Project 4.

Fourier Series.

This project is concerned with problem 14 on page 178. Thus you should first work this problem by the methods intended by the textbook. You probably worked it in MA141, but do it again, it will be a good review for you. Now in this problem we do not have a formula for the velocity function, so we will try to approximate it by finding a truncated Fourier series. Refer to page 643 and satisfy yourself that the appropriate formulas are as follows:

$$f(x) = a_0 + a_1\cos(2\pi x) + a_2\cos(4\pi x) + a_3\cos(6\pi x) + \ldots$$

$$+ b_1\sin(2\pi x) + b_2\sin(4\pi x) + b_3\sin(6\pi x) + \ldots$$

where the coefficients are given by the formulas,

$$a_0 = \int_0^1 f(x)dx, \qquad (1)$$

$$a_n = 2\int_0^1 f(x)\cos(2n\pi x)dx, n = 1,2,3\ldots \qquad (2)$$

$$b_n = 2\int_0^1 f(x)\sin(2n\pi x)dx, n = 1,2,3\ldots \qquad (3)$$

What we will be doing here may be described as approximating an approximation. This is because we do not know a formula for $f(x)$ and so we will have to evaluate the integrals in equations (1), (2) and (3) numerically. So complete the following steps:

(a) By using the trapezoid rule find, with 12 subintervals, approximations for $a_0, a_1, a_2, a_3, b_1, b_2, b_3$.

(b) With your values for these, write down the truncated Fourier series

$$f(x) \approx a_0 + a_1\cos(2\pi x) + a_2\cos(4\pi x) + a_3\cos(6\pi x) +$$
$$+ b_1\sin(2\pi x) + b_2\sin(4\pi x) + b_3\sin(6\pi x)$$

(c) Graph this Fourier series. Is it like the graph in the textbook?

(d) Rework the problem using your Fourier series. What are the differences in your answers.