T}_{>\alpha}, \mathcal{A}_{>\alpha})$). The *inconsistency degree* of $\mathcal{B}$, denoted $Inc(\mathcal{B})$, is defined as $Inc(\mathcal{B}) = max\{\alpha_i : \mathcal{B}_{\geq\alpha_i} \textit{ is inconsistent}\}$.

We use the following example as a running example throughout this paper.

Example 1. Suppose we have a possibilistic DL knowledge base $\mathcal{B} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$, where $\mathcal{R} = \emptyset$, $\mathcal{T} = \{(Bird \sqsubseteq Fly, 0.8), (HasWing \sqsubseteq Bird, 0.95)\}$ and $\mathcal{A} = \{(Bird(chirpy), 1), (HasWing(tweety), 1), (\neg Fly(tweety), 1)\}$. The TBox $\mathcal{T}$ states that it is rather certain that birds can fly and it is almost certain that something with wing is a bird. The ABox $\mathcal{A}$ states that it is certain that tweety has wing and it cannot fly, and chirpy is a bird. Let $\alpha = 0.8$. We then have $\mathcal{B}_{\geq 0.8} = (\mathcal{R}_{\geq 0.8}, \mathcal{T}_{\geq 0.8}, \mathcal{A}_{\geq 0.8})$, where $\mathcal{R}_{\geq 0.8} = \emptyset$, $\mathcal{T}_{\geq 0.8} = \{Bird \sqsubseteq Fly, HasWing \sqsubseteq Bird\}$ and $\mathcal{A}_{\geq 0.8} = \{HasWing(tweety), \neg Fly(tweety), Bird(chirpy)\}$. It is clear that $\mathcal{B}_{\geq\alpha}$ is inconsistent. Now let $\alpha = 0.95$. Then $\mathcal{B}_{\geq\alpha} = (\mathcal{R}_{\geq 0.95}, \mathcal{T}_{\geq 0.95}, \mathcal{A}_{\geq 0.95})$, where $\mathcal{R}_{\geq 0.95} = \emptyset$, $\mathcal{T}_{\geq 0.95} = \{HasWing \sqsubseteq Bird\}$ and $\mathcal{A}_{\geq 0.95} = \{HasWing(tweety), \neg Fly(tweety), Bird(chirpy)\}$. So $\mathcal{B}_{\geq\alpha}$ is consistent. Therefore, $Inc(\mathcal{B}) = 0.8$.

3.2 Semantics

The semantics of possibilistic DL is defined by a *possibility distribution* π over the set $\mathbf{I}$ of all classical description logic interpretations, i.e., $\pi : \mathbf{I} \rightarrow [0, 1]$. $\pi(I)$ represents the degree of compatibility of interpretation I with available information. For two interpretations I_1 and I_2, $\pi(I_1) > \pi(I_2)$ means that I_1 is preferred to I_2 according to the available information. Given a possibility distribution π, we can define the possibility measure Π and necessity measure N as follows: $\Pi(\phi) = max\{\pi(I) : I \in \mathbf{I}, I \models \phi\}$ and $N(\phi) = 1 - \Pi(\neg\phi)$, where $\neg\phi$ is the consistency negation defined in [9][2]. Given two possibility distributions π and π', we say that π is more specific (or more informative) than π' iff $\pi(I) \leq \pi'(I)$ for all $I \in \Omega$. A possibility distribution π satisfies a possibilistic axiom (ϕ, α), denoted $\pi \models (\phi, \alpha)$, iff $N(\phi) \geq \alpha$. It satisfies a possibilistic DL knowledge base $\mathcal{B}$, denoted $\pi \models \mathcal{B}$, iff it satisfies all the possibilistic axioms in $\mathcal{B}$.

Given a possibilistic DL knowledge base $\mathcal{B} = \langle \mathcal{R}, \mathcal{T}, \mathcal{A} \rangle$, we can define a possibility distribution from it as follows: for all $I \in \mathbf{I}$,

$$\pi_{\mathcal{B}}(I) = \begin{cases} 1 & \text{if } \forall \phi_i \in \mathcal{R}^* \cup \mathcal{T}^* \cup \mathcal{A}^*, I \models \phi_i, \\ 1 - max\{\alpha_i | I \not\models \phi_i, (\phi_i, \alpha_i) \in \mathcal{R} \cup \mathcal{T} \cup \mathcal{A}\} & \text{otherwise.} \end{cases} \quad (1)$$

As in possibilistic logic, we can also show that the possibility distribution defined by Equation 1 is most specific possibility distribution satisfying $\mathcal{B}$. Let us consider Example 1 again. $I = \langle \Delta^I, \circ^I \rangle$ is an interpretation, where $\Delta^I = \{tweety, chirpy\}$ and $Bird^I = \{tweety, chirpy\}$, $Fly^I = \{chirpy\}$, and $HasWing^I = \{tweety\}$. It is clear that I satisfies all the axioms except $Bird \sqsubseteq Fly$ (whose weight is 0.8), so $\pi_{\mathcal{B}}(I) = 0.2$.

