A New Class of Elastic Body Splines for Nonrigid Registration of Medical Images

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Abstract. We introduce an elastic registration approach based on point landmarks and elastic body splines (*EBS*). Since EBS are derived from the physics of elastic objects this approach allows to approximate the deformations of human tissue. In this contribution we introduce a new family of EBS using Gaussian forces. In comparison to earlier work these forces are physically more realistic and they allow us to to derive a transformation that is well–suited to cope with local as well as global differences in medical images. We demonstrate the advantages of the new family of EBS using synthetic as well as real tomographic images.

1 Introduction

Nonrigid registration of medical images is a key method for improving clinical diagnosis and planning of neurosurgical interventions. In contrast to affine and rigid registration techniques, elastic registration approaches allow to cope with local differences between corresponding images (e.g. see [1]). These differences may be caused, for example, by physical deformations of human tissue due to surgical interventions or pathological processes (e.g., growth of tumors). In these cases a registration approach based on a physical model is well-suited to cope with these deformations.

In this contribution we introduce a point–based elastic registration approach for medical imaging that is derived from the Navier equation which serves as our physical model. Since the Navier equation describes the deformations of elastic objects the resulting approach allows to approximate physical deformations of human tissue. Here, we consider elastic registration of medical images based on a physical model and point landmarks. Point landmarks are corresponding anatomical points in the source and target images and as a main advantage, they allow to develop computationally efficient registration schemes.

In previous work, Davis et al. [2] introduced a point–based elastic registration scheme using elastic body splines (EBS), which is based on the theory of linear elasticity. By assuming polynomial forces $\mathbf{f}(\mathbf{x}) = \mathbf{c}r$, $r = ||x||$ and $\mathbf{f}(\mathbf{x}) = \mathbf{c}/r$, Davis et al. derived analytical solutions of the Navier equation using the Galerkin vector method. These functions are then used as basis functions for their interpolation-based registration scheme. Unfortunately, the application of the forces $\mathbf{f}(\mathbf{x}) = \mathbf{c}r$ and $\mathbf{f}(\mathbf{x}) = \mathbf{c}/r$ which are used in [2] have several drawbacks. Either they increase with increasing distance to the origin, which is physically not plausible, or they have a singularity at the origin.

2 A new class of Elastic Body Splines

In this contribution, we introduce an extension of the EBS approach of Davis et al. [2] and we thereby define a new class of elastic body splines. Analogously to [2] we use the Navier equation for the derivation of our registration approach. However, in contrast to [2], we here use a Gaussian force

$$\mathbf{f}(\mathbf{x}) = \mathbf{c}_i \frac{1}{(\sqrt{2\pi}\sigma)^3} e^{-\frac{r^2}{2\sigma^2}} \ , \tag{1}$$

which, on the one hand, considerably complicates the derivation of the basis function but, on the other hand, have several advantages over polynomial forces as used in [2]. Since Gaussian functions have a definite value at the origin and show a fast decay to zero with increasing distance to the origin they do not suffer from a singularity at the origin as the polynomial force $\mathbf{f}(\mathbf{x}) = \mathbf{c}/r$ do and they are physically more plausible. By using Gaussian forces, we have a parameter (the standard deviation of the Gaussian) which can be used to control the local influence of the EBS transformation. Thus, by varying this parameter we can cope with local as well as global differences in the corresponding source and target images.

To match two images we first apply an initial affine transformation that is followed by an independent pure elastic transformation

$$\mathbf{u}(\mathbf{x}) = \mathbf{x} + \sum_{i=1}^{N} \mathbf{G}(\mathbf{x} - \mathbf{p}_i)\mathbf{c}_i \ , \tag{2}$$

which is in contrast to [2] where the affine and elastic transformation is performed in one single step. In brain image registration such a two–step approach has the advantage that the affine transformation can be used to initially align the contours of the rigid skull. The pure elastic transformation is then used to cope with the elastic deformations of the brain tissue. In (2) $\mathbf{p}_i$ are the positions of the i–th landmarks in the source image. $\mathbf{G}(\mathbf{x}-\mathbf{p}_i)$ denotes the basis function and N is the overall number of landmarks. Note, that the coefficients $\mathbf{c}_i$ in (2) correspond to the strength of the Gaussian forces. The coefficients are computed by solving a system of linear equations that results from the interpolation constraints $\mathbf{q}_i = \mathbf{u}(\mathbf{p}_i)$ and the displacements of corresponding landmarks.

