# Lab 5

## Euler's Method.

Refer to section 9.3 in your book.

Euler's method is a numerical scheme or algorithm which approximates values of the solution to differential equation. To be definite suppose we wish to know the solution, that is to say the value of $y$ at $t = b$ to the initial value problem:

$$\frac{dy}{dt} = f(t, y),$$

$$y(a) = y_0.$$

We divide the interval [*a*,*b*] into *N* equal subintervals $a = t_0, t_1 \ldots t_N = b$, then Euler's method says that an approximation to $y(t_{k+1})$ is $y_{k+1}$ where

$$y_{k+1} = y_k + hf(t_k, y_k), \quad k = 0..N - 1,$$

and $h$ is the length of each subinterval, which in this case is *(b-a)/N*. Thus we use $y_0 = y(a)$ to compute $y_1$, $y_1$ to compute $y_2$, $y_2$ to compute $y_3$, and so on until we have $y_N \approx y(b)$.

A Maple V program to do this could be as follows:

First we input the function *f* as a function of *t* and *y*.

```
>f:=(t,y)-> whatever your function is.
>endpoint:=proc(a,b,N,y0)
local k,u,h;
h:=(b-a)/N;
u:=y0;
for k to N do u:=evalf(u+h*f(a+(k-1)*h,u));od;
end;
```

For example if we had the system:

$$\frac{dy}{dt} = \sqrt{t^2 + y^2}, y(0) = 0.5,$$

and we wanted to know an approximation to $y(1)$ we would write:

```
>f:=(t,y)->sqrt(t^2+y^2);
endpoint(0,1,100,0.5);
```

You should get the answer 1.538871403. If you change N to 500 you should get 1.546627413.

This program can be easily modified to plot the points Euler's method generates:

```
>f:=(t,y)-> whatever your function is.
```

```
>endpoint:=proc(a,b,N,y0)
local k,u,h,values;
h:=(b-a)/N;
u:=y0;
values:=a,y0;
for k to N do u:=evalf(u+h*f(a+(k-1)*h,u));
values:=values,k*h,u;od;
plot([values],t=a..b,style=point);
end;
```

Now work the following problems using the appropriate algorithm.

(1) For the logistic equation $\frac{dp}{dt} = 3p - p^2$, find Euler's estimate of $p(2)$ if $p(0) = 0.125$, using 100 intervals. This differential equation can be solved analytically, see page 535 of your text. Solve this equation and compute $p(2)$ to see how close Euler's estimate is. Now use the second algorithm to plot the Euler points and the analytic solution simultaneously.