We have the following theorem which says that consistency of a possibilistic DL knowledge bases can be equivalently defined by the possibility distribution associated with it.

Theorem 1. *Let $\mathcal{B}$ be a possibilistic DL knowledge base and $\pi_{\mathcal{B}}$ be the possibility distribution obtained by Equation 1. Then $\mathcal{B}$ is consistent if and only if $\pi_{\mathcal{B}} \models \mathcal{B}$, where $\pi_{\mathcal{B}}$ is the possibility distribution defined by Equation 1.*

The proof of is clear by considering Condition (i) of the consistency negation.

Similar to possibilistic logic, we have the following result.

Proposition 1. *Let $\mathcal{B}$ be a possibilistic DL knowledge base and $\pi_{\mathcal{B}}$ be the possibility distribution obtained by Equation 1. Then $Inc(\mathcal{B}) = 1 - max_{I \in \mathbf{I}} \pi_{\mathcal{B}}(I)$.*

Proposition 1 shows that the inconsistency degree of a possibilistic DL knowledge base can be equivalently defined by the possibility distribution.

3.3 Inference in possibilistic DLs

We consider the following inference services in possibilistic DLs.

- Instance checking: an individual a is a *plausible* instance of a concept C with respect to a possibilistic DL knowledge base $\mathcal{B}$, written $\mathcal{B} \models_P C(a)$, if $\mathcal{B}_{>Inc(\mathcal{B})} \models C(a)$.

[2] There are two kinds of negations defined in [9]: consistency negation and coherence negation. An axioms ψ is said to be a consistency-negation of an axiom ϕ, written $\neg\phi$, iff it satisfies the following two conditions: (i) $\{\phi, \psi\}$ is inconsistent and (ii) there exists no other ψ' such that ψ' satisfies condition (i) and $Cn(\{\psi'\}) \subset Cn(\{\psi\})$.

- Subsumption: a concept C is *plausible* subsumed by a concept D with respect to a possibilistic DL knowledge base $\mathcal{B}$, written $\mathcal{B} \models_P C \sqsubseteq D$, if $\mathcal{B}_{>Inc(\mathcal{B})} \models C \sqsubseteq D$.
- Instance checking with necessity degree: an individual a is an instance of a concept C to degree α with respect to $\mathcal{B}$, written $\mathcal{B} \models_\pi (C(a), \alpha)$, if the following conditions hold: (1) $\mathcal{B}_{\geq\alpha}$ is consistent, (2) $\mathcal{B}_{\geq\alpha} \models C(a)$, (3) for all $\beta > \alpha$, $\mathcal{B}_{\geq\beta} \not\models C(a)$.
- Subsumption with necessity degree: a concept C is subsumed by a concept D to a degree α with respect to a possibilistic DL knowledge base $\mathcal{B}$, written $\mathcal{B} \models_\pi (C \sqsubseteq D, \alpha)$, if the following conditions hold: (1) $\mathcal{B}_{\geq\alpha} \models C \sqsubseteq D$, (2) $\mathcal{B}_{\geq\alpha} \models C \sqsubseteq D$, (3) for all $\beta > \alpha$, $\mathcal{B}_{\geq\beta} \not\models C \sqsubseteq D$.

We illustrate the inference services by reconsidering Example 1.

Example 2. (Example 1 continued) According to Example 1, we have $Inc(\mathcal{B}) = 0.8$ and $\mathcal{B}_{>0.8} = (\mathcal{R}_{>0.8}, \mathcal{T}_{>0.8}, \mathcal{A}_{>0.8})$, where $\mathcal{R}_{>0.8} = \emptyset$, $\mathcal{T}_{>0.8} = \{HasWing \sqsubseteq Bird\}$ and $\mathcal{A}_{>0.8} = \{HasWing(tweety), \neg Fly\ (tweety), Bird(chirpy)\}$. Since $\mathcal{B}_{>0.8} \models Bird\ (tweety)$, we can infer that *tweety* is plausible to be a bird from $\mathcal{B}$. Furthermore, since $\mathcal{B}_{\geq 0.95} \models Bird(tweety)$ and $\mathcal{B}_{\geq 1} \not\models Bird(tweety)$, we have $\mathcal{B} \models_\pi (Bird(tweety), 0.95)$. That is, we are almost certain that tweety is a bird.

4 Algorithms for Inference in Possibilistic DLs

In this section, we give algorithms for implementing possibilistic inference in possibilistic DLs and analyze the computational complexity of the algorithms.

