Lab 6

Hyperbolic Functions.

Some common transcendental functions which your book does not discuss, but which are in common use especially by engineers are the hyperbolic functions. The two basic ones are:

$$\cosh(x) = \frac{e^x + e^{-x}}{2}, \quad \sinh(x) = \frac{e^x - e^{-x}}{2}.$$

In a manner similar to the usual trig functions we can define several other functions in terms of these basic two, namely:

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}, \coth(x) = \frac{1}{\tanh(x)}, \sec h(x) = \frac{1}{\cosh(x)}, \csc h(x) = \frac{1}{\sinh(x)}.$$

When you first studied trig. you will recall that there is a seemingly endless list of identities involving the trig functions. The same is true for the hyperbolic functions. For example, let's compute $\cosh^2(x) - \sinh^2(x)$;

(a)

$$\cosh^2(x) - \sinh^2(x) = \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 = \left(\frac{e^{2x} + 2 + e^{-2x}}{4}\right) - \left(\frac{e^{2x} - 2 + e^{-2x}}{4}\right) = 1.$$

Some other common ones which you can derive are:

(b) $\sinh(u+v) = \sinh(u)\cosh(v) + \cosh(u)\sinh(v)$,

(c) $\cosh(u+v) = \cosh(u)\cosh(v) + \sinh(u)\sinh(v)$,

(d) $\sinh(-u) = -\sinh(u)$, hence sinh is an odd function.

(e) $\cosh(-u) = \cosh(u)$, hence cosh is an even function.

Now do the following, without the help of Maple V using your knowledge of the exponential function, and then check your answers with Maple V:

1) $\frac{d}{dx}\cosh(x) = ?$, 2) $\frac{d}{dx}\sinh(x) = ?$, 3) $\frac{d}{dx}\cosh(ax) = ?$,

4) $\frac{d}{dx}\sinh(ax) = ?$ 5) $\int \cosh(x)dx = ?$, 6) $\int \sinh(x)dx = ?$,

7) $\int \cosh(ax)dx = ?$, 8) $\int \sinh(ax)dx = ?$.

Now use Maple V to simultaneously plot the following on the interval (-1,1):

9) $\{\cosh(x/2), \cosh(x), \cosh(2x)\}$, 10) $\{\sinh(x/2), \sinh(x), \sinh(2x)\}$.

What point do the graphs of the functions in (9) all pass through?
What point do the graphs of the functions in (10) all pass through?

11) What is $\frac{d^2}{dx^2}\cosh(x)$?

12) What is $\frac{d^2}{dx^2}\sinh(x)$?

13) Can you now write down a general solution to the differential equation $y'' - y = 0$, using hyperbolic functions?