Since the transformation (2) of our new EBS approach is an analytical solution of the Navier equation, our transformation models real physical deformations. The Gaussian forces are centred at the positions of the landmarks and elastically deform the image in a way that prescribed landmark correspondences (displacements) are preserved. In the case of small values for the standard deviation the Gaussian forces well approximate point forces. Actually, we have shown that the transformation function assuming Gaussian forces converges to a transformation function assuming ideal point forces if in the limit the standard deviation of the Gaussian force approaches zero. Note, that the transformation function derived on the basis of ideal point forces diverges at the positions of the landmarks and is therefore not suited for use in medical image registration. By using Gaussian forces instead, the transformation function is bounded and therefore does not suffer from this drawback.

3 Experimental results

Experimentally we have compared the performance of the EBS approaches based on both polynomial and Gaussian forces using synthetic as well as real tomographic images. In particular, we have compared the approaches considering physical deformations in the case of a resection of a brain tumor.

In the first experiment we use a simple model for a tumor resection for which there exists an analytical solution of the Navier equation. In our model (Fig. 1 (a)) the outer circle corresponds to the skull bone, which is assumed to be rigid. The inner circle represents the boundary of the resection area. The space between the inner and the outer circle is assumed to be filled with an elastic material, in our case we assume brain tissue. In order to apply the EBS approaches we placed equidistant landmarks at the inner and outer circle. We have chosen the smallest value for the standard deviation of the Gaussian forces for which we observed nearly radial symmetric deformations. Fig. 1 (b) shows the analytically obtained result. The results using the EBS approaches are shown in Fig. 1 (c)-(e). Applying the EBS approaches using the forces $\mathbf{f}(\mathbf{x}) = \mathbf{c}r$ (Fig. 1 (c)) and $\mathbf{f}(\mathbf{x}) = \mathbf{c}/r$ (Fig. 1 (d)) turns out that these approaches do not perform well in comparison to the analytically obtained result (Fig. 1 (b)). In contrast, our EBS approach using Gaussian forces (Fig. 1 (e)) very well approximates the analytically obtained solution.

In the second experiment we compare the results of the EBS approaches applied to pre– and postsurgical MR images of the human brain. These images are slices of 3D MR images of a patient before (Fig. 2 (a)) as well as after the resection of a brain tumor (Fig. 2 (b)). The 3D images have initially been registered by an affine transformation. For the application of the EBS approach we used 10 landmarks at the contour of the tumor as well as at the contour of the resection area and 12 landmarks at contours in the vicinity of the tumor. We used the smallest value for the standard deviation of the Gaussian forces for which we observed a smooth transformation. The transformed presurgical images using the EBS approaches are presented in Fig. 2 (c)-(e). In order to compare the quality of the registration results we have computed the absolute difference of the gray values of these images. These difference images allow us to analyse the registration accuracy and are presented in Fig. 2 (f)-(h). The application of the EBS approach based on the force $\mathbf{f}(\mathbf{x}) = \mathbf{c}r$ leads for this landmark distribution to an unusable result (Fig. 2 (c)). The result using the EBS approach based on the force $\mathbf{f}(\mathbf{x}) = \mathbf{c}/r$ is better (Fig. 2 (d)), but as demonstated by the difference

Fig. 1. Model for tumor resection (a), Resulting deformations according to an analytical solution of the Navier equation (b). Results obtained by applying EBS using $\mathbf{f}(r) = \mathbf{c}r$ (c), $\mathbf{f}(r) = \mathbf{c}/r$ (d), and Gaussian forces (e).