Algorithm 1 computes the inconsistency degree of a possibilistic DL knowledge base using a binary search. The function *Asc* takes a finite set of numbers in $(0, 1]$ as input and returns a vector which contains those distinct numbers in the set in an ascending order. For example, $Asc(0.2, 0.3, 0.3, 0.1) = (0.1, 0.2, 0.3)$. Let $W = (\beta_1, ..., \beta_n)$ is a vector consisting of n distinct numbers, then $W(i)$ denotes β_i. If the returned inconsistency degree is 0, that is $W(-1) = 0$, it shows the ontology to be queried is consistent.

Since Algorithm 1 is based on binary search, to compute the inconsistency degree, it is easy to check that the algorithm requires $\ulcorner log_2 n \urcorner + 1$ satisfiability checks using a DL reasoner in the worst case.

Algorithm 2 returns the necessity degree of an axiom inferred from a possibilistic DL knowledge base *w.r.t* the possibilistic inference. We compute the inconsistency degree of the input ontology. If the axiom is a plausible consequence of a possibilistic DL knowledge base, then we compute its necessity degree using a binary search (see the first "if" condition). Otherwise, its necessity degree is 0, i.e., the default value given to w. Note that our algorithm is different from the algorithm given in [15] for computing the necessity of a formula in possibilistic logic (this algorithm needs to compute the negation of a formula, which is computationally hard in DLs according to [9]). We consider only subsumption checking here. However, the algorithm can be easily extended to reduce instance checking as well.

Algorithm 1: Compute the inconsistency degree

Data: $\mathcal{B} = \langle \mathcal{T}, \mathcal{A} \rangle$, where $\mathcal{T} \cup \mathcal{A} = \{(\phi_i, \alpha_i) : \alpha_i \in (0,1], i = 1, ..., n\}$, where n is the number of axioms in the testing ontology $\mathcal{B}$;
Result: The inconsistency degree d
begin
  $b := 0$ // b is the begin pointer of the binary search
  $m := 0$ // m is the middle pointer of the binary search
  $d := 0.0$ // The initial value of inconsistency degree d is set to be 0.0
  $W = Asc(\alpha_1, ..., \alpha_n)$
  $W(-1) = 0.0$ // The special element -1 of W is set to be 0.0
  $e := |W| - 1$ // e is the end pointer of the binary search
  if $\mathcal{B}_{\geq W(0)}$ *is consistent* **then**
    d:=0.0
  else
    while $b \leq e$ **do**
      if $b = e$ **then**
        return b
      $m := \ulcorner (b+e)/2 \urcorner$
      if $\mathcal{B}_{\geq W(m)}$ *is consistent* **then**
        $e := m - 1$
      else
        $b := m + 1$
    $d := W(b)$
end

Proposition 2. *Let $\mathcal{B}$ be a possibilistic DL knowledge base and ϕ be a DL axiom. Deciding whether $\mathcal{B} \models_P \phi$ requires $\ulcorner log_2 n \urcorner + 1$ satisfiability check using a DL reasoner, where n is the number of distinct certainty degrees in $\mathcal{B}$. Furthermore, deciding whether $\mathcal{B} \models_\pi (\phi, \alpha)$ requires at most $\ulcorner log_2 n \urcorner + \ulcorner log_2 n - l \urcorner + 1$ satisfiability check using a DL reasoner, where where n is the number of distinct certainty degrees in $\mathcal{B}$ and l is the inconsistency degree of $\mathcal{B}$.*

5 Conclusions and Future Work

We gave a possibilistic extension of description logics in this paper. Two kinds of inference services were considered: one is a plausible consequence relation and the other is a possibilistic consequence relation. Algorithms were given to check the inference services and we implemented the algorithms in Java using KAON2 [3] as the basic reference service. The source codes and some ontologies used for testing can be downloaded from

http://radon.ontoware.org/incoquery.zip

To represent the weight for each axiom, we use an annotation property "Rating" to associate one value with the class defined. Thus the axioms starting

[3] http://kaon2.semanticweb.org/

Algorithm 2: Possibilistic inference with certainty degrees

Data: $\mathcal{B} = \langle \mathcal{T}, \mathcal{A} \rangle$, where $\mathcal{T} \cup \mathcal{A} = \{(\phi_i, \alpha_i) : \alpha_i \in (0, 1], i = 1, ..., n\}$; a DL axiom ϕ.
Result: The certainty degree w associated with a query ϕ

```
begin
    m := 0
    w := 0.0 // The initial certainty degree of φ is set to be 0.0
    W = Asc(α_1, ..., α_n)
    W(−1) = 0.0
    e := |W| − 1
    compute l such that W(l) = Inc(B) //Inc(B) is computed by Algorithm 1
    b := l + 1
    if B_{≥W(b)} ⊨ φ then
        while b ≤ e do
            if b = e then
                return b
            m := ⌜(b + e)/2⌝
            if B_{≥W(m)} ⊭ φ then
                e := m − 1
            else
                b := m + 1
        w := W(b)
end
```

with this class also have the same value as their weights. Take the following code as an example. The axioms $Messaging \sqsubseteq \neg Kerberos$ and $Messaging \sqsubseteq \neg GeneralReliabilityUsernamePolicy$ will have the same weight 0.345.

```
<owl:Class rdf:about="#Messaging">
  <Rating rdf:datatype="http://www.w3.org/2001/XMLSchema#double">0.345
  </Rating>
  <owl:disjointWith rdf:resource="#Kerberos"/>
  <owl:disjointWith rdf:resource="#GeneralReliabilityUsernamePolicy"/>
</owl:Class>
```

The advantage of this way to represent confidence values is that confidence values and the ontology can be kept in the same owl file. So far, we only support some simple queries like instance checking $A(a)$ and subsumption $A \sqsubseteq B$, where a is an instance and A, B are concepts.

Possibilistic inference has been criticized for the "drowning problem", i.e., all the axioms whose necessity degrees which are less than or equal to the inconsistency degree of the possibilistic DL knowledge base do not contribute to the inference. Several variants of possibilistic inference have been proposed in classical logic to solve the drowning problem [2]. We plan to implement these approaches in our future work.

Acknowledgements Research presented in this paper was partially supported by the European Commission NeOn (IST-2006-027595, http://www.neon-project.org/), the Knowledge Web projects (IST-2004-507842, http://knowledgeweb.semanticweb.org/), and the X-Media project (www.x-media-project.org) sponsored by the European Commission as part of the Information Society Technologies (IST) programme under EC grant number IST-FP6-026978.

References

1. Franz Baader, Diego Calvanese, Deborah McGuinness, Daniele Nardi, and Peter Patel-Schneider. *The Description Logic Handbook: Theory, implementation and application*. Cambridge University Press, 2003.
2. Salem Benferhat, Claudette Cayrol, Didier Dubois, Jérôme Lang, and Henri Prade. Inconsistency management and prioritized syntax-based entailment. In *Proc. of IJCAI'93)*, pages 640–647. Morgan Kaufmann, 1993.
3. Salem Benferhat, Didier Dubois, and Henri Prade. Representing default rules in possibilistic logic. In *Proc. of KR'92*, pages 673–684, 1992.
4. Salem Benferhat, Sylvain Lagrue, and Odile Papini. Reasoning with partially ordered information in a possibilistic logic framework. *Fuzzy Sets and Systems*, 144(1):25–41, 2004.
5. Didier Dubois, Jérôme Lang, and Henri Prade. Possibilistic logic. In *Handbook of Logic in Artificial Intelligence and Logic Programming*, pages 439–513. Oxford University Press, 1994.
6. Didier Dubois, Jérôme Mengin, and Henri Prade. Possibilistic uncertainty and fuzzy features in description logic: A preliminary discussion. In *Capturing Intelligence: Fuzzy Logic and the Semantic WEb*, pages 101–113. Elsevier, 2006.
7. Didier Dubois and Henri Prade. Epistemic entrenchment and possibilistic logic. *Artif. Intell.*, 50(2):223–239, 1991.
8. Didier Dubois and Henri Prade. Possibility theory: qualitative and quantitative aspects. In *Handbook of Defeasible Reasoning and Uncertainty Management Systems*, pages 169–226, 1998.
9. Giorgos Flouris, Zhisheng Huang, Jeff Z. Pan, Dimitris Plexousakis, and Holger Wache. Inconsistencies, negations and changes in ontologies. In *Proc. of AAAI'06*, 2006.
10. Rosalba Giugno and Thomas Lukasiewicz. P-shoq(d): A probabilistic extension of shoq(d) for probabilistic ontologies in the semantic web. In *Proc. of JELIA'02*, pages 86–97, 2002.
11. Peter Haase and Johanna Völker. Ontology learning and reasoning - dealing with uncertainty and inconsistency. In *Proc. of URSW'05*, pages 45–55, 2005.
12. Jochen Heinsohn. Probabilistic description logics. In *Proc. of UAI'94*, pages 311–318, 1994.
13. Bernhard Hollunder. An alternative proof method for possibilistic logic and its application to terminological logics. *Int. J. Approx. Reasoning*, 12(2):85–109, 1995.
14. Manfred Jaeger. Probabilistic reasoning in terminological logics. In *Proc. of KR'94*, pages 305–316, 1994.
15. Jérôme Lang. Possibilistic logic: complexity and algorithms. In *Handbook of Defeasible Reasoning and Uncertainty Management Systems*, pages 179–220. 2000.
16. Umberto Straccia. Reasoning within fuzzy description logics. *J. Artif. Intell. Res.*, 14:137–166, 2001.