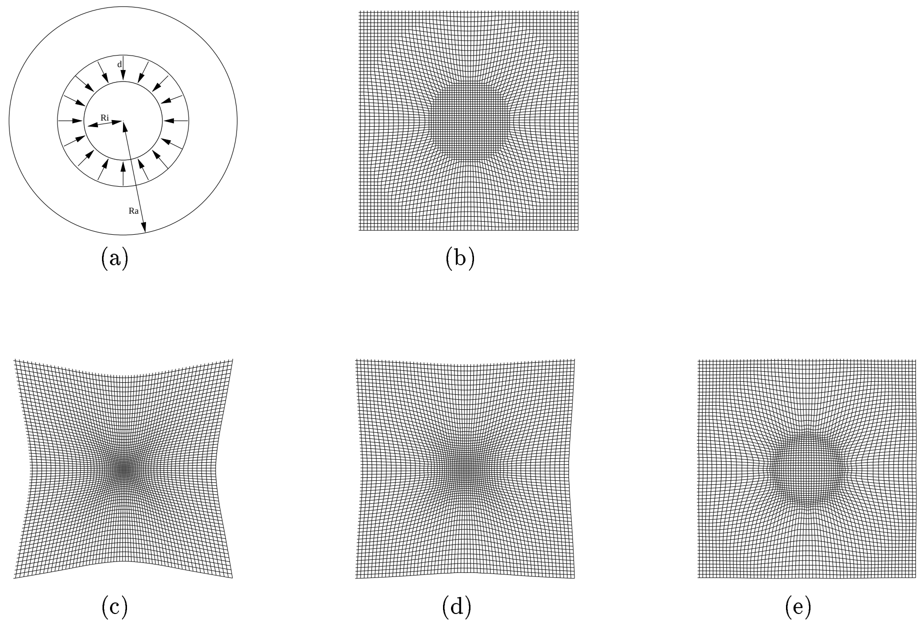
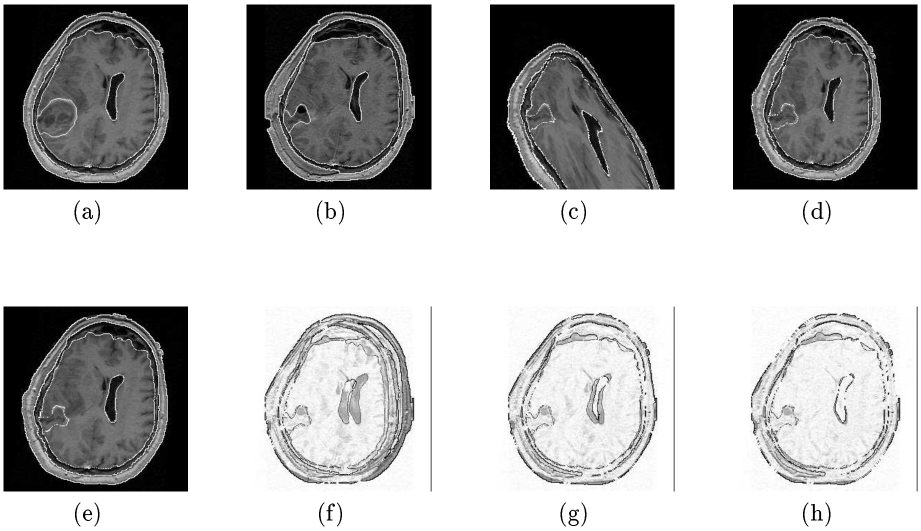


image in Fig. 2 (f) the contours of the ventricular system and the skull bones differ significantly. Placing additional landmarks at the contours of the skull leads to well–aligned skull contours, but however, the contours of the ventricular system still differ significantly (Fig. 2 (g)). Fig. 2 (h) shows the result using our EBS approach. It can be seen that the contours of the skull bones and the ventricular system are well–aligned. Additional landmarks at the contours of the skull have no influence on the quality of the registration result. Thus the EBS using Gaussian forces are better suited to approximate the tissue deformations caused by the considered tumor resection than the EBS using polynomial forces.

4 Summary and conclusion

In this contribution we have introduced a new class of elastic body splines for elastic registration of medical images that is based on a physical model and can cope with global as well local differences between images. We have compared the performance of the EBS approaches using synthetic as well as real tomographic images and focussed on deformations caused by the resection of a brain tumor.

Fig. 2. Registration of 2D MR brain images: presurgical image (a), postsurgical image (b). Transformed presurgical images obtained by applying EBS using $\mathbf{f}(\mathbf{x}) = \mathbf{c}r$ (c), $\mathbf{f}(\mathbf{x}) = \mathbf{c}/r$ (d), and Gaussian forces (e). Differences between postsurgical image and transformed presurgical image obtaind by applying EBS using $\mathbf{f}(\mathbf{x}) = \mathbf{c}/r$ (f), $\mathbf{f}(\mathbf{x}) = \mathbf{c}/r$ using additional landmarks (g), and Gaussian forces (h).



It turns out that our EBS approach using Gaussian forces is well suited to approximate these deformations and is superior to the previous EBS